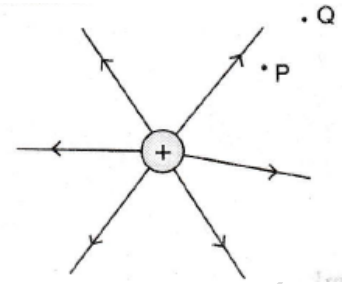
CLASS – 12

WORKSHEET- ELCTROSTAT POTENTIAL AND CAPACITANCE

A. ELECTROSTATIC POTENTIAL ENERGY

(1 Mark Questions)

1. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason.



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2. What is the amount of work done in moving a point charge around a circular arc of radius r at the centre of which another point charge is located?

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(2 Marks Questions)

3. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 A, and the electron is roughly 1 A from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy. [Ans. 19.2 eV]

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(3 Marks Questions)

4. An electric dipole consists of two opposite charges each of magnitude 1μc separated by 2cm. The dipole is placed in an external electric field of 10^5 NC^{-1}. Find (i) the maximum

torque exerted by the field on the dipole (ii) the work which the external agent will have to do in turning the dipole through 180° starting from the position $\theta = 0°$.

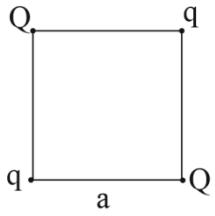
[Ans. 4×10^{-3} J]

5. Two point charges A and B of values + 15μC and +9μC are kept 18cm apart in air. Calculate the work done when charge B is moved by 3cm towards A. [Ans. 1.35J]

6. Derive an expression for the potential energy of a system of two point charges and write its relation with electric potential; of a charge.

7. In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 A: **(a)** Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton. **(b)** What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)? **(c)** What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 A separation? [Ans. 27.2 eV, 13.6 eV, 13.6 eV]

8. Four point charge Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.



Find the
(a) resultant electric force on a charge Q, and
(b) potential energy of this system

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(5 marks Questions)

9. Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

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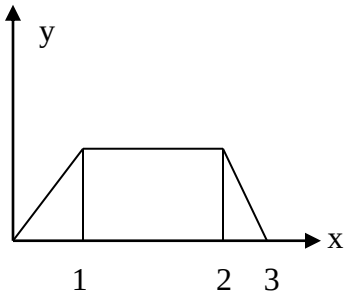
10. The electric potential as a function of distance x is shown in the figure. Draw a graph of the electric field E as a function of x



B. POTENTIAL

(1 Mark Questions)

1. The physical quantity having SI unit NC^{-1} m is _____.

2. What is the geometrical shape of equipotential surface due to a single isolated point charge?

3. Two charges 2 μC and –2μC are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system.

4. Can there be a potential difference between two adjacent conductors carrying the same charge?

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5. Can the potential function have a maximum or minimum in free space?

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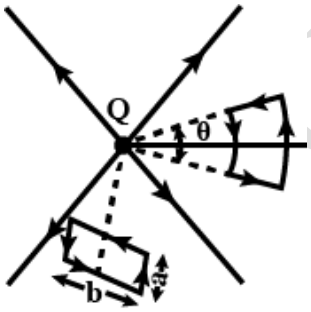
6. A test charge q is made to move in the electric field of a point charge Q along two different closed paths (Fig.). First path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?



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(2 Marks Questions)

7. Draw a plot showing variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q.

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8. Define electric potential. Derive an expression for the electric potential at a distance r from a charge q.

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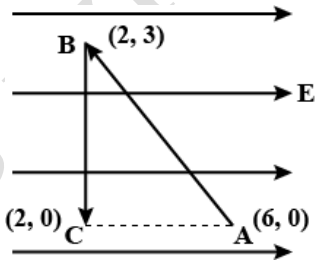
9. Show that the electric field at any point is equal to the negative of the potential gradient at that point.

10. The point charge +Q is place at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero?

11. A test charge 'q' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure . (i) Calculate the potential difference between A and C. (ii) At which point (of the two) is the electric potential more and why?



12. Two uniformly large parallel thin plates having densities $+\sigma$ and $-\sigma$ are kept in the X-Z plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge –q remains stationary between the plates, what is the magnitude and direction of this field?

13. (a) Draw equipotential surfaces due to point Q > 0.
(b) Are these surfaces equidistant from each other ? if no, explain why ?

14. Two charges –q and +q are located at point A(0, 0, –a) and B(0, 0, +a) respectively. How much work is done in moving a test charge from point P(7,0,0) to Q (–3,0, 0)?

15. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the center of the hexagon. [Ans. 2.7×10^{6}V]

16. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P(0,0,3 cm) to a point Q (0,4 cm, 0), via a point R (0,6 cm, 9 cm). [Ans. 1.2 J]

17. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube. [Ans. $\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$]

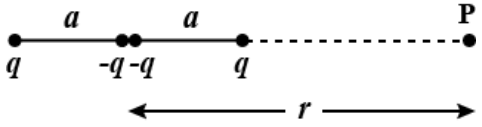
(3 Marks Questions)

18. Write a relation between potential energy of a charge and electric potential. Hence define electric potential.

19. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:
(a) at the mid-point of the line joining the two charges, and
(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point. [Ans. 2.4×10^5V, 2×10^5V]

20. Two charges $5 \times 10^{-8}8$ C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. [Ans. 40cm]

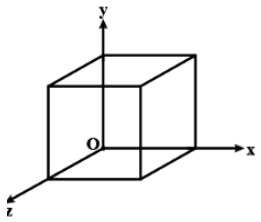
21. Figure shows a charge array known as an electric quadrupole. For a point on the axis of quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



(5 marks Questions)

22. Derive the potential due to a dipole at angular position.

23. A cube to side 20cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by V = 10x + 5, where V is in volt and x is in m.



(i) electric field $\vec{E}$, and
(ii) total electric flux through the cube.

24. The magnitude of electric field (in NC^{-1}) in a region varies with the distance r (in m) as

$$E = 10r + 5$$

By how much does the electric potential increase in moving from point at r = 1 m to a point at r = 10 m.

25. Two charges -q and +q are located at points (0,0, -a) and (0,0, a), respectively.
(a) What is the electrostatic potential at the points (0,0,z) and (x,y,0)?
(b) Obtain the dependence of potential on the distance r of a point from the origin when r/a»1.
(c) How much work is done in moving a small test charge from the point (5, 0, 0) to (-7, 0, 0) along the x-axis?
Does the answer change if the path of the test charge between the some points is not along the x-axis? [Ans. 0, $1/r^2$. 0]

26. Describe schematically the equipotential surfaces corresponding to

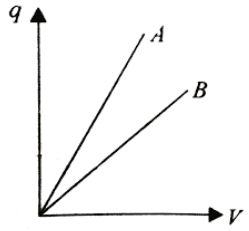
- A constant electric field in the z-direction.
- a field that uniformly increases in magnitude but remains in a constant (say, z) direction.
- a single positive charge at the origin, and
- a uniform grid consisting of long equally spaced parallel charged wires in a plane.

27. Draw an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

C. CAPACITOR

(1 Mark Questions)

1. The given graph shows variation of charge 'q' versus potential difference 'V for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than the of C_1. Which line (A or B) corresponds of C_1 and why?



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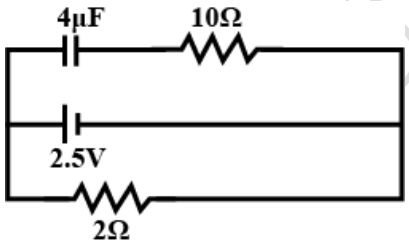
2. A capacitor of 4 μ F is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω . The amount of charge on the capacitor plates will be



(a) 0 (b) 4 μ C (c) 16 μ C (d) 8 μ C

(2 Marks Questions)

3. What is law of capacitance?

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4. What is the area of the plats of 2F parallel plate capacitor having separation between the plates is 0.5 cm?

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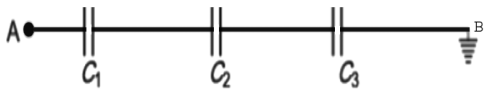
5. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure . Given potential at A is 90V. C_1 = 20 μF, C_2 = 30 μf and C_3 = 15 μf.

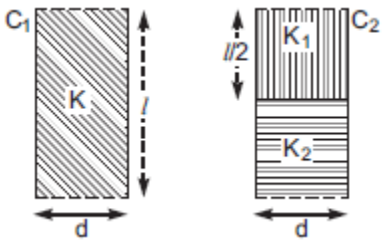


6. A 12 μF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? [Ans. 1.5×10^{-8} J]

7. A 600 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 μF capacitor. How much electrostatic energy is lost in the process? [Ans. 6×10^{-6}J]

8. What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [Ans. 1130 km^2]

9. Two identical parallel plate (air) capacitor C_1 and C_2 have capacitances C each. The area between their plates is now filled with dielectrics as shown.



