

WORKSHEET- ELECTRIC CHARGE AND FIELD

A. ELECTRIC CHARGE

(1 Mark Questions)

1. If a body contains n_1 electrons and n_2 protons then what is the total charge on the body?

Sol. $Q = q_1 + q_2 + \dots + q_n$. (Additive property of charge)

$$Q = (n_2 - n_1)e$$

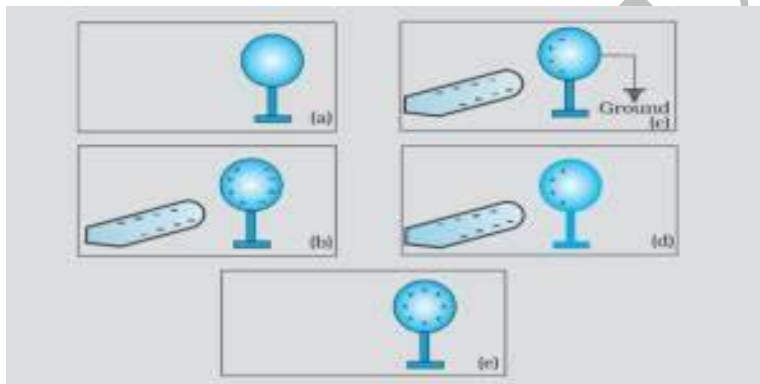
2. What is the total positive or negative charge present in 1 molecule of water?

Sol. H_2O has 10 electrons (2 of hydrogen and 8 of oxygen) Total charge = $10e$

(2 Marks Questions)

3. How can you charge a metal sphere positively without touching it?

Sol. The following sequence of figures shows the technique.



4. If 10^9 electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the other body?

Sol. The charge given out in one second = $1.6 \times 10^{-19} \times 10^9 C = 1.6 \times 10^{-10} C$

$$\text{Time required to accumulate 1C charge} = \frac{1C}{1.6 \times 10^{-10} C/s} = 6.25 \times 10^9 s$$

$$= \frac{6.25 \times 10^9}{(365 \times 24 \times 3600)} \text{ years} = 198 \text{ years}$$

5. What is electric charge? Is it scalar or vector? Name its SI unit.

Sol. Electric charge is an intrinsic property of the elementary particles like electrons, protons, etc. of which all the objects are made up of. Electric charge is a scalar quantity. Its SI unit is coulomb (C).

6. What is meant by quantization of electric charge? What is its cause?

Sol. It is found experimentally that the electric charge of any body large or small, is always an integral multiple of a certain minimum amount of charge. The basic charge is the charge on an electron, which is denoted by e and has magnitude 1.6×10^{-19} coulomb. Thus the charge on an electron is $-e$, on a proton is $+e$ and that on α -particle is $+2e$.

Causes: The basic cause of quantization of electric charge is that during rubbing only an integral number of electrons can be transferred from one body to another.

7. State the law of conservation of charge. Give its two examples to illustrate it.

Sol. If some amount of matter is isolated in a certain region of space and no matter either enters or leaves this region by moving across its boundary, then whatever other changes may occur in the matter inside, its total charge will not change with time.

Examples: 1. When a glass rod is rubbed with a silk cloth, it develops a positive charge. But at the same time, the silk cloth develops an equal negative charge. Thus the net charge of the glass rod and the silk cloth is zero, as it was before rubbing.

2. The rock salt ionizes in aqueous solution as follows: $\text{NaCl} \rightleftharpoons \text{Na}^+ + \text{Cl}^-$. As the total charge is zero before and after the ionization, so charge is conserved.

(3 Marks questions)

8. How much positive and negative charge is there in a cup of water?

Sol. Suppose the mass of water contained in a cup is 250g. The molecular mass of water is 18g.

Number of molecules present in 18g of water = Avogadro's number = 6.02×10^{23}

\therefore Number of molecules present in a cup (or 250g) of water

$$n = \frac{6.02 \times 10^{23} \times 250}{18} = 8.36 \times 10^{24}$$

Each molecule of water (H_2O) contains $2 + 8 = 10$ electrons as well as 10 protons.

Total number of electrons or protons present in a cup of water

$$n' = n \times 10 = 8.36 \times 10^{25}$$

Total negative charge carried by electrons or total positive charge carried by protons in a cup of water. $q = n'e$

$$= 8.36 \times 10^{25} \times 1.6 \times 10^{-19} \text{C} = 1.33 \times 10^7 \text{C}$$

9. (i) Explain the meaning of the statement 'electric charge of a body is quantised'.
(ii) Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Sol. (i) Electric charge of a body is quantized. This means that only integral (1, 2, ..., n) number of electrons can be transferred from one body to the other. Charges are not transferred in fraction. Hence, a body possesses total charge only in integral multiples of electric charge.

(ii) We can ignore the quantization of electric charge when dealing with macroscopic charges because the charge on one electron is $1.6 \times 10^{-19}\text{C}$ in magnitude which is very small as compared to the large scale change.

10. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Sol. It is observed that the positive that the positive charge developed on the glass rod has the same magnitude as the negative charge developed on silk cloth. So total charge after rubbing is zero as before rubbing. Hence the law of conservation charge is being obeyed here.

11. A polythene piece rubbed with wool is found to have a negative charge of $3.2 \times 10^{-7}\text{C}$. (i) Estimate the number of electrons transferred (from which to which?) (ii) Is there a transfer of mass from wool to polythene?

Sol. (i) Here $q = 3.2 \times 10^{-7}\text{C}$, $e = 1.6 \times 10^{-19}\text{C}$
As $q = ne$, therefore, Number of electrons transferred

$$n = \frac{q}{e} = \frac{3.2 \times 10^{-7}}{1.6 \times 10^{-19}} = 2 \times 10^{12}$$

Since polythene has negative charge, so electrons are transferred from wool to polythene during rubbing.

(ii) Yes, there is a transfer of mass from wool to polythene because each electron has a finite mass of $9.1 \times 10^{-31}\text{kg}$.

$$\begin{aligned} \text{Mass transferred} &= m_e \times n = 9.1 \times 10^{-31} \times 2 \times 10^{12} \\ &= 1.82 \times 10^{-18} \text{ kg} \end{aligned}$$

Clearly, the amount of mass transferred is negligibly small.

12. It is now believed that protons and neutrons are themselves built out to more elementary units called quarks. A proton and neutron consists of three quarks, the so called 'up' quark (denoted by u of charge $+(2/3)e$, and the 'down' quark (denoted by d) of charge $(-1/3)e$, together with electrons build up ordinary matter. Suggest a positive quark composition of a proton and neutron.

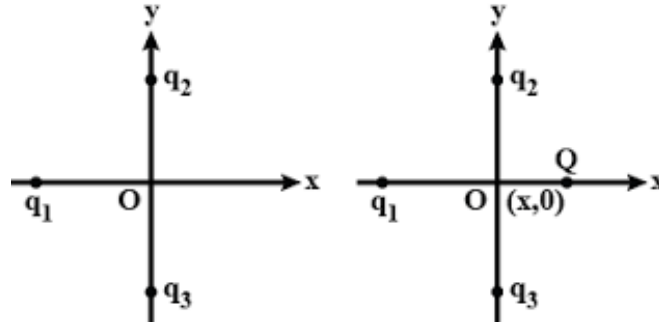
Sol. The net charge on proton is $+e$. This can be obtained by $p = (2/3 + 2/3 - 1/3)e = +e$
Hence, proton has two 'up' quark and a 'down' quark'.

The net charge on neutron is 0. This can be obtained by $n = (2/3 - 1/3 - 1/3) = 0$
Hence, neutron has one 'up' quark and two 'down' quarks.

B. COULOMB'S LAW

(1 Mark Questions)

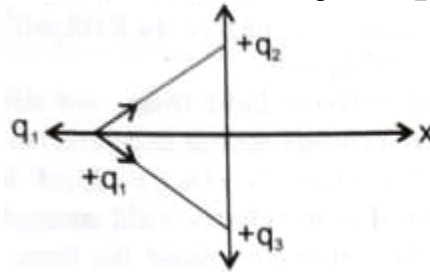
1. In Fig., two positive charges q_2 and q_3 fixed along the y axis, exert a net electric force in the + x direction on a charge q_1 fixed along the x axis. If a positive charge Q is added at $(x, 0)$, the force on q_1 .



- (a) shall increase along the positive x-axis. (b) shall decrease along the positive x-axis.
 (c) shall point along the negative x-axis.
 (d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3 .

Sol.

(a) The force on q_1 depend on the force acting between q_1 and q_2 and q_1 and q_3 so that the net force acting on q_1 by q_2 and q_1 by q_3 is along the +x direction so the force acting between q_1 , q_2 and q_1 , q_3 is attractive force as shown in figure.



The attractive force between these charges states that q_1 is a negative charge (since q_2 and q_3 are positive)

Then the force acting between q_1 and charge Q (positive) is also known as attractive force and then the net force on q_1 by q_2 , q_3 and Q are along the same direction as shown in the figure.

2. Two identical conducting balls A and B have charges $-Q$ and $+3Q$ respectively. They are brought in contact with each other and then separated by a distance d apart. Find the nature of the Coulomb force between them.

Sol. Final charge on each ball = $\frac{q_A + q_B}{2} = \frac{-Q + 3Q}{2} = +Q$

As both the balls have same nature of charges, hence nature of the Coulomb force is repulsive.

