CLASS – 11

WORKSHEET- SYSTEMS OF PERICLES AND ROTATIONAL MOTION

(1 mark questions)

1. Define centre of mass.

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2. The velocity of centre of mass of the system remains constant, if the total external force acting on the system is

(a) minimum (b) maximum (c) unity (d) zero

3. Show that the centre of mass of an isolated system moves with a uniform velocity along a straight line path.

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4. The angle between two vectors $\vec{A}$ and $\vec{B}$ is 60°. Then the ratio of $\vec{A} \cdot \vec{B}$ and $|\vec{A} \times \vec{B}|$ is

(a) ½ (b) $1/\sqrt{3}$ (c) 1 (d) $\sqrt{3}/2$

5. The SI unit of angular momentum is

(a) kg ms^{-1} (b) Nm (c) kg m^2s^{-1} (d) Nm^2

6. When a torque acting upon a system is zero. Which of the following will be constant?

(a) Force (b) Linear impulse (c) Linear momentum (d) angular momentum

7. A rigid body is said to be in partial equilibrium, when it is in

(a) transitional equilibrium only (b) rotational equilibrium only

(c) either (a) or (b) (d) neither (a) nor (b)

8. The moment of inertia of a body depends upon

(a) mass of the body (b) axis of rotation of the body

(c) shape and size of the body (d) all of these

9. Define 1 kg m^2.

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10. Two masses each of mass M are attached to the end of a rigid massless rod of length L. The moment of inertia of the system about an axis passing through centre of mass and perpendicular to its length is

(a) $ML^2/4$ (b) $ML^2/2$ (c) ML^2 (d) $2ML^2$

11. State the theorem of perpendicular axes.

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12. A flywheel rotating at 420 rpm shows down at a constant rate of 2 rad/s. The time required to stop the flywheel is

(a) 22s (b) 11s (c) 44s (d) 12s

13. A disc is rotating with angular velocity $\vec{\omega}$ about its axis. A force $\vec{F}$ acts at a point whose position vector with respect to the axis of rotation is $\vec{r}$. The power associated with the torque die the force is given by

(a) $\left(\vec{r}\times\vec{F}\right).\vec{\omega}$ (b) $\left(\vec{r}\times\vec{F}\right)\times\vec{\omega}$ (c) $\vec{r}.\left(\vec{F}\times\vec{\omega}\right)$ (d) $\vec{r}\times\left(\vec{F}\times\vec{\omega}\right)$

14. Which of the following principles a circuit acrobat employs in his performance?

(a) Conservation of energy (b) Conservation of linear momentum

(c) Conservation of mass (d) Conservation of angular momentum

15. State right hand rule to find the direction of angular momentum.

16. Why do we prefer to use wrench of longer arm?

17. Is it difficult to open the door by pushing it or pulling it at the hinge. Why?

18. Why a force is applied at right angles to the heavy door at the outer edge while closing or opening it?

19. A faulty balance with unequal arms has its beam horizontal. Are the weights of the two pans equal?

20. Which physical quantities are expressed by the following: (i) the rate of change of angular momentum, and (ii) moment of linear momentum?

21. If the earth were to shrink suddenly, what would happen to the length of the day?

22. Two identical particles move towards each other with velocity 2v and v respectively. What is the velocity of the centre of mass?

23. What is a rigid body?

24. What is torque? Give its SI unit.

25. Which physical quantity is represented by the product of moment of inertia and angular velocity?

26. Define the term moment of momentum.

(2 marks Qestions)

27. Two point mass of 1kg and 2kg lie at (1, 2) and (2, - 3) respectively. Calculate the coordinates of the centre of mass of the system.

28. Three identical spheres each of radius ‘r’ and mass ‘m’ are placed on a vertical plane such that each spheres touching each other and stay in equilibrium. Find the position of centre of mass.

29. What is the value of linear velocity, if $\vec{r} = 3\hat{i} + 4\hat{j} + 6\hat{k}$ and $\vec{\omega} = -5\hat{i} + 3\hat{j} + 5\hat{k}$?

30. If $\vec{r} = 2\hat{i} + 3\hat{j}$ and $\vec{F} = 4\hat{i} - 3\hat{j}$, then find the magnitude of torque.

31. The position of a particle is given by $\vec{r} = \hat{i} + 2\hat{j} - \hat{k}$ and its linear momentum is given by $\vec{p} = 3\hat{i} + 4\hat{j} - 2\hat{k}$. In which axis its angular momentum, about the origin is perpendicular?