If the two capacitors still have equal capacitance, obtained the relation between dielectric constants K, K_1 and K_2.

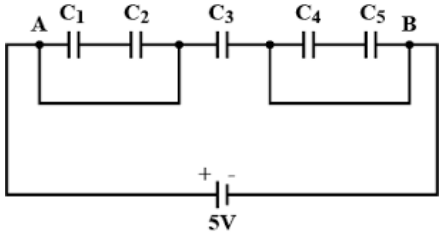
(3 Marks Questions)

10. Derive the capacitance of a parallel plate capacitor.

11. Three capacitors of capacitances C_1, C_2 and C_3 are connected in series. Find their equivalent capacitance.

12. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this, capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor? [Ans. 15pF, 1.8×10^{-9}C]

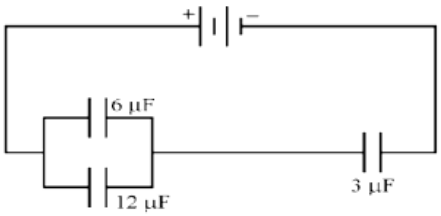
13. In the figure given below, find the



(a) equivalent capacitance of the network between points A and B.
Given : $C_1 = C_5 = 4\ \mu F$, $C_2 = C_3 = C_4 = 2\ \mu F$
(b) Maximum charge supplied by the battery and
(c) total energy stored in the network

14. A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor ? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor.

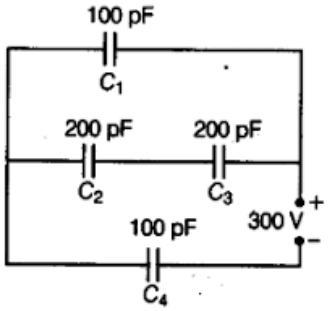
15. In the following arrangement of capacitors, the energy stored in the 6 µF capacitor is E. Find the value of the following.
(i) Energy stored in 12µF capacitor
(ii) Energy stored in 3 µF capacitor
(iii) Total energy drawn from the battery



16. Three capacitors each of capacitance 9 µF are connected in series.
(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitors if the combination is connected to a 120 V supply? [3pF, 40 V]

17. Obtain the equivalent capacitance of the network in figure. For a 300 V supply, determine the charge and voltage across each capacitor. [Ans. 10^{-8}C]



18. Show that the force on each plate of a parallel plate capacitor has magnitude equal to ½ QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor 1/2.

19. In a Van-de-Graff type generator, a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 V m^1. What is the minimum radius of the spherical shell required? [Ans. 30 cm]

(5 marks Questions)

20. Three capacitors of capacitances 2 μF, 3 μF and 4 μF are connected in parallel.
(a) What is the total capacitance of the combination.
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply. [Ans. (a) 9pF, (b) 2×10^{-10}C, 3×10^{-10}C, 4×10^{-10}C]

21. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

D. EFFECT OF DIELECTRIC ON CAPACITOR

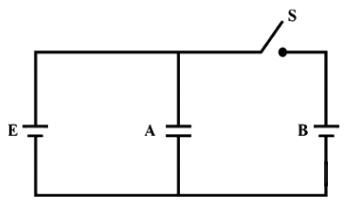
(3 Marks Questions)

1. Explain why the polarization of a dielectric reduces the electric field inside the dielectric. Hence define dielectric constant.

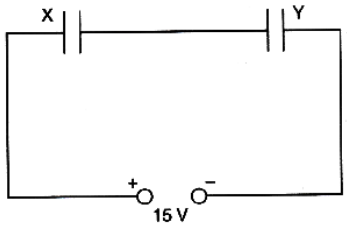
2. Define 'dielectric constant' of a medium. Briefly why the capacitance of a parallel plate capacitor increases, on introducing a dielectric medium between the plates?

3. Explain what would happen if in the capacitor given in above question, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates.
(a) While the voltage supply remained connected.
(b) After the supply was disconnected. [Ans. 108pF, 16.6V]

4. Two identical parallel plate capacitors A and B are connected to a battery of V volt with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



5. Two parallel plate capacitors X and Y have the same area of separation between them. X has air between the plates while Y contains a dielectric of $\varepsilon_r = 4$.



(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4μF.
(ii) Calculate the potential difference between the plates of X and Y.
(iii) Estimate the ratio of electrostatic energy stored in X and Y.