3. Two equal balls having equal positive charge 'q' coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two?

Sol. As in air, $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$

In medium, $F' = \frac{1}{4\pi\epsilon_{0K}} \frac{q^2}{r^2}$

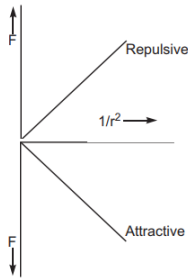
$\therefore F' = F/K$ where K is dielectric constant of materials
Hence, the force is reduced, when a plastic sheet is inserted.

(2 Marks Questions)

4. Two point charges repel each other with a force F when placed in water of dielectric constant 81. What will be the force between them when placed the same distance apart in air?

Sol. $F_w = F_{air}/k$
 $F_{air} = k.F_w = 81F = 81F$

5. Plot a graph showing the variation of coulomb force (F) versus ($1/r^2$), where r is the distance between the two charges of each pair of charges: ($1\mu\text{C}, 2\mu\text{C}$) and ($2\mu\text{C}, -3\mu\text{C}$), interpret the graphs obtained.



Sol.

(i) Pair ($1\mu\text{C}, 2\mu\text{C}$): From upper graph it is clear that the force of repulsion increases with the reducing between two charges.

(ii) Pair ($2\mu\text{C}, -3\mu\text{C}$): From lower graph it is clear that the force of attraction increases as the distance between two charges reduces.

6. What is the force between two small charged spheres having charges of $2 \times 10^{-7}\text{C}$ and $3 \times 10^{-7}\text{C}$ placed 30 cm apart in the air?

Sol. $q_1 = 2 \times 10^{-7}\text{C}, q_2 = 3 \times 10^{-7}\text{C}, r = 30\text{cm} = 0.3\text{m}$

From coulomb's, in air

$$F = \frac{1}{4\pi\epsilon_0} \times \frac{q_1 q_2}{r^2}$$

$$= 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.3)^2}$$

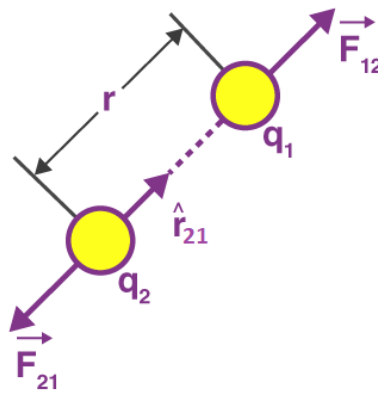
$$= 6 \times 10^{-3}\text{N.}$$

(3 Marks questions)

7. State Coulomb's law of force between two electric charges and state its limitations. Also define the SI unit of electric charge.

8. Write Coulomb's law in vector form. What is the importance of expressing it in vector form?

Sol. As shown in figure, consider two positive point charges q_1 and q_2 placed in vacuum at distance r from each other. They repel each other.



In vector form, Coulomb's law may be expressed as \vec{F}_{21} = Force on charge q_2 due to q_1
 $= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12}$ where $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$, is a unit vector in the direction from q_1 to q_2

Similarly, \vec{F}_{12} = Force on charge q_1 due to q_2

$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21}$ where $\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$, is a unit vector in the direction from q_2 to q_1 .

Importance of vector form: The vector form of Coulomb's law gives the following additional information:

1. As $\hat{r}_{21} = -\hat{r}_{12}$, therefore $\vec{F}_{21} = -\vec{F}_{12}$.

This means that two charges exert equal and opposite forces on each other. So Coulombian forces obey Third law of motion.

2. As the Coulombian forces act along \vec{F}_{12} or \vec{F}_{21} , i.e., along the line joining the centres of two charges, so they are central forces.

9. The electrostatic force on a small sphere of charge $0.4 \mu\text{C}$ due to another small sphere of charge $-0.8 \mu\text{C}$ in the air is 0.2 N .
 (a) What is the distance between the two spheres?
 (b) What is the force on the second sphere due to the first?

Sol. Using Coulomb's Law,

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$r^2 = 9 \times 10^9 \times \frac{0.4 \times 0.8 \times 10^{-12}}{0.2}$$

$$r^2 = 14.4 \times 10^{-3} = 144 \times 10^{-4} \quad r = 12 \times 10^{-2} = 0.12 \text{ m}$$

(b) Force on second sphere due to first is 0.2N but in opposite direction, because the charges are due to same and distance is same; so force is also same.

10. Check that the ratio ke^2/Gm_em_p is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Sol. The given ratio is ke^2/Gm_em_p . Where, G = Gravitational constant. Its unit is $\text{N m}^2 \text{kg}^{-2}$, m_e and m_p = Masses of electron and proton and their unit is kg. e = Electric charge. Its unit is C.

$$k = \frac{1}{4\pi\epsilon_0} \text{ and its unit is } \text{N m}^2 \text{C}^{-2}.$$

Therefore, unit of the given ratio

$$\frac{ke^2}{Gm_em_p} = \frac{[\text{Nm}^2\text{C}^{-2}][\text{C}^{-2}]}{[\text{Nm}^2\text{kg}^{-2}][\text{kg}][\text{kg}]} = M^0L^0T^0$$

Hence, the given ratio is dimensionless.

$$e = 1.6 \times 10^{-19} \text{ C}$$

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$m_p = 1.66 \times 10^{-27} \text{ kg}$$

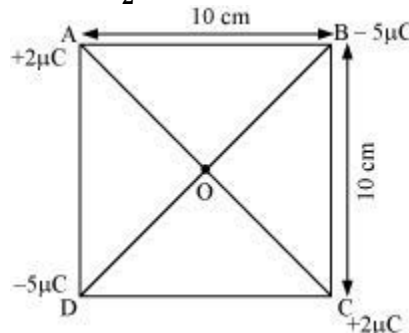
Hence, the numerical value of the given ratio is

$$\frac{ke^2}{Gm_em_p} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{6.67 \times 10^{-11} \times 9.1 \times 10^{-31} \times 1.67 \times 10^{-27}} \approx 2.3 \times 10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

11. **Four point charges** $q_A = 2 \mu\text{C}$, $q_B = -5 \mu\text{C}$, $q_C = 2 \mu\text{C}$, and $q_D = -5 \mu\text{C}$ **are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 μC placed at the centre of the square?**

Sol. Here $OA = OB = OC = OD = \frac{\sqrt{10^2+10^2}}{2} = 5\sqrt{2} \text{ cm} = 5\sqrt{2} \times 10^{-2} \text{ m}$



Force exerted on the charge of 1 μC located at the centres are

$$\vec{F}_A = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 1 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} = 3.6 \text{ N, along } \vec{OC}$$

$$\vec{F}_B = \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 1 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} = 9 \text{ N, along } \vec{OB}$$

$$\vec{F}_C = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 1 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} = 3.6 \text{ N, along } O\vec{A}$$

$$\vec{F}_D = \frac{9 \times 10^9 \times 5 \times 10^{-6} \times 1 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2} = 9 \text{ N, along } O\vec{D}$$

Clearly, $\vec{F}_C = -\vec{F}_A$ and $\vec{F}_D = -\vec{F}_B$

Hence total force on $1 \mu\text{C}$ charge is

$$\begin{aligned} \vec{F} &= \vec{F}_A + \vec{F}_B + \vec{F}_C + \vec{F}_D \\ &= \vec{F}_A + \vec{F}_B - \vec{F}_A - \vec{F}_B = \text{zero N} \end{aligned}$$

12. (i) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is $6.5 \times 10^{-7} \text{ C}$ each? The radii of A and B are negligible compared to the distance of separation.
 (ii) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Sol. Here $q_1 = 6.5 \times 10^{-7} \text{ C}$, $q_2 = 6.5 \times 10^{-7} \text{ C}$, $r = 50 \text{ cm} = 0.50 \text{ m}$

$$\begin{aligned} \text{Using Coulomb's law, } f_{\text{air}} &= k \frac{q_1 q_2}{r^2} = 9 \times 10^9 \cdot \frac{6.5 \times 10^{-7} \times 6.5 \times 10^{-7}}{(0.50)^2} \text{ N} \\ &= 1.5 \times 10^{-2} \text{ N} \end{aligned}$$

$$\text{The mutual gravitational attraction, } F_G = G \frac{m_1 m_2}{R^2} = \frac{6.67 \times 10^{-11} \times 0.5 \times 0.5}{(0.5)^2} = 6.67 \times 10^{-11} \text{ N}$$

Clearly, $F_G \ll F_{\text{air}}$.

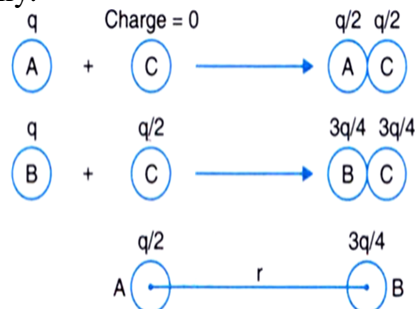
(ii) When charge on air on each sphere is doubled, and the distance between them is halved, the force of repulsion becomes

$$F_{\text{air}} = k \frac{2q_1 \cdot 2q_2}{\left(\frac{r}{2}\right)^2} = 16k \frac{q_1 q_2}{r^2}$$

13. Suppose the spheres A and B in Exercise 12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Sol. Charge on each of the spheres A and B is, $q = 6.5 \times 10^{-7} \text{ C}$

When similar but uncharged sphere C is placed in contact with sphere A, each sphere shares a charge $q/2$ equally.