32. Define radius of gyration of a body rotating about an axis. Derive an expression for it.

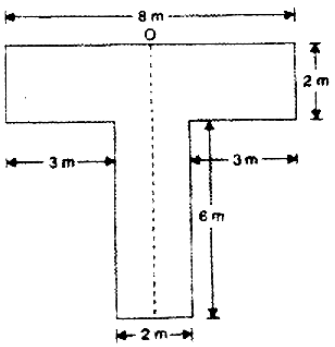
33. If two circular discs A and B are of same mass but of radii r and 2r respectively, then what is the moment of inertia of A in terms that of B?

34. State parallel axis theorem of moment of inertia. What is the moment of inertia of a (i) ring (ii) disc about diameter?

35. How a ballet dancer does take the advantage of the principle of conservation of angular momentum?

36. A solid sphere rolls down an inclined plane. Find the ratio of its rotational kinetic energy, the total kinetic energy.

37. Find the position of the centre of mass of the T-shaped plate from O in figure.



[Ans. 2.71m]

38. Find the torque of force $7\hat{i}-3\hat{j}-5\hat{k}$ about the origin which acts on a particle whose position vector is $\hat{i}+\hat{j}-\hat{k}$.

39. Three mass points m_1, m_2 and m_3 are located at the vertices of an equilateral triangle of length a. What is the moment of inertia of the system about an axis along the altitude of the triangle passing through m_1?

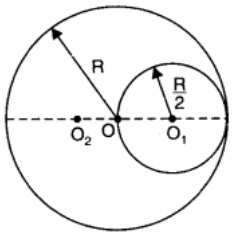
40. State the factors on which the moment of inertia of a body depends.

41. Show that the area of the triangle contained between the vectors $\vec{a}$ and $\vec{b}$ is one half of the magnitude $\vec{a}\times\vec{b}$.

42. Show that $\vec{a}.(\vec{b}\times\vec{c})$ is equal in magnitude to the volumes of the parallelepiped formed on the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$.

43. To maintain a rotor at a uniform angular speed of 200 rad s^{-1}, an engine needs to transmit a torque of 180 Nm. What is the power required by the engine?

44. From a uniform disk of radius R, a circular hole of radius R/2 is cut out. The centre of the hole is at R/2 from the centre of the original disc. Locate the centre of gravity of the resulting flat body. [Ans. R/6]

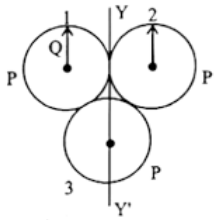


45. A metre stick is balanced on a knife edge at its centre. When two coins, each of mass 5g are put one on top of the other at the 12.0 cm mark, the stick is found to be balanced at 45.0 cm. What is the mass of the metre stick? [Ans. 66.0g]

(3 marks Questions)

46. Define torque. Derive an expression for it in Cartesian coordinates.

47. Three identical rings, each of mass M and radius R are arranged as shown in figure. What is the moment of inertia of the arrangement about YY'?



48. Calculate the MI of a uniform circular disc of mass 500gm and radius 10cm about
(i) Diameter (ii) Axis tangent to the disc and parallel to diameter (iii) Axis passing through centre and perpendicular to its plane.

49. State the theorem of parallel axis and perpendicular axes.

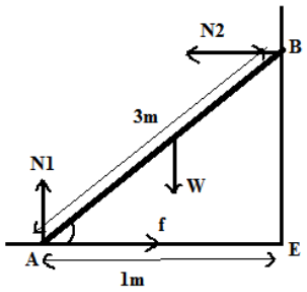
50. A constant power is supplied to a rotating disc. How is angular velocity (ω) of disc varies with number of rotations (n) made by the disc?

51. A cylinder of mass 10kg and radius 15cm is rolling perfectly on a plane of inclination 30°. The coefficient of friction $\mu = 0.25$. (a) How much is the force of friction on acting on the cylinder? (b) What is the work done against friction rolling down? Take $g=10ms^{-2}$.

52. Three masses 3, 4 and 5kg are located at the corners of an equilateral triangle of side 1m. Locate the centre of mass of the system. [Ans. 0.54m, 0.36m]

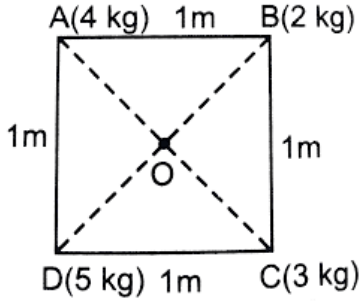
53. Two bodies of masses 10kg and 2kg are moving with velocities $2\hat{i}-7\hat{j}+3\hat{k}$ and $-10\hat{i}+35\hat{j}-3\hat{k}$ ms^{-1} respectively. Find the centre of mass of the system.

