CLASS – 11

WORKSHEET- OSCILLATIONS

(1 mark questions)

1. The equation of motion of a particle is $x = A\cos(\alpha t)^2$. The motion is
   (a) periodic but not oscillatory (b) periodic and oscillatory
   (c) oscillatory but not periodic (d) neither periodic nor oscillatory

2. Can a motion be periodic but not oscillatory?
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3. The time period of simple harmonic motion depends upon
   (a) amplitude (b) energy (c) phase constant (d) mass

4. What is the (a) distance moved (b) displacement of a particle executing SHM in one vibration?
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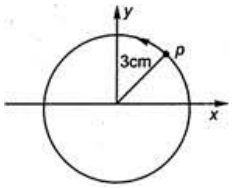
5. Simple Harmonic motion is the projection of uniform motion on
   (a) x-axis (b) y-axis (c) reference circle
   (d) any diameter of reference circle

6. The figure shows circular motion of a reference particle to represent simple harmonic motion. What is the amplitude of simple harmonic motion?



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7. A particle executing SHM. The phase difference between acceleration and displacement is
   (a) 0 (b) $\pi/2$ (c) π (d) $1/2\ \pi$

8. Can velocity and acceleration be in the same direction in a SHM?

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9. What is phase relationship between displacement, velocity and acceleration in SHM?

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10. The equation of motion in a simple harmonic motion is

(a) $\frac{d^2x}{dt^2} = -\omega^2 x$ (b) $\frac{d^2x}{dt^2} = -\omega^2 t$ (c) $\frac{d^2x}{dt^2} = -\omega x$ (d) $\frac{d^2x}{dt^2} = -\omega t$

11. Which of the following relationships between the acceleration a and the displacement x of a particle executing simple harmonic motion?

(a) $a = 2x^2$ (b) $a = -2x^2$ (c) $a = 2x$ (d) $a = -2x$

12. The total energy of a simple harmonic oscillator is proportional to

(a) amplitude (b) square of amplitude (c) frequency (d) velocity

13. The amplitude of a simple harmonic oscillator is doubled. How does this affect: (a) the period (b) the total energy (c) the maximum velocity of the oscillator?

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14. Write an expression for PE of a harmonic oscillator at any point.

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15. The length of the simple pendulum which ticks seconds is

(a) 0.5m (b) 1m (c) 1.5m (d) 2m

16. What is the effect on the time period of a simple pendulum if the mass of the bob is doubled?

(a) Halved (b) Doubled (c) becomes 8 times (d) no effect

17. There are two springs, one delicate and another hard or stout one. For which spring, the frequency of the oscillator will be more?

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18. The girl sitting on swing stands up. What will be the effect on periodic time of swing?

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19. What will be the time period of a second pendulum inside an artificial satellite?
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20. Write the expression for time period of a simple pendulum.
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21. Define force constant of a spring.
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22. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?
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23. A particle oscillating under a force is a $\vec{F} = -k\vec{x} - b\vec{v}$ is a(k and b are constants).
(a) simple harmonic motion (b) linear oscillator
(c) damped oscillator (d) forced oscillator

24. At resonance, the amplitude of forced oscillations is
(a) minimum (b) maximum (c) zero (d) none of these

25. Marching troops are asked to break their steps while crossing the bridge. Why?
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26. Why is loud sound heard at resonance?
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27. At a certain speed of a bus, its whole body starts vibrating strongly. Explain.
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28. Which of the following examples represent periodic motion?

(i) A swimming completing one (return) trip from one bank of a river to the other and back.
(ii) A freely suspended bar magnet displaced from its N_S direction and released.
(iii) A hydrogen molecule rotating about its centre of mass
(iv) An arrow released from a bow

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29. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
(i) The rotation of earth about its axis.
(ii) Motion of an oscillating mercury column in a U-tube.
(iii) Motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.
(iv) General vibrations of a polyatomic molecule about its equilibrium position

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(2 marks Questions)

30. Prove that the equation $x = a \sin \omega t + b \cos \omega t$ shows SHM.

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31. Show that motion of a particle represented by $y = \sin \omega t - \cos \omega t$ is a simple harmonic motion with time period $2\pi/\omega$.