Now, when the sphere C (with charge $q/2$) is placed in contact with sphere B (with charge q), the charge is redistributed equally, so that

$$\text{Charge on sphere B or C} = \frac{1}{2} \left(q + \frac{q}{2} \right) = \frac{3q}{4}$$

∴ New force of repulsion between A and B is

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{3q \cdot q}{r^2}$$

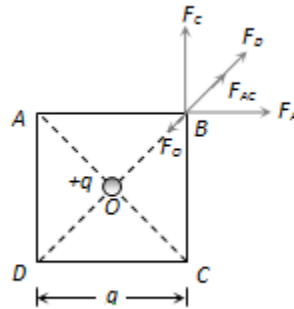
$$= \frac{3}{8} \times 1.5 \times 10^{-2} \text{N} = 0.5625 \times 10^{-2} \text{N}$$

$$= 5.7 \times 10^{-3} \text{N}$$

(5 Marks Questions)

14. Four equal point charges each of $16 \mu\text{C}$ are placed on the four corners of a square of side 0.2m . Calculate the force on any of the charges.

Sol. As shown in figure, suppose the four charges are placed at the corners of the square ABCD. Let us calculate the total force on q_4 .



Here $AB = BC = CD = AD = 0.2\text{m}$

$$q_1 = q_2 = q_3 = q_4 = 16\mu\text{C} = 16 \times 10^{-6} \text{C}$$

Force exerted on q_4 by q_1 is, $F_1 = \frac{9 \times 10^9 \times 16 \times 10^{-6} \times 16 \times 10^{-6}}{(0.2)^2} = 57.6\text{N}$, along AD produced

Force exerted on q_4 by q_2 is $F_2 = \frac{9 \times 10^9 \times 16 \times 10^{-6} \times 16 \times 10^{-6}}{(0.2)^2 + (0.2)^2} = 28.8\text{N}$, along BD produced

Force exerted on q_4 by q_3 is $F_3 = \frac{9 \times 10^9 \times 16 \times 10^{-6} \times 16 \times 10^{-6}}{(0.2)^2} = 57.6\text{N}$, along CD produced

As F_1 and F_3 are perpendicular to each other, so their resultant force is

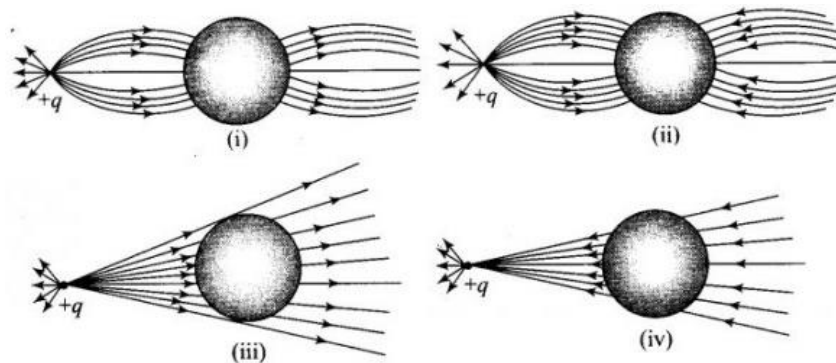
$$F' = \sqrt{F_1^2 + F_3^2} = \sqrt{57.6^2 + 57.6^2} = 57.6\sqrt{2} = 81.5\text{N}$$
, in the direction of F_2 .

Hence total force on q_4 is $F = F_2 + F' = 28.8 + 81.5 = 110.3\text{N}$ along BD produced.

15. Two identical point charges, q each, are kept 2m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q .

Sol. Let us suppose that the third charge ' Q ' is placed on the line joining the first and second charge such that $AO = x$ and $OB = (2 - x)$.

Net force on each of the three charges must be zero for the system of charges to be in equilibrium.



(a) Fig (i)

(b) Fig (ii)

(c) Fig (iii)

(d) Fig (iv)

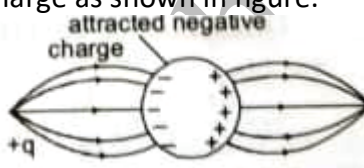
Sol.

(a)

If a positive point charge is brought near an isolated conducting sphere without the sphere, then the free electrons in the sphere are attracted towards the positive charge and electric field passes through a charged body. This leaves an excess of positive charge on the (right) surface of sphere due to the induction process.

Both type of charges are bound in the (isolated conducting) sphere and cannot escape. They, therefore, reside on the surface.

Thus the left surface has an excess of negative charge and the right surface of sphere has an excess of positive charge as shown in figure.

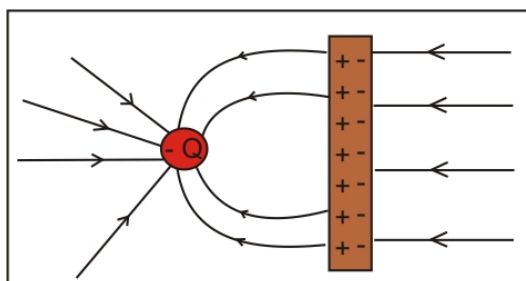


An electric field lines start from positive charge and ends at negative charge.

Also electric field line emerges from a positive charge, in case of single charge and ends at infinity shown in figure (a)

2. Draw the pattern of electric field lines when a point charge $+q$ is kept near an unchanged conducting plate.

Sol.



3. Why do the electrostatic field lines not form closed loops?

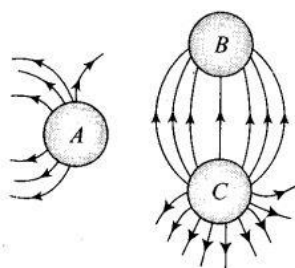
Sol. Because the direction of an electric field is from positive to negative charge. So, one can regard a line of force starting from a positive charge and ending on a negative charge. This indicates that an electric line of force cannot form closed loops.

4. Why do the electric field lines never cross each other?

Sol. In case two electric lines of forces intersect at a point, then two tangents may be drawn at the point of intersection which means two different directions of the electric field at a point. As this is not possible, lines of forces do not intersect.

(2 Marks Questions)

5. Figure shows the electric field lines around three point charges A, B and C.



(i) Which charges are positive? (ii) Which charge has the largest magnitude?

Why?

(iii) In which region or regions of the picture could the electric field be zero? Justify your answer.

(a) Near A

(b) Near B

(c) Near C

(d) Nowhere

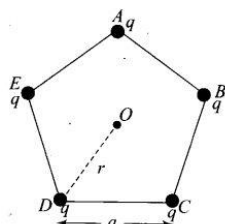
Sol. (i) Charges A and C are positive since lines of force emanate from them.

(ii) Charge C has the largest magnitude since maximum number of field lines are associated with it.

(iii) (a) near A. There is no neutral point between a positive and a negative charge. A neutral point may exist between two like charges. From the figure we see that a neutral point exists between charges A and C. Also between two like charges the neutral point is closer to the charge with smaller magnitude. Thus, electric field is zero near charge A.

(3 Marks questions)

6. Five charges, q each are placed at the corners of a regular pentagon of side a .



- (a) (i) What will be the electric field at O, the centre of the pentagon?
 (ii) What will be the electric field at O if the charge from one of the corners (say A) is removed?
 (iii) What will be the electric field at O if the charge q at A is replaced by $-q$?
 (b) How would your answer to (a) be affected if pentagon is replaced by n -sided regular polygon with charge q at each of its corners?

Sol. (a) (i) zero (ii) $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ along \overline{OA} (iii) $\frac{1}{4\pi\epsilon_0} \frac{2q}{r^2}$ along \overline{OA}

(b) Same as (a)

7. Two point charges of $+16\mu\text{C}$ and $-9\mu\text{C}$ are placed 8cm apart in air. Determine the position of the point at which the resultant field is zero.

Sol. Let P be the point at distance x from A, where the net field is zero.

At point P, $E_1 + E_2 = 0$

$$\frac{k \times 16 \times 10^{-6}}{(x \times 10^{-2})^2} + \frac{k \times (-9) \times 10^{-6}}{[(8-x) \times 10^{-2}]^2} = 0$$

$$\text{Or } \frac{16}{x^2} = \frac{9}{(8-x)^2}$$

$$\text{Or } \frac{4}{x} = \pm \frac{3}{8-x}$$

$$\text{Or } x = 32/7 \text{ cm, } 32 \text{ cm}$$

At $x = 32/7$ cm, both E_1 and E_2 will be in the same direction, therefore, net electric field cannot be zero. Hence $x = 32$ cm, i.e. electric field is zero at a point 24cm to the right of $-9\mu\text{C}$ charge.

8. A particle of charge $2\mu\text{C}$ and mass 1.6g is moving with a velocity $4\hat{i} \text{ ms}^{-1}$. At $t = 0$ the particle enters in a region having an electric field \vec{E} (in NC^{-1}) $= 80\hat{i} + 60\hat{j}$. Find the velocity of the particle at $t = 5$ s.