54. A 3m long ladder weighing 20kg leans on a frictionless wall. Its feet rest on the floor 1m from the wall as shown in figure. Find the reaction forces of the wall and the floor.



55. Four points of masses 4kg, 2kg, 3kg and 5kg are respectively located at the four corners A, B, C and D of a square of side 1m as shown in figure. Calculate the moment of inertia

of the system about (i) an axis passing through the point of intersection of the diagonals and perpendicular to the plane of the square. (ii) the side AB and (iii) the diagonal BD.



[Ans. (i) 7kg m^2 (ii) 8 kg m^2 (iii) 3.5 kg m^2]

56. Three particles, each of 10g are located at the corners of an equilateral triangle of side 5cm. Determine the moment of inertia of this system about an axis passing through one corner of the triangle and perpendicular to the plane of the triangle. [Ans. 5×10^{-5} kgm^2]

57. A flywheel of mass 25kg has a radius of 0.2m. What force would be applied tangentially to the rim of the flywheel so that it acquires an angular acceleration of 2 rad s^{-1}?

[Ans. 5N]

58. A grindstone has moment of inertia of 6 kg m^2. A constant torque is applied and the grindstone is found to have a speed o f150 rpm, 10 seconds after starting from rest. Calculate the torque. [3π Nm]

59. A ring of diameter 0.4m and mass 10kg is rotating about its axis at the rate of 2100 rpm. Find (i)moment of inertia (ii) angular momentum and (iii) rotational KE of the ring.
[Ans. 0.4 kg m^2, 88 kg m^2s^{-1}, 9680J]

60. Explain if the ice on the polar caps of the earth melts, how will it affect the duration of the day?

61. Establish the relation between torque and angular acceleration. Hence define moment of inertia.

62. Establish the relation between moment of inertia and torque on a rigid body.

63. Establish the relation between angular momentum and moment of inertia for a rigid body.

64. State and prove the principle of conservation of angular momentum.

65. Give the location of the centre of mass of a (i) sphere, (ii) cylinder, (iii) ring, and (iv) cube, each of uniform mass density. Does the centre of mass of a body necessarily lie inside the body?

66. In the HCl molecule, the separation between the nuclei of the two atoms is about 1.27 Å (1 Å = 10^{-10}m). Find the approximate location of the CM of the molecule, given that a chlorine atom is about 35.5 times as massive as a hydrogen atom and nearly all the mass of an atom is concentrated in its nucleus. [Ans. 1.235Å]

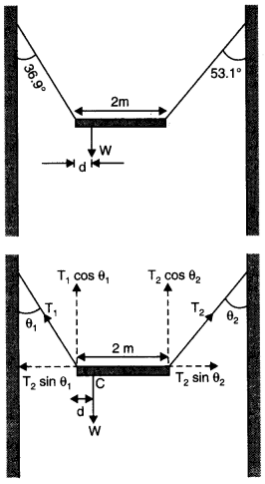
67. A child sits stationary at one end of a long trolley moving uniformly with a speed V on a smooth horizontal floor. If the child gets up and runs about on the trolley in any manner, what is the speed of the CM of the (trolley + child) system?

68. Find the components along the x, y, z-axes of the angular momentum l of a particle, whose position vector is r with components x, y, z and momentum is p with components

p_x, p_y and p_z. Show that if the particle moves only in the x-y plane the angular momentum has only a z- component.

69. Two particles, each of mass m and speed v, travel in opposite directions along parallel lines separated by a distance d. Show that the vector angular momentum of the two particle system the same whatever be the point about which the angular momentum is taken.

70. A non-uniform bar of weight W is suspended at rest by two strings of negligible weight as shown in Fig. The angles made by the strings with the vertical are 36.9° and 53.2° respectively. The bar is 2 m long. Calculate the distance d of the centre of gravity of the bar from its left end.



[Ans. 72cm]

71. A car weighs 1800 kg. The distance between its front and back axles is 1.8 m. Its centre of gravity is 1.05 m behind the front axle. Determine the force exerted by the level ground on each front wheel and each back wheel. [Ans. 3675N, 5145N]

72. (a) Find the moment of inertia of a sphere about a tangent to the sphere, given the moment of inertia of the sphere about any of its diameters to be $2\ MR^2/5$, where M is the mass of the sphere and R is the radius of the sphere.
(b) Given the moment of inertia of a disc of mass M and radius R about any of its diameters to 1 be $1/4\ MR^2$, find the moment of inertia about an axis normal to the disc passing through a point on its edge. [Ans. $7/5\ MR^2$, $3/2\ MR^2$]

73. Torques of equal magnitude are applied to a hollow cylinder and a solid sphere, both having the same mass and radius. The cylinder is free to rotate about its standard axis of symmetry, and the sphere is free to rotate about an axis passing through its centre. Which of the two will acquire a greater angular speed after a given time?

74. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad s^{-1}. The radius of the cylinder is 0.25 m. What is the kinetic energy associated with the rotation of the cylinder? What is the magnitude of angular momentum of the cylinder about its axis? [Ans. 3125J, 62.5 kg m^2 s^{-1}]

75. (a) A child stands at the centre of a turntable with his arms outstretched. The turntable is set rotating with an angular speed of 40 rev/min. How much is the angular speed of the child if he folds his hands back and thereby reduces his moment of inertia to 2/5 times the initial value? Assume that the turntable rotates without friction, (b) Show that the child's new kinetic energy of rotation is more than the initial kinetic energy of rotation. How do you account of this increase in kinetic energy? [Ans. 100 rpm, 2.5]

76. A rope of negligible mass is wound round a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of

30 N? What is the linear acceleration of the rope? Assume that there is no slipping.
[Ans. 25 rad s^{-2}, 10 ms^{-2}]

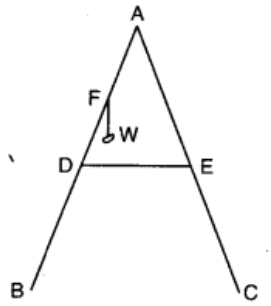
77. A solid sphere rolls down two different inclined planes of the same heights but different angles of inclination, (a) Will it reach the bottom with the same speed in each case? (b) Will it take longer to roll down one plane than the other? (c) If so, which one and why?

78. A hoop of radius 2 m weighs 100 kg. It rolls along a horizontal floor so that its centre of mass has a speed of 20 cm/s. How much work has to be done to stop it? [Ans. 4J]

79. The oxygen molecule has a mass of 5.30×10^{-26} kg and a moment of inertia of 1.94×10^{-45} kg m^2 about an axis through its centre perpendicular to the lines joining the two atoms. Suppose the mean speed of such a molecule in a gas is 500 m/s and that its kinetic energy of rotation is two thirds of its kinetic energy of translation. Find the average angular velocity of the molecule. [Ans. 6.75×16^{12} rad s^{-1}]

80. A solid cylinder rolls up an inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m/s.
(a) How far will the cylinder go up the plane?
(b) How long will it take to return to the bottom? [Ans. 3.8m, 3.0s]

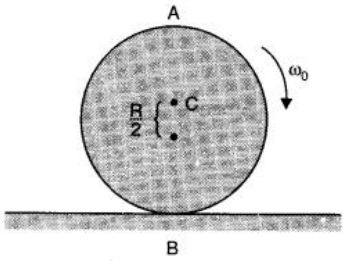
81. As shown in Fig. the two sides of a step ladder BA and CA are 1.6 m long and hinged at A. A rope DE, 0.5 m is tied halfway up. A weight 40 kg is suspended from a point F, 1.2 m from B along the ladder BA. Assuming the floor to be friction less and neglecting the weight of the ladder, find the tension in the rope and forces exerted by the floor on the ladder. (Take g = 9.8 m^2)(Hint: Consider the equilibrium of each side of the ladder separately.) [Ans. 147N, 97N]



82. A man stands on a rotating platform, with his arms stretched horizontally holding a 5 kg weight in each hand. The angular speed of the platform is 30 revolutions per minutes. The man then brings his arms close to his body with the distance of each weight from the axis changing from 90 cm to 20 cm. The moment of inertia of the man together with the platform may be taken to be constant and equal to 7.6 kg m^2.(a) What is his new angular speed? (Neglect friction)(b) Is kinetic energy conserved in the process? If not, from where does the change come about? [Ans. 59rpm, 1.97]

83. A bullet of mass 10 g and speed 500 m/s is fired into a door and gets embedded exactly at the centre of the door. The door is 1.0 m wide and weighs 12 kg. It is hinged at one end and rotates about a vertical axis practically without friction. Find the angular speed of the door just after the bullet embeds into it.(Hint: The moment of inertia of the door about the vertical axis at one end is $ML^2/3$.) [Ans. 0.625 rad s^{-1}]

84. A disc rotating about its axis with angular speed w_o is placed lightly (without any translational push) on a perfectly friction less table. The radius of the disc is R. What are the linear velocities of the points A, B and C on the disc shown in Fig.? Will the disc roll in the direction indicated?