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32. The displacement of particle in SHM may be given by a $y = a \sin (\omega t + \phi)$. Show that if the time t is increased by $2\pi/\omega$, the value of y remains the same.

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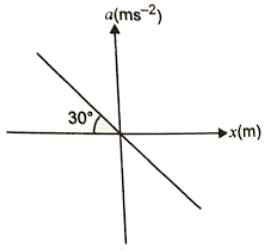
33. Figure shows the acceleration displacement graph of a particle in SHM. Find the time period (in second).



34. Find the period of vibrating particle (SHM) which ahs acceleration of 45 cm s^{-1}, when displacement from mean position is 5cm.

35. Describe the motion of a particle acted upon by force $F = -(x - 3)^3$.

36. Show that the acceleration of a particle in SHM is proportional to its displacement from the mean position.

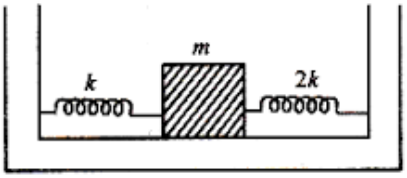
37. A body is executing a simple harmonic motion such that its potential energy is U_1 at x and U_2 at y. When the displacement is x + y, calculate the potential energy.

38. A point particle of mass 0.1kg is executing SHM of amplitude 0.1m. When the particle passes through the mean position, its kinetic energy is 0.008J. If each is 45°, then what is the equation of motion of this particle?

39. A particle is moving on x-axis and has potential energy $U = 2 – 20x + 5x^2$ joule where x is position. The particle is released at x = - 3. If the mass of the particle is 0.1kg, then the maximum velocity (in m/s) of the particle is 25β.If the amplitude is 5m, find the value of β.

40. A 0.2 kg of mass hangs at the end of a spring. When 0.02 kg more mass is added to the end of the spring, it stretches 7cm more. If the 0.02kg mass is removed, what will be the period of vibration of the system?

41. Two springs of force constants K and 2K are connected in a block of m as shown in the figure. What is the frequency of oscillation of this block?



42. The formula for time period T for a loaded spring, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$. Foes the time period depend on the length of the spring?

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43. Two pendulums of length 100cm and 121cm start oscillating. At some instant the two are at the mean position in the same phase. After how many oscillations of the longer pendulum will the two be in the same phase the mean position again?

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44. A massless spring of spring constant k is attached with a mass ma and is made to oscillate vertically. Deduce the expression for its time period.

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45. In a forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. What is the relation between ω_1 and ω_2?

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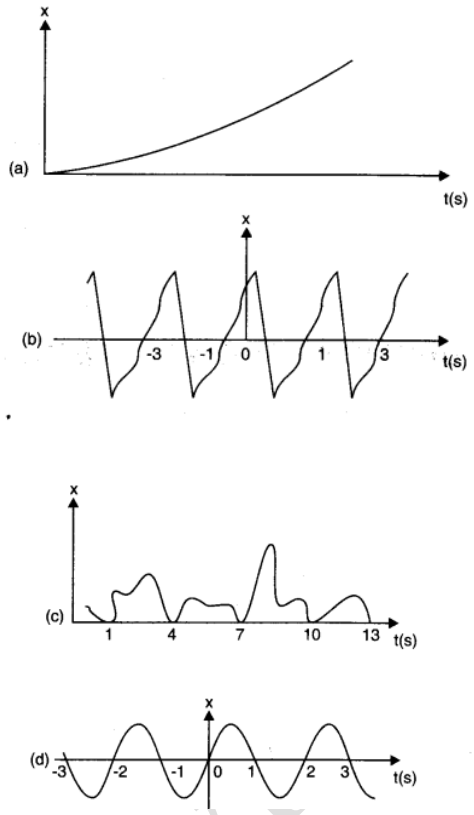
46. Figure depicts four x-t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?