Sol. Given $q = 2\mu\text{C}$, $m = 1.6\text{g} = 1.6 \times 10^{-3} \text{ kg}$, $u = 4\hat{i} \text{ ms}^{-1}$, $E = 80\hat{i} + 60\hat{j}$ and $t = 5$ s

$F = m\vec{a}$ (From Newton's law)

$$\Rightarrow 2 \times 10^{-6}(80\hat{i} + 60\hat{j}) = (1.6 \times 10^{-3}) \vec{a}$$

$$\Rightarrow \vec{a} = 100 \times 10^{-3}\hat{i} + 75 \times 10^{-3} \hat{j}$$

Now, from equations of motion,

$$\vec{v} = \vec{u} + \vec{a}t = 4\hat{i} + (10^{-3}\hat{i} + 75 \times 10^{-3} \hat{j})5$$

$$= 4.5\hat{i} + 0.375\hat{j}$$

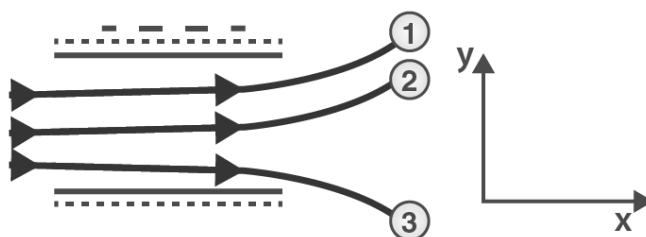
9. Two electric field lines cannot cross each other. Also, they cannot form closed loops. Give reasons.

Sol. Electric field lines do not form closed loops due to conservative nature of electric field. At the point of intersection of two field lines, there will be two directions for the resultant electric field. This is not acceptable.

10. (i) An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?
 (ii) Explain why two field lines never cross each other at any point.

Sol. An electrostatic field line is a continuous curve because a charge experiences a continuous force when placed in an electrostatic field. The field line cannot have sudden breaks because the charge moves continuously and does not jump from one point to another.

11. **The figure below shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?**



Sol. Particles 1 and 2 have negative charges because they are being deflected towards the positive plate of the electrostatic field.

Particle 3 has positive charge because it is being deflected towards the negative plate.

Acceleration acting on charge q in y -direction, $a = \frac{F}{m} = \frac{qE}{m}$

Therefore deflection of charged particle in time t in y direction.

$$h = 0 \times t + \frac{1}{2} at^2 = \frac{1}{2} at^2 = \frac{1}{2} \frac{qE}{m} t^2$$

i.e., $h \propto \frac{q}{m}$

As the particle 3 suffers maximum deflection in y direction, so it has highest charge to mass (q/m) ratio.

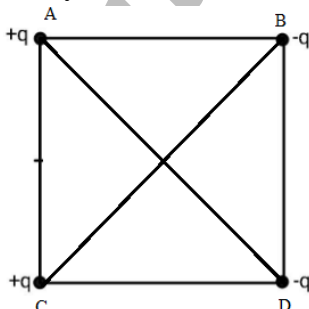
12. An oil drop of 12 excess electrons is held stationary under a constant electric field of $2.55 \times 10^4 \text{ Vm}^{-1}$ in Millikan's oil drop experiment. The density of the oil is 126 g cm^{-3} . Estimate the radius of the drop ($g = 9.81 \text{ ms}^{-2}$; $e = 1.60 \times 10^{-19} \text{ C}$)

Sol. Total charge on the oil drop = $12e$
 $= 12 \times 1.6 \times 10^{-19} \text{ C}$
 $= 1.92 \times 10^{-18} \text{ C}$
 Density of oil = $1.26 \text{ gm/cm}^3 = 1.26 \times 10^3 \text{ kg/m}^3$
 Mass of oil drop = $V \times \rho = \frac{4}{3} \pi r^3 \rho$
 $E = 2.55 \times 10^4 \text{ V/m}$
 $F_e = (12e)E$
 $= 1.92 \times 10^{-18} \times 2.55 \times 10^4 \text{ N}$
 Gravitational force on the oil drop = $F_g = F_e$
 $= \frac{4}{3} \pi r^3 \rho g = 1.92 \times 2.55 \times 10^{-14}$
 $r = \left[\frac{3 \times 1.92 \times 2.55 \times 10^{-14}}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.8} \right]^{1/3}$
 $r = 9.8 \times 10^{-7} \text{ m}$

(5 Marks Questions)

13. Four charges $+q, +q, -q, -q$ are placed respectively at the four corners A, B, C and D of a square of side 'a'. Calculate the electric field at the centre of the square.

Sol. Let E_A, E_B, E_C and E_D be the electric fields at the centre O of the square due to the charges A, B, C and D respectively. Their directions are shown in figure.



Since all the charge are of equal magnitude and at the same distance from the centre O so

$$E_A = E_B = E_C = E_D = k \frac{q}{r^2} = \frac{q}{\left(\frac{a}{\sqrt{2}}\right)^2} = 2 \frac{kq}{a^2} [\because r^2 + r^2 = a^2]$$

Because E_A and E_C act in the same direction, so their resultant is

$$E_1 = E_A + E_C = \frac{2kq}{a^2} + \frac{2kq}{a^2} = \frac{4kq}{a^2}$$

Similarly resultant E_B and E_D is

$$E_2 = E_B + E_D = \frac{4kq}{a^2}$$

$$\text{Now the resultant of } E_1 \text{ and } E_2 \text{ will be } E = \sqrt{E_1^2 + E_2^2} = \sqrt{\left(\frac{4kq}{a^2}\right)^2 + \left(\frac{4kq}{a^2}\right)^2} = 4\sqrt{2} k \frac{q}{a^2},$$

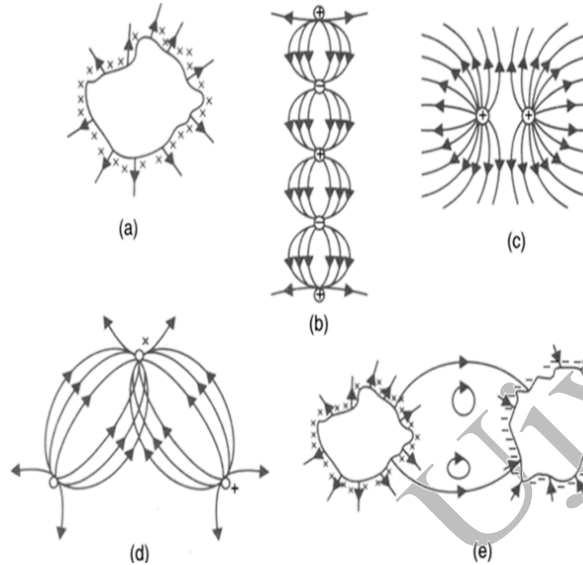
directed parallel to AD or BC

$$\cos \beta = E_1/E = 1/\sqrt{2}$$

$$\therefore \beta = 45^\circ$$

i.e., the resultant field is inclined at an angle of 45° with AC.

14. Which among the curves shown in Figure cannot possibly represent electrostatic field lines?



- Sol. Only figure (c) is right and the remaining figures cannot represent the electrostatic field lines.

Figure (a) is wrong because field lines must be normal to a conductor.

Figure (b) is wrong because lines of force cannot start from a negative charge.

Figure (c) is right because it satisfies all the properties of lines of force.

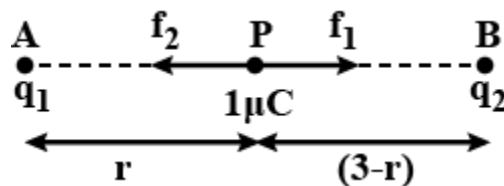
Figure (d) is wrong because lines of force cannot intersect each other.

Figure (e) is wrong because electrostatic field lines cannot form closed loops.

15. Two point charges of $+1 \mu\text{C}$ and $+4\mu\text{C}$ are kept 30cm apart. How far from the $+1\mu\text{C}$ charge on the line joining the two charges, will the net electric field be zero?

- Sol. Let the point P, the net electric field is zero, then

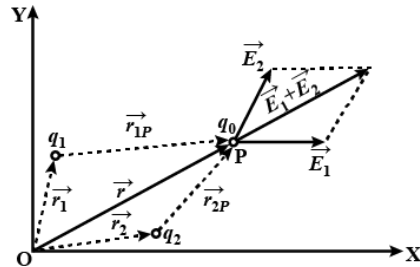
$$\frac{1}{4\pi\epsilon_0} \frac{q_1}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{(30-x)^2}$$



$$\frac{1}{x^2} = \frac{4}{(30-x)^2} \Rightarrow x = 10\text{cm}$$

16. Consider a system of n charges q_1, q_2, \dots, q_n with position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3, \dots, \vec{r}_n$ relative to some origin 'O'. Deduce the expression for the net electric field \vec{E} at a point P with position vector \vec{r}_P due to this system of charges.