85. Explain why friction is necessary to make the disc roll (refer to Q. 84) in the direction indicated.
(a) Give the direction of frictional force at B, and the sense of frictional torque, before perfect rolling begins.
(b) What is the force of friction after perfect rolling begins?

86. A solid disc and a ring, both of radius 10 cm are placed on a horizontal table simultaneously, with initial angular speed equal to 10π rad/s. Which of two will start to roll earlier? The coefficient of kinetic friction is $u_k = 0.2$. [Ans. 0.53s, 0.80s]

87. A cylinder of mass 10 kg and radius 15 cm is rolling perfectly on a plane of inclination 30°. The coefficient of static friction us = 0.25.
(a) How much is the force of friction acting on the cylinder?
(b) What is the work done against friction during rolling?

(c) If the inclination O of the plane is increased, at what value of 8 does the cylinder begin to skid, and not roll perfectly? [Ans. 16.33N, 0J, 36°52']

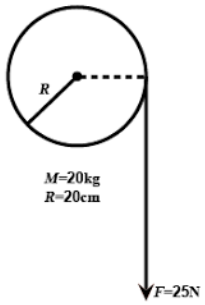
(5 marks Questions)

88. (a) Derive an expression for torque on polar coordinates.
(b) A torque of 20 Nm is applied on a wheel initially are rest. Calculate the angular momentum at the wheel after 3s.

89. Obtain the expression for the linear acceleration of a solid cylinder of radius 'R' rolling down an inclined plane. Also find the frictional force acting between the solid cylinder and the plane.

90. A flywheel of mass 25 kg has a radius of 0.2m. It is making 240rpm. What is the torque necessary to bring it to rest tin 20s? If the torque is due to a force applied tangentially on the rim of the flywheel, what is the magnitude of the force?

91. A cord of negligible mass is wound round the rim of flywheel of mass 20kg and radius 20cm. A steady pull of 25N is applied on the cord as shown in figure. The flywheel is mounted on a horizontal axis with frictionless bearings.



(a) Compute the angular acceleration of the wheel.
(b) Find the work done by the pull, when 2m of the cord is unwind.
(c) Find also the kinetic energy of the wheel at this point. Assume that the wheel starts from rest.
(d) Compare answers to parts (b) and (c).

92. Define rotational motion of a body. Derive the following equations of rotational motion under constant angular acceleration.

(a) $\omega = \omega_0 + \alpha t$ (b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ (c) $\omega^2 = \omega_0^2 + 2\alpha\theta$

93. Prove that the rate of change of total angular momentum of a system of particles about a reference point is equal to the total torque acting on the system.

94. Derive an expression for the total work done on a rigid body executing both translational and rotational motions. Hence deduce the condition for the equilibrium of the rigid body.

95. Obtain an expression for the linear acceleration of a cylinder rolling down an inclined plane and hence find the condition for the cylinder to roll down without slipping.

96. Two discs of moments of inertia I_1 and I_2 about their respective axes (normal to the disc and passing through the centre), and rotating with angular speed w_1 and w_2 are brought into contact face to face with their axes of rotation coincident, (a) What is the angular speed of the two-disc system? (b) Show that the kinetic energy of the combined system is less than the sum of the initial kinetic energies of the two discs. How do you account for this loss in energy? Take w_1 not equal to w_2.

97. (a) Prove the theorem of perpendicular axes.
Hint: Square of the distance of a point (x, y) in the x-y plane from an axis through the origin perpendicular to the plane is $x^2 + y^2$]
(b) Prove the theorem of parallel axes.
Hint: If the centre of mass of chosen the origin [$\Sigma m_i r_i = 0$]

98. Prove the result that the velocity v of translation of a rolling body (like a ring, disc, cylinder or sphere) at the bottom of an inclined plane of a height h is given by, $v^2=2gh/(1+k^2/R^2)$ using dynamical consideration (i.e., by consideration of forces and torques). Note k is the radius of gyration of the body about its symmetry axis, and R is the radius of the body. The body starts from rest at the top of the plane.

99. Read each statement below carefully, and state, with reasons, if it is true or false:
(a) During rolling, the force of friction acts in the same direction as the direction of motion of the CM of the body.
(b) The instantaneous speed of the point of contact during rolling is zero.
(c) The instantaneous acceleration of the point of contact during rolloing is zero.
(d) For perfect rolling motion, work done against friction is zero.
(e) A wheel moving down a perfectly frictionless inclined plane will undergo slipping (not rolling) motion.