47. Which of the following functions of time represent (a) simple harmonic motion (b) periodic but not simple harmonic motion, and (c) non-periodic motion? Give period for each case of periodic motion [ω is any positive constant]

(i) $\sin \omega t - \cos \omega t$ (ii) $\sin^3 \omega t$ (iii) $3 \cos [\pi/4 - 2 \omega t]$ (iv) $\cos \omega t + \text{cps } 3 \omega t + \cos 5 \omega t$ (v) $\exp (-\omega^2 t^2)$ (vi) $1 + \omega t + \omega^2 t^2$

(3 marks Questions)

48. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite direction when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. Find the phase difference.

49. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. What will be the minimum time taken by the particle to travel half of the amplitude from the equilibrium position?

50. Plot the reference circle for each of the following simple harmonic motion. Indicate the initial (t= 0) opposition of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s)

(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right)$ (b) $x = \cos\left(\frac{\pi}{6} - t\right)$

51. A block is resting on a piston which is moving vertically with a SHM of period 1.0s. At what amplitude of vibration will the block and the piston separate? What is the maximum velocity of the piston at this amplitude?

52. A body is describing SHM has a maximum acceleration of 8π m/s^2 and maximum speed of 1.6m/s. Find the time period and the amplitude.

53. If the displacement x and velocity v of a particle executing SHM are related through the experiment $4v^2 = 25 - x^2$, then determine its time period.

54. What is Simple Harmonic Motion? What is phase difference between displacement and acceleration in SHM. A simple harmonic motion is described by a = -25x where a is acceleration (m/s) and x is displacement (m). What I the time period?

55. Show that the total energy of a particle executing SHM is directly proportional to the square of its amplitude and frequency.

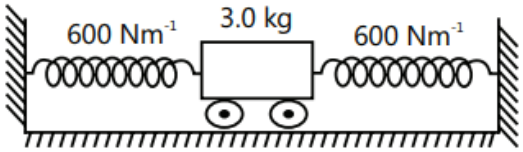
56. A simple harmonic motion is described by $y = A \sin \omega t$. Find th time at which kinetic energy and potential energy of the simple harmonically oscillating particle are equal to each other.

57. Find the displacement of a simple harmonic oscillator at which its PE is half of the maximum energy of the oscillator.

58. A man of mass 60kg is standing on a platform executing SHM in vertical direction. The displacement from mean position of platform varies as $y = 0.5 \sin (2\pi\upsilon t)$. What will be the minimum value of υ, for which the man will feel weightlessness at the highest point?

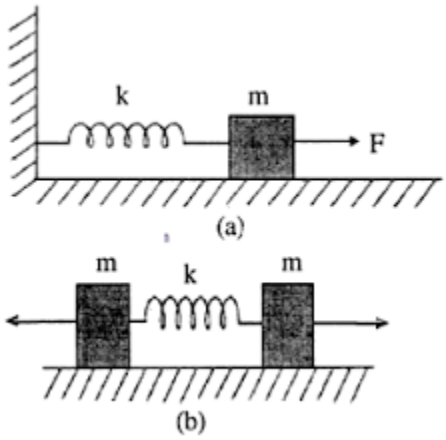
59. Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period. What would be the time period of simple pendulum at the centre of the earth. Justify your answer.

60. A trolley of mass 3.0kg is connected to two identical springs, each of force constant 600 N/m as shown in the figure. If the trolley is displaced from its equilibrium position by 5.0cm and released, what is



(a) the time period of ensuing oscillations (b) the maximum speed of the trolley? (c) How much is the total energy dissipated as heat by the time the trolley comes to rest due to damping forces?

61. Figures (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F is applied to the free end stretches the spring. Figure (b) shows the same spring with both ends attached to mass m at either end. Each end of the spring in figure (b) is stretched by the same force F. What will be the maximum extension of the spring in both cases? Also, find out the time period for each case.