6. A parallel plate capacitor of capacitance 'C' is charged to 'V' volt by a battery. After sometime the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant 1 < K < 2 is introduced to fill the space between the plates. How will the following be affected?
(i) the electric field between the plate of the capacitor?
(ii) The energy stored in the capacitor.
Justify your answer in each case.

7. A parallel plate capacitor with air between the plates has a capacitance of 8 μF. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6? [Ans. 96 pF]

8. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 V m^{-1}. For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 μF? [Ans. 19 cm^2]

(5 Marks Questions)

9. Two capacitors with capacity C_1 and C_2 are charged to potential V_1 and V_2 respectively and then connected in parallel. Calculate the common potential across the combination, the charge on each capacitor, the electrostatic energy stored in the system and the change in the electrostatic energy from its initial value.

10. Derive the expression for the capacitance of a parallel plate capacitor of area A and plate separation d if a dielectric slab of thickness t (where t<d) is introduced one by one between the plates of the capacitor.

11. State the working of principle of Van-de-graft generator with the help of neat and clean diagram.

E. CHALLENGING PROBLEMS

1. Answer the following:
(i). The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 V m^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage, so there is no field inside).

(ii) A man fixes outside house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2. Will he get an electric shock if he touches the metal sheet next morning?

(iii) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(iv) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

2. A long charged cylinder of linear charge density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

3. Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $\left(\vec{E}_2 - \vec{E}_1\right).\hat{n} = \frac{\sigma}{\varepsilon_0}$ where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of
n is from side 1 to side 2). Hence show that just outside a conductor, the electric field is $\sigma\frac{\hat{n}}{\varepsilon_0}$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

4. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

5. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e. bending of the field lines at the ends). [Ans. 2.9×10^4V]

6. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessary flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

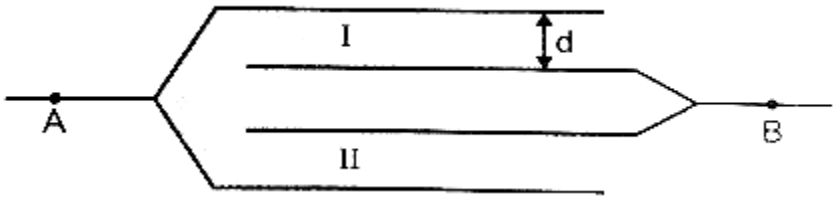
7. A spherical capacitor consists of two concentric spherical conductors held in position by suitable insulating supports. Show that the capacitance of a spherical capacitor is given by $C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$ where r_1 and r_2 are the radii of outer and inner spheres, respectively.

8. Answer carefully:

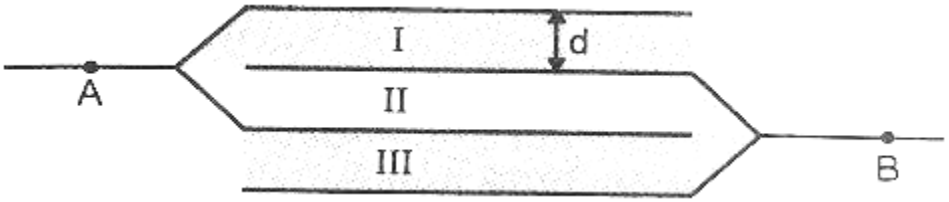
1. Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other, is the magnitude of electrostatic force between them exactly given by $\frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ r is the distance between their centres?
2. If coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?
3. A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?
4. What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

5. We know that electric field is discontinuous across the surface of a charged conductor. `Is electric potential also discontinuous there?
6. What meaning would you give to the capacitance of a single conductor?
7. Guess a possible reason why water has a much greater dielectric constant {- 80) then say, mica (= 6).

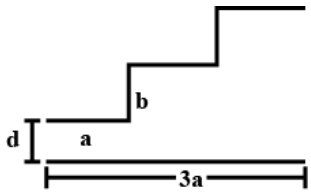
9. What is the capacitance of arrangement of a plates of area A at distance d in air in figure? [Ans. $2\varepsilon_0 A/d$]



10. What is the capacitance of arrangement of a 4 plates of area A at distance d in air in figure? [Ans. $3\varepsilon_0 A/d$]



11. A capacitor is made of flat plate area A and a second plate having a stair like structure, as shown in figure. The width of each stair is a and height is b. Find the capacitance of the assembly.



SPACE FOR ROUGH WORK

SPACE FOR NOTES