Sol. According to Coulomb's law, the force on charge test q_0 due to charge q_1 is



Where \hat{r}_{1P} is a unit vector in the direction from q_1 to P and r_{1P} is the distance between q_1 and P. Hence the electric field at point P due to charge q_1 is

$$E_1 = \frac{\vec{F}_1}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_{1P}^2} \hat{r}_{1P}$$

Similarly electric field at P due to charge q_2 is

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{2P}^2} \hat{r}_{2P}$$

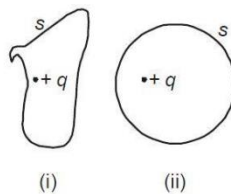
If \vec{E} is the electric field at point p due to the system of charges, then by the principle of superposition of electric fields.

$$\begin{aligned} \vec{E} &= \vec{E}_1 + \vec{E}_2 + \dots + \vec{E}_n \\ &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{r_{1P}^2} \hat{r}_{1P} + \frac{q_2}{r_{2P}^2} \hat{r}_{2P} + \dots + \frac{q_n}{r_{nP}^2} \hat{r}_{nP} \right] \\ \text{Or } \vec{E} &= \frac{1}{4\pi\epsilon_0} \sum_{i=1}^n \frac{q_i}{r_i^2} \hat{r}_{iP} \end{aligned}$$

D. ELECTRIC FLUX AND GAUSS THEOREM

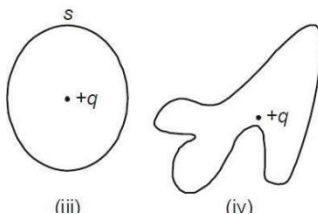
(1 Mark Questions)

1. The Electric flux through the surface



(i)

(ii)



(iii)

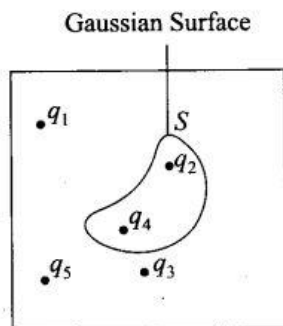
(iv)

- (a) Fig. (iv) is the largest. (b) in Fig. (iii) is the least.
 (c) in Fig. (ii) is same as Fig.(iii) but is smaller than Fig. (iv)
 (d) is the same for all the figures.

Sol. (d)

By Gauss Law: The total of the electric flux out of a closed surface is equal to the charge enclosed to the charge enclosed divided by the permittivity i.e., $\phi = Q/\epsilon_0$. Thus electric flux through a surface doesn't depend on the shape, size of area of a surface but it depends on the no. of charges enclosed by the surface. So all the given figures have same electric flux as all of them also have same single positive charge.

2. Five charges q_1 , q_2 , q_3 , q_4 , and q_5 are fixed at their positions as shown in Fig. S is a Gaussian surface. The Gauss's law is given by : $\oiint_s \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$



Which of the following statements is correct?

- (a) E on the LHS of the above equation will have a contribution from q_1 , q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only.
 (b) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.
 (c) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1 , q_3 and q_5 only.
 (d) Both E on the LHS and q on the RHS will have contributions from q_2 and q_4 only.

Sol. (b)

Gauss law states that total electric flux of an enclosed surface is given by

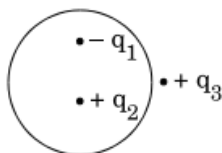
$$\oiint_s \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

includes the sum of all charges enclosed by the surface.

The charge may be located anywhere inside the surface, and outside the surface. Then, the electric field on the left side of equation is due to all the charges, both inside and outside S.

So, E on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

3. Electric flux through a spherical surface shown in the figure, is _____



Sol. $\phi = \frac{q_{in}}{\epsilon_0} = \left(\frac{q_2 - q_1}{\epsilon_0} \right)$

4. A point charge is placed at the centre of a hollow conducting sphere of internal radius 'r' and outer radius '2r'. The ratio of the surface charge density of the inner surface to that of the outer surface will be _____.

Sol - 4: 1.

5. If the net electric flux through a closed surface is zero, then we can infer
- no net charge is enclosed by the surface
 - uniform electric field exists within the surface
 - electric potential varies from point to point inside the surface.
 - charge is present inside the surface

Sol. (b), (c)

6. The electric flux through a closed Gaussian surface depends upon
- net charge enclosed and permittivity of the medium
 - net charge enclosed, permittivity of the medium and the size of the Gaussian surface.
 - net charge enclosed only
 - permittivity of the medium only

Sol. (a)

7. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased?

Sol. According to Gauss's law, the electric flux is given by

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{enclosed}}{\epsilon_0}$$

When radius of spherical Gaussian surface is increased, its surface area will be increased but the charge enclosed in the sphere remains same. Hence there will be no change in the electric flux.

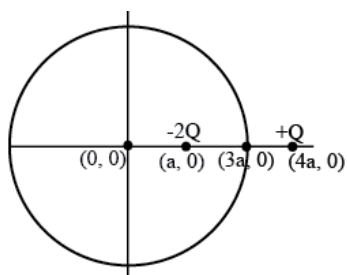
8. A charge 'q' is placed at the centre of a cube of side l . What is the electric flux passing through each face of the cube?

Sol. By Gauss's theorem, total flux through whole of the cube, $\phi = \frac{q}{\epsilon_0}$ where q is the total charge enclosed by the cube. As charge is at centre, therefore electric flux is symmetrically distributed on all 6 faces.

9. Does the charge given to a metallic sphere depend on whether it is hollow or solid. Give reason for your answer.

Sol. No the charge given to a metallic sphere does not depend on whether it is hollow or solid because all the charges will move to the outer surface of the sphere. Charge will be distributed uniformly over the surface of the surface.

10. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$ respectively. What is the electric flux due to these charges through a sphere of radius ' $3a$ ' with its centre at the origin?



Sol.

$$\text{Electric flux } \phi = \frac{q_{\text{inside}}}{\epsilon_0} = \frac{-2Q}{\epsilon_0}$$

(2 Marks Questions)

11. (i) Define the term 'electric flux'? Write its SI unit.
 (ii) What is the flux due to electric field $\vec{E} = 3 \times 10^3 \hat{i}$ N/C through a square of side 10cm, when it is held normal to \vec{E} ?

Sol. (i) Electric flux: Total number of electric field lines crossing a surface normally is called electric flux. SI unit of electric flux is $\text{N m}^2 \text{C}^{-1}$.

(ii) The area of the surface can be represented as a vector along normal to the surface.

$$\text{Here } \vec{E} = 3 \times 10^3 \hat{i} \text{ NC}^{-1}.$$

$$\text{Area of the square} = \Delta S = 10 \times 10 \text{ cm}^2$$

$$\Delta S = 10^{-2} \hat{i} \text{ m}^2$$

$$\text{Since normal to the square is along x axis, we have } \Delta S = 10^{-2} \hat{i} \text{ m}^2$$

$$\text{Electric flux through the square, } \phi = \vec{E} \cdot \Delta \vec{S} = (3 \times 10^3 \hat{i}) \cdot (10^{-2} \hat{i})$$

$$\phi = 30 \text{ Nm}^2 \text{C}^{-1}.$$

12. Given a uniform electric field $\vec{E} = 5 \times 10^3 \hat{i}$ N/C. Find the flux of this field through a square of 10cm on a side whose plane is parallel to the y-z plane. What would be the flux through the same square if the plane makes a 30° angle with the x-axis?

Sol. Here $\vec{E} = 5 \times 10^3 \hat{i}$ N/C, side of square, $a = 10\text{cm} = 0.1\text{m}$.

$$\text{Area of square, } S = a^2 = (0.1)^2 = 0.01\text{m}^2$$

Case I: Area vector is along x axis, $S = 0.01\hat{i}\text{m}^2$

$$\text{Required flux, } \phi = \vec{E} \cdot \vec{S}$$

$$\Rightarrow \phi = (5 \times 10^3 \hat{i}) \cdot (0.01 \hat{i}) \Rightarrow \phi = 50 \text{ Nm}^2/\text{C}$$

Case II: Plane of square makes a 30° angle with the x axis.

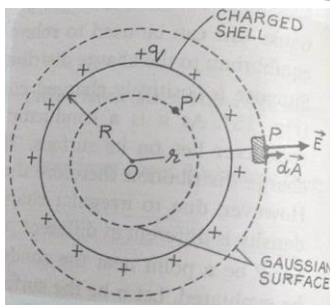
Here angle between area vector and the electric field is 60°

So, required flux $\phi' = E.S \cos\theta$

$$= (5 \times 10^3)(10^{-2})\cos 60^\circ = 25 \text{ N}^2/\text{C}$$

13. Apply Gauss's law to show that for a charged spherical shell, the electric field outside the shell is, as if the entire charge were concentrated at the centre.

Sol. Using Gauss's theorem at point P



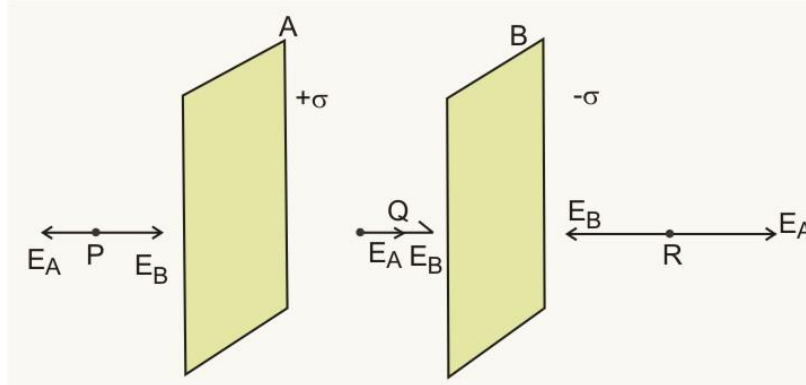
$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0} \quad (\because E \text{ is constant throughout the surface})$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

This is the same as electric field due to a point charge which can be assumed to be concentrated at the centre.