62. An infinite number of springs with spring constant k, 2k, 4k, 8k, 16k, …∞. Respectively are connected in series. What is the equivalent spring constant?

63. The vertical motion of a huge piston in a machine is approximately SHM with a frequency of 0.5/s. A block of 10kg is placed on piston, what is the maximum amplitude for the block and piston to remain together?

64. Determine the period of small oscillations of a pendulum that is bob suspended by a thread L = 20cm in length, if it is located in a liquid whose density is 3 times less than that of bob.

65. A 21.2 kg object oscillates at the end of a vertical spring that has a spring constant 20500 N/m. The effect of air resistances is represented by the damping coefficient b = 2kg/s. Fine the time interval that elapses while the energy of the system drops to 10% of its initial value (given ln 10 = 2.302).

66. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion? (a) = 0.7x (b) $a = - 200x^2$ (c) a = - 10x (d) $a = 100 x^3$.

67. (a) A particle in SHM is described by the displacement function,

$$x(t) = A \cos(\omega t + \phi),\ \omega = 2\pi/T$$

If the initial (t = Q) position of the particle is 1cm and the initial velocity π cm/s , what are its amplitude and initial phase angle? The angular frequency of the particle is πs^{-1}.

(b) A particle in SHM is described by the displacement function,

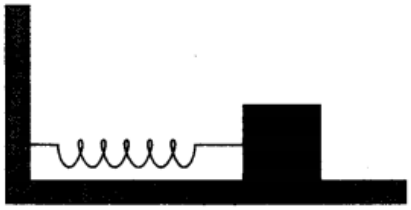
$$x(t) = B \sin(\omega t + \alpha),\ \omega = 2\pi/T$$

If the initial (t = Q) position of the particle is 1cm and the initial velocity π cm/s , what are its amplitude and initial phase angle? The angular frequency of the particle is πs^{-1}.

68. A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20cm. A body suspended from this spring, when displaced and released, oscillates with a period of 0.60s. What is the weight of the body? [Ans. 219.13N]

69. A spring of force constant 1200 Nm^{-1} is mounted horizontally on a horizontal table. A mass of 3.0 kg is attached to the free end of the spring, pulled sideways to a distance of

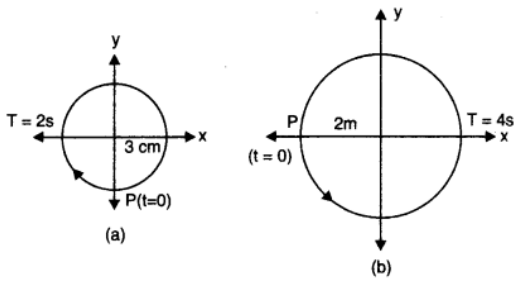
2.0cm and released (i) What is the frequency of oscillation of the mass? (ii) What is the maximum acceleration of the mass? (iii) What is the maximum speed of the mass?



[Ans. (i) 3.2 s^{-1} (ii) 8.0 m/s^2 (iii) 0.40 m/s]

70. In Qs 69, let us take the position of the mass, when the spring is unstretched, as x = 0, and the direction from left to right as the positive direction of X-axis. Give x as a function of time t for the oscillating mass, if at the moment we start the stop watch (t = 0) the mass is (i) at the mean position (ii) at the maximum stretched position (iii) at the maximum compressed position.
In what do these different functions of SHM differ? Frequency, amplitude or initial phase? [Ans. 2sin 20t, 2cos 20t, - 2cos 20t]

71. Figure corresponds to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e., clockwise or anti clockwise) are indicated on each figure.



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

72. The piston in the cylinder head of locomotive has a stroke (twice the amplitude) of 1.0m. If the piston moves with simple harmonic motion with an angular frequency of 200 rev/min, what is the maximum speed? [Ans. 100m/min]

73. The acceleration due to gravity on the surface of the moon is 1.7 m/s^2. What is the time period of a simple pendulum on the moon if its time period on the earth is 3.5s? Given g on earth = 9.8 m/s^2. [Ans. 8.4s]

74. A simple pendulum of length l and having a bob mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

75. A cylindrical piece of cork of base area A and height h floats in a liquid of density ρ_1. The work is depressed slightly and then released> Show that the work oscillates up and down simple harmonically with period $T = 2\pi\sqrt{h\rho/\rho_1 g}$, where ρ is the density of cork. (Ignore dumping due to viscosity of the liquid).