14. Two large parallel plane sheets have uniform charge densities $+\sigma$ and $-\sigma$. Determine the charge electric field (i) between the sheets, and (ii) outside the sheets.

Sol. Let us consider the two parallel sheets A and B of charges $+\sigma$ and $-\sigma$ and the points P, Q, R.



Now, the electric field due to the plane sheet of charge is given as

$$E = \frac{\sigma}{2\epsilon_0}$$

Where, σ is the surface charge density.

Such that,

$$E_A = +\frac{\sigma}{2\epsilon_0} \quad \text{and} \quad E_B = -\frac{\sigma}{2\epsilon_0}$$

then,

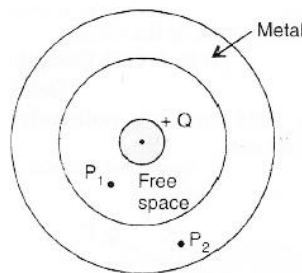
(i) The electric field between the sheets at point Q is ;

$$\begin{aligned} \vec{E}_\sigma &= \vec{E}_A - \vec{E}_B \\ &= \frac{\sigma}{2\epsilon_0} - \left(-\frac{\sigma}{2\epsilon_0}\right) \\ &= \frac{2}{2} \left(\frac{\sigma}{\epsilon_0}\right) \\ \vec{E} &= \frac{\sigma}{\epsilon_0} \end{aligned}$$

(ii) The electric field outside the sheets at point P or R

$$\begin{aligned} \vec{E} &= \vec{E}_A + \vec{E}_B \\ \vec{E} &= \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} \\ \vec{E} &= 0 \end{aligned}$$

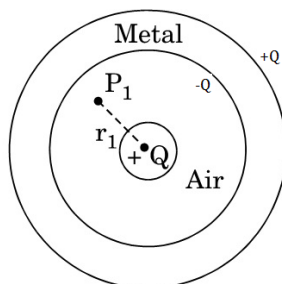
15. A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2 .



Sol. Using Gauss's theorem, electric field at P_1 , $E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1^2} = 0$

Again at field P_2 , $E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{r_2^2} = 0$

Because electric field inside a conductor is zero.



16. An infinite line charge produces a field of $9 \times 10^4 \text{ N/C}$ at a distance of 2 cm. Calculate the linear charge density. [Ans. $0.1 \mu\text{Cm}^{-1}$]

Sol. Electric field produced by the infinite line charges at a distance d having linear charge density λ is given by the relation,

$$E = \frac{\lambda}{2\pi\epsilon_0 d}$$

$$\Rightarrow \lambda = 2\pi\epsilon_0 dE$$

Where, $d = 2 \text{ cm} = 0.02 \text{ m}$

$$E = 9 \times 10^4 \text{ N/C}$$

$\epsilon_0 =$ Permittivity of free space and $1/4\pi\epsilon_0 = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$ Therefore,

$$\lambda = \frac{0.02 \times 9 \times 10^4}{2 \times 9 \times 10^9}$$

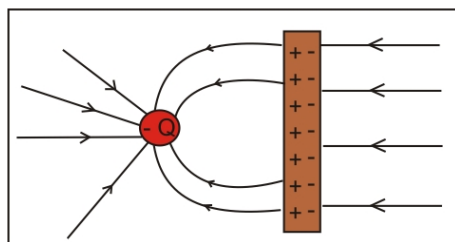
$$= 10 \mu\text{C/m}$$

Therefore, the linear charge density is $10 \mu\text{C/m}$.

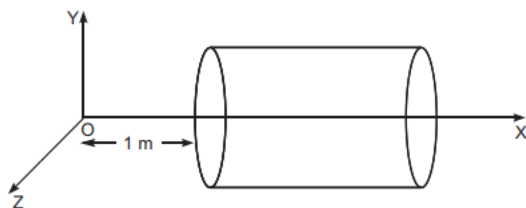
(3 Marks questions)

17. A point charge (+Q) is kept in the vicinity of an uncharged conducting plate. Sketch the electric field lines between the charge and the plate.

Sol.

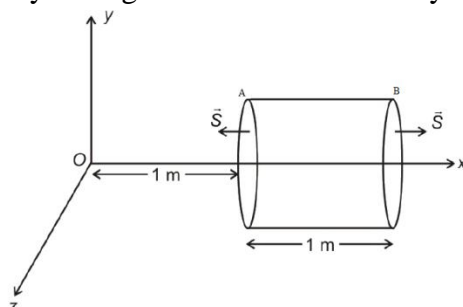


18. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\vec{E} = 50x\hat{i}$, where E is in NC^{-1} and x is in metres. Find



- (i) net flux through the cylinder (ii) charge enclosed by the cylinder.

- Sol. (i) Given $\vec{E} = 50x\hat{i}$ and $A = 25\text{cm}^2 = 25 \times 10^{-4}\text{m}^2$. As the electric field is only along the x axis, so flux will pass only through the cross section of cylinder.



Magnitude of electric field at cross section A, $E_A = 50 \times 1 = 50 \text{ NC}^{-1}$

Magnitude of electric field at cross section B, $E_B = 50 \times 2 = 100 \text{ NC}^{-1}$

The corresponding electric fluxes are:

$$\phi_A = \vec{E}_A \cdot \vec{A} = 50 \times 25 \times 10^{-4} \cos 180^\circ = -0.125 \text{ N m}^2 \text{ C}^{-1}$$

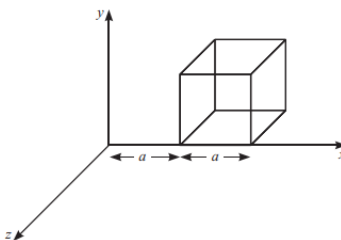
$$\phi_B = \vec{E}_B \cdot \vec{A} = 100 \times 25 \times 10^{-4} \cos 0^\circ = 0.25 \text{ N m}^2 \text{ C}^{-1}$$

So the net flux through the cylinder, $\phi = \phi_A + \phi_B = -0.125 + 0.25 = 0.125 \text{ N m}^2 \text{ C}^{-1}$.

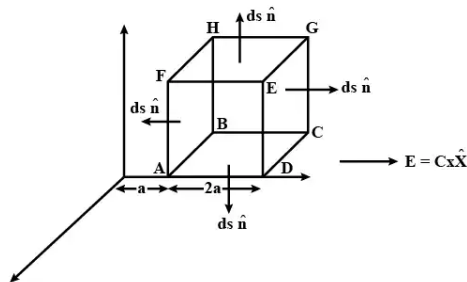
$$(ii) \text{ Using Gauss's law, } \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0} \Rightarrow 0.125 = \frac{q}{8.85 \times 10^{-12}}$$

$$\Rightarrow q = 8.85 \times 0.125 \times 10^{-12} = 1.1 \times 10^{-12} \text{ C}$$

19. State Gauss's law in electrostatic. A cube with each side 'a' is kept in an electric field given by $\vec{E} = Cx\hat{i}$ (as shown in figure) where C is a positive dimensional constant. Find out



- (i) the electric flux through the cube (ii) the net charge inside the cube.
- Sol. Gauss's law in electrostatics states that the total electric flux through a closed surface enclosing a charge is equal to $1/\epsilon_0$ times the magnitude of that charge

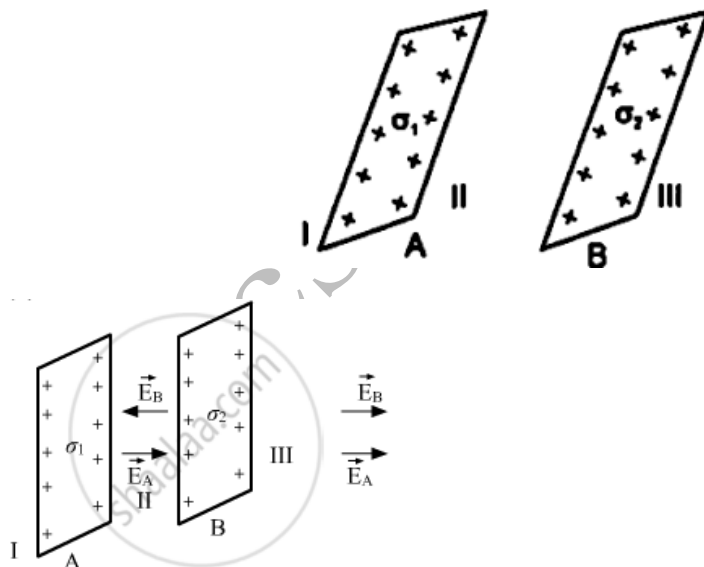


$$\phi = \oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

(i) Net flux, $\phi = \phi_1 + \phi_2$ where $\phi_1 = 2aC \, dS \cos 0^\circ = 2aC \times a^2 = 2a^3C$, $\phi_2 = aC \times a^2 \cos 180^\circ = -a^3C$, $\phi = 2a^3C + (-a^3C) = a^3C \, \text{Nm}^2\text{C}^{-1}$

(ii) Net charge (q) = $\epsilon_0 \times \phi = a^3C \, \epsilon_0$ coulomb

20. Two infinitely large plane thin parallel sheets having surface charge densities σ_1 and σ_2 ($\sigma_1 > \sigma_2$) are shown in the figure. Write the magnitudes and directions of the net electric fields in the regions marked II and III.