76. One end of U-tube containing mercury is connected to a suction pump and the other end is connected to the atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid in U-tube executes SHM.

77. An air chamber of volume V has a neck of area of cross section A into which a ball of mass m can move without friction. Show that when a ball is pressed down through some

distance and released, the ball executes SHM. Obtain the formula for the time period of this SHM, assuming pressure volume variations of the air to be (i) isothermal (ii) adiabatic.

78. You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristic of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant and (b) the dumping constant b for the spring and shock absorber system of one wheel, assuming that each wheel supports 750 kg. [Ans. 1351.4 kg/s]

79. Show that for a particle in linear SHM, the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

80. A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillation is found to be 1.5s. The radius of the disc is 15cm. Determine the torsional spring constant of the wire.

[Ans. 2.0 Nm rad^{-1}]

81. A body describes SHM with an amplitude of 5cm and a period 0.2s. Find the acceleration and velocity of the body when the displacement is (a) 5cm (b) 3cm (c) 0 cm.

[$5\pi^2 ms^{-2}$, $-3\pi^2 ms^{-2}$,0]

82. A mass attached to a spring is free to oscillate, with angular velocity ω in a horizontal plane without friction or dumping. It is pulled to a distance x_0 and pushed towards the centre with a velocity v_0 at time t = 0. Determine the amplitude of the resulting oscillations in terms of the parameters ω, x_0 and v_0.

(5 marks Questions)

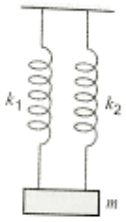
83. A body oscillates with SHM along the x-axis. Its displacement varies with time according to the equation $x = (4.00m)\cos(\pi t + \pi/4)$. Calculate (a) displacement (b) velocity (c) acceleration at t = 1.00s (d) the maximum speed and maximum acceleration and (e) phase at t = 2.00s.

84. Deduce an expression for the (a) displacement (b) velocity (c) acceleration of a particle executing SHM.

85. (a) Draw a graph showing the variation of kinetic energy and potential energy of a particle executing SHM with its displacement from mean position.
(b) Show that total mechanical energy of a particle executing simple harmonic motion remains conserved with time, when dissipative forces are neglected.

86. (a) One end of a U tube containing mercury is conned to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid column of mercury in the U-tube executes SHM.

(b) An arrangement of springs for SHM is shown in the figure. If mass of block is m, then find frequency of oscillation.



87. Show that the time period of a oscillation of a liquid column in a U tube is independent of the mass of the liquid column, the density of the liquid and the cross section area of U tube but depends only upon the length of the liquid column and on the value of the acceleration due to gravity.

88. Explain damped harmonic oscillation and the equation of such oscillations.

89. Discuss driven oscillations.

90. A particle is in linear simple harmonic motion between two points A and B, 10 cm apart. Take the direction from A to B as the positive direction and give the signs of velocity, acceleration and force on the particle when it is (a) at the end A (b) at the end B (c) at the mid point of AB going towards A (d) at 2cm away from B going towards A (e) at 3cm away from A going towards B and (f) at 4cm away from A going towards A

91. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t = 0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case (x is in cm and t is in s).

(i) $x = -2\sin(3t + \pi/3)$ (ii) $x = \cos(\pi/6 - t)$ (iii) $x = 3\sin(2\pi t + \pi/4)$

(iv) $x = 2\cos \pi t$

92. Answer the following questions:

(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle: $T = 2\pi\sqrt{m/k}$. A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater that $2\pi\sqrt{l/g}$. Think of a qualitative argument to appreciate this result.

(c) A man with wrist watch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?