Sol.

In region II: The electric field due to the sheet of charge A will be from left to right (along the positive direction) and that due to the sheet of charge B will be from right to left (along the negative direction). Therefore in region II, we have

$$E = \frac{\sigma_1}{\epsilon_0} + \left(-\frac{\sigma_2}{\epsilon_0} \right)$$

$$\vec{E} = \frac{1}{\epsilon_0} (\sigma_1 - \sigma_2) \text{ along positive direction}$$

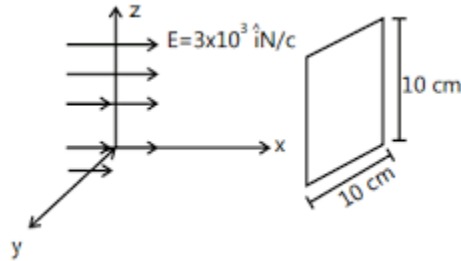
In region III: The electric fields due to both the charged sheets will be from left to right, i.e. along the positive direction. Therefore, in region III we have

$$E = \frac{\sigma_1}{\epsilon_0} + \frac{\sigma_2}{\epsilon_0}$$

$$\vec{E} = \frac{1}{\epsilon_0}(\sigma_1 + \sigma_2) \text{ along positive direction}$$

21. Consider a uniform electric field $E = 3 \times 10^3 \hat{i} \text{ N/C}$.
- (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the yz – plane?
- (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the x-axis?

Sol. Given electric field $\vec{E} = 3 \times 10^3 \hat{i} \text{ NC}^{-1}$. Magnitude of area, $S = 10 \text{ cm}^2 = 1 \times 10^{-3} \text{ m}^2$



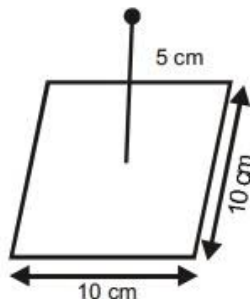
- (a) When the surface is parallel to y-z plane, the normal to plane is along x axis. In this case $\cos \theta = 1$; so electric flux, $\phi = \vec{E} \cdot \vec{S} = (3 \times 10^3 \hat{i}) \cdot (1 \times 10^{-3} \hat{i}) = 3 \text{ Nm}^2 \text{ C}^{-1}$.
- (b) In this case $\theta = 60^\circ$, so electric flux $\phi = \vec{E} \cdot \vec{S} \cos \theta = 3 \times 10^3 \times 1 \times 10^{-3} \cos 60^\circ = 3 \times 1/2 = 1.5 \text{ Nm}^2 \text{ C}^{-1}$.

22. What is the net flux of the uniform electric field of Question 21 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

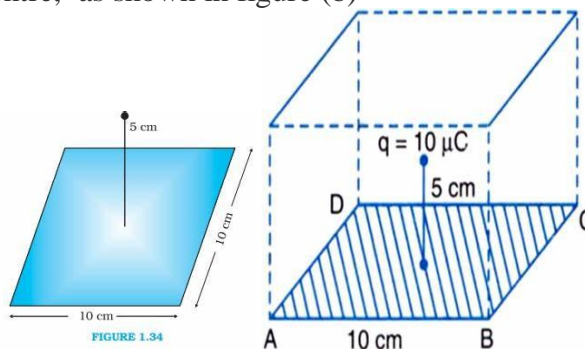
23. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is $8.0 \times 10^3 \text{ N m}^2 / \text{C}$.

- (a) What is the net charge inside the box?
- (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not? [Ans. $0.07 \mu\text{C}$, No]

24. A point charge $+10\ \mu\text{C}$ is at a distance $5\ \text{cm}$ directly above the centre of a square of side $10\ \text{cm}$, as shown in Fig. What is the magnitude of the electric flux through the square?



- Sol. We can imagine the square as face of a cube with edge $10\ \text{cm}$ and with the charge $+10\ \mu\text{C}$ placed at its centre, as shown in figure (b)



Symmetry of six faces of a cube about its centre ensures that the flux ϕ_s through each square face is same when the charge q is placed at the centre.

Therefore total flux, $\phi_E = 6 \times \phi_s = \frac{q}{\epsilon_0}$

$$\text{Or } \phi_s = \frac{q}{\epsilon_0} = \frac{1}{6} \times 10 \times 10^{-6} \times 4\pi \times 9 \times 10^9 = 1.88 \times 10^5 \text{ Nm}^2\text{C}^{-1}.$$

25. A point charge of $2.0\ \mu\text{C}$ is at the centre of a cubic Gaussian surface $9.0\ \text{cm}$ on edge. What is the net electric flux through the surface?

- Sol. Here $q = 2.0\ \mu\text{C} = 2.0 \times 10^{-6}\text{C}$, $\epsilon_0 = 8.85 \times 10^{12}\text{C}^2\text{N}^{-1}\text{m}^{-2}$

By Gauss' theorem, electric flux is

$$\phi_E = \frac{q}{\epsilon_0} = \frac{2.0 \times 10^{-6}}{8.85 \times 10^{-12}} = 2.26 \times 10^5 \text{ Nm}^2 \text{ C}^{-1}.$$

26. A point charge causes an electric flux of $-1.0 \times 10^3 \text{ N m}^2/\text{C}$ to pass through a spherical Gaussian surface of $10.0\ \text{cm}$ radius centred on the charge.

(a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?

(b) What is the value of the point charge?

- Sol. (i) $\phi_E = -1.0 \times 10^3 \text{ Nm}^2\text{C}^{-1}$ because the charge enclosed is the same in both the cases.

(ii) Charge, $q = \epsilon_0 \phi_E$

$$\begin{aligned} &= \frac{1}{4\pi \times 9 \times 10^9} \times (-1.0 \times 10^3) \\ &= -8.84 \times 10^{-9}\text{C} = 8.84 \text{ nC} \end{aligned}$$

27. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is $1.5 \times 10^3 \text{ N/C}$ and points radially inward, what is the net charge on the sphere?

Sol. Electric field intensity (E) at a distance (d) from the centre of a sphere containing net charge q is given by the relation,

$$E = q/4\pi\epsilon_0 d^2$$

Where, q = Net charge = $1.5 \times 10^3 \text{ N/C}$

$$d = 20 \text{ cm} = 0.2 \text{ m}$$

$$\text{And, } 1/4\pi\epsilon_0 = 9 \times 10^9 \text{ Nm}^2\text{C}^{-2}$$

$$\therefore q = E(4\pi\epsilon_0)d^2$$

$$= 1.5 \times 10^3 \times (0.2)^2 / 9 \times 10^9$$

$$= 6.67 \times 10^9 \text{ C}$$

$$= 6.67 \text{ nC}$$

28. A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of $80.0 \mu\text{C}/\text{m}^2$. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Sol (a) $d = 2.4 \text{ m}$, $r = 1.2 \text{ m}$

Surface charge density, $\sigma = 80.0 \mu\text{C}/\text{m}^2 = 80 \times 10^{-6} \text{ C}/\text{m}^2$

Total charge on surface of sphere,

$$Q = \text{Charge density} \times \text{Surface area} = \sigma \times 4\pi r^2$$

$$= 80 \times 10^{-6} \times 4 \times 3.14 \times (1.2)^2 = 1.447 \times 10^{-3} \text{ C}$$

Therefore, the charge on the sphere is $1.447 \times 10^{-3} \text{ C}$.

$$(b) \phi_{\text{total}} = Q/\epsilon_0$$

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$$

$$Q = 1.447 \times 10^{-3} \text{ C}$$

$$\phi_{\text{total}} = (1.44 \times 10^{-3}) / (8.854 \times 10^{-12}) = 1.63 \times 10^8 \text{ NC}^{-1} \text{ m}^2$$

Therefore, the total electric flux leaving the surface of the sphere is $1.63 \times 10^8 \text{ NC}^{-1} \text{ m}^2$

29. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22} \text{ C}/\text{m}^2$

What is E :

(a) In the outer region of the first plate,

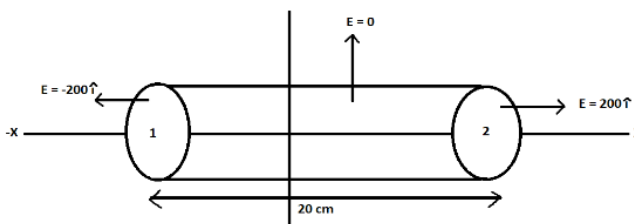
(b) In the outer region of the second plate, and (c) between the plates?

[Ans. Zero, zero, $19.2 \times 10^{-10} \text{ NC}^{-1}$]

(5 Marks Questions)

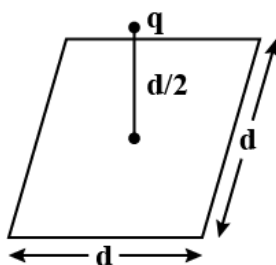
30. An electric field is uniform and acts along +x direction in the region of positive x. It is also uniform with the same magnitude but acts in -x direction in the region of negative x. The value of field is $E = 200 \text{ NC}^{-1}$ for $x > 0$ and $E = -200 \text{ NC}^{-1}$ for $x < 0$. A right circular cylinder of length 20cm and radius 5cm has its centre at the origin and its axis along the x-axis so that one flat face is at $x = +10\text{cm}$ and the other is at $x = -10\text{cm}$. Find (i) the net outward flux through the cylinder. (ii) the net charge present inside the cylinder.

Sol. (i) Given $l = 20\text{cm}$, $r = 5\text{cm} = 0.05\text{m}$, Net flux $\phi = \int E \cdot da + \int E \cdot dA = 200\pi(0.05)^2 \cos 0 \times 2 = \pi \text{Nm}^2\text{C}^{-1}$



(ii) The net charge enclosed, $q = \phi_1 \epsilon_0 = \pi \text{Nm}^2\text{C}^{-1} \times 8.85 \times 10^{-12} \text{C}^2 \text{N}^{-1} \text{m}^{-2} = 27.789 \times 10^{-12} \text{C}$

31. (a) Define electric flux, Is it a scalar or a vector quantity? A point charge q is at a distance $d/2$ directly above the centre of a square of side d , as shown in the figure. Use Gauss's law to obtain the expression for the electric flux through the square.



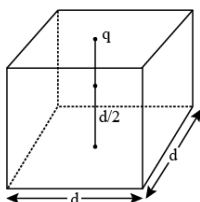
(b) If the net point charge is now moved to a distance 'd' from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected?

Sol (a) Electric flux with a surface is the number of electric lines of force cutting through a surface normally and is measured as surface integral of electric field over that surface, S

$$\text{i.e. } \phi = \int_S \vec{E} \cdot d\vec{S}$$

It is a scalar quantity.

Let us assume that the given square be one face of the cube of edge d cm. As charge of q is at distance of $d/2$ above the centre of a square, so it is enclosed by the cube. Hence by Gauss's theorem, electric flux, linked with the cube is



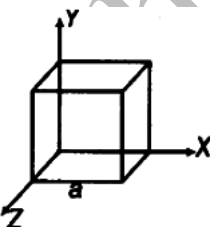
$$\phi = \frac{q}{\epsilon_0}$$

So, magnitude of the electric flux through the square is $\phi_{eq} = \frac{\phi}{6} = \frac{q}{6\epsilon_0}$

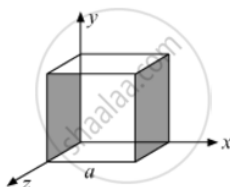
(b) Here distance of point charge becomes doubled and also sides of square gets doubled. Same kind of symmetry is still here with sides of cube $2d$, hence electric flux through the square will not be affected, i.e., $\phi_{eq} = \frac{q}{6\epsilon_0}$.

Hence there will be no change in electric flux.

32. Given the electric field in the region $\vec{E} = 2x\hat{i}$, find the electric flux through the cube and the charge enclosed by it.



Sol. $\vec{E} = 2x\hat{i}$



So flux passes through faces of cube which are perpendicular to x axis.

The magnitude of electric field at the left face ($x = 0$), $E_L = 0$. The magnitude of electric field at the right face, ($x = a$), $E_R = 2a$

$$\begin{aligned} \text{So, net flux } \phi &= \vec{E} \cdot \Delta\vec{S} \\ &= E_L \Delta s \cos 180^\circ + E_R \Delta s \cos 0^\circ \\ &= 0 + 2a \times a^2 = 2a^3 \end{aligned}$$

Assume enclosed charge is q .

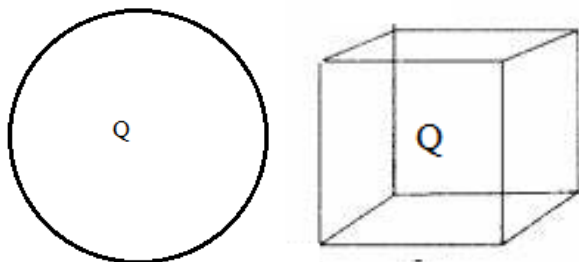
$$\text{Use Gauss's law, } \phi = \frac{q}{\epsilon_0}; q = \epsilon_0 \phi, \therefore q = 2a^3 \epsilon_0.$$

33. Define electric flux. Write its SI unit. 'Gauss's law in electrostatics is true for any closed surface, no matter what its shape or size is'. Justify this statement with the help of a suitable example.

Sol. Electric flux linked with a surface is the number of electric lines of force cutting through the surface normally.

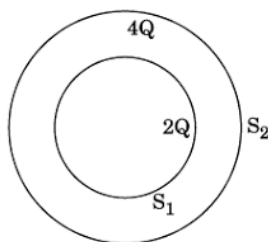
Its SI unit is $\text{N m}^2\text{C}^{-1}$ or Vm . On decreasing the radius of spherical surface to half there will be no effect on the electric flux.

Let us take a charge Q inside a cube or a sphere.



The flux through both the closed surfaces will be same i.e. $\phi_{\text{net}} = \frac{Q}{\epsilon_0}$

34. Consider two hollow concentric spheres S_1 and S_2 enclosing charges $2Q$ and $4Q$ respectively as shown in the figure.



- (i) Find out the ratio of the electric flux through them.
 (ii) How will the electric flux through the sphere S_1 change if a medium of dielectric constant ' ϵ_r ' is introduced in the space outside S_1 in place of air? Deduce the necessary expression.

Sol. (i) By Gauss's Theorem,

$$\text{Flux through } S_1 \text{ is } \phi_1 = \frac{2Q}{\epsilon_0}$$

$$\text{Flux through } S_2 \text{ is } \phi_2 = \frac{2Q+4Q}{\epsilon_0} = \frac{6Q}{\epsilon_0}$$

Ratio of electric flux through S_1 and S_2 are

$$\frac{\phi_1}{\phi_2} = \frac{2Q/\epsilon_0}{6Q/\epsilon_0} = \frac{1}{3} = 1:3.$$

- (ii) If a medium of dielectric constant ' ϵ_r ' is introduced in the space inside S_1 , then flux through S_1 becomes

$$\phi_1 = \oint \vec{E} \cdot \vec{dS} = \frac{2Q}{\epsilon_0}$$

Now when a material is a dielectric constant ϵ_r is introduced then $\epsilon_r = \frac{\epsilon}{\epsilon_0}$

$$\text{Now flux through } S_1 \text{ is } \phi_1' = \frac{2Q}{\epsilon_0}$$

But $\epsilon = \epsilon_r \epsilon_0$ where ϵ = permittivity of medium, ϵ_0 = permittivity of air

$$\text{So, } \phi_1' = \frac{2Q}{\epsilon_0}, \text{ but } \frac{2Q}{\epsilon_0} = \phi_1$$

$$\text{So, } \phi_1' = \frac{\phi_1}{\epsilon_r} \text{ or } \phi_1' = \frac{2Q}{\epsilon_0 \epsilon_r}$$

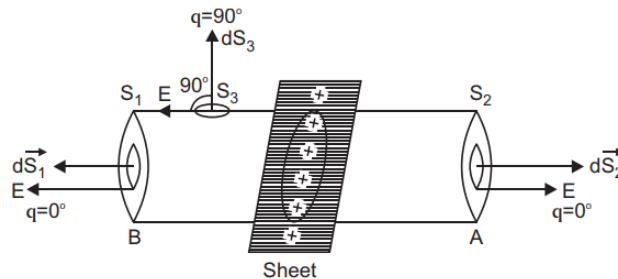
So, now flux is reduced ϵ_r times when placed in a dielectric medium of dielectric constant ϵ_r .

35. State Gauss's law on electrostatics and derive an expression for the electric field due to a long straight thin uniformly charged wire (linear charge density λ) at a point lying at a distance r from the wire.

Sol According to Gauss's law, total flux over a closed surface S in vacuum is $1/\epsilon_0$ times the total charge enclosed by closed surface S .

$$F = \oint_S \vec{E} \cdot \vec{dS} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Assume a cylinder Gaussian surface S with charged wire on its axis and point P on its surface, then net electric flux through surface S is



$$\phi = \oint_S \vec{E} \cdot \vec{dS} = \int_{\text{upper plane face}} E dS \cos 90^\circ + \int_{\text{curved surface}} E dS \cos 0^\circ + \int_{\text{lower plane face}} E dS \cos 90^\circ$$

$$\text{Or } \phi = 0 + EA + 0 \text{ or } \phi = E \cdot 2\pi r l$$

$$\text{But by Gauss's theorem } \phi = \frac{q}{\epsilon_0} = \frac{\lambda l}{\epsilon_0}$$

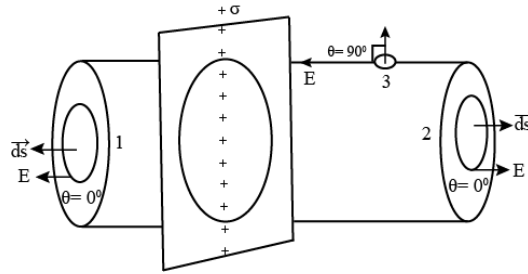
Where q is the charge on length l of wire enclosed by cylindrical surface S , and λ is uniform linear charge density of wire.

$$\therefore E \times 2\pi r l = \frac{\lambda l}{\epsilon_0} \text{ or } E = \frac{\lambda}{2\pi \epsilon_0 r}$$

directed normal to the surface of charged wire

36. Use Gauss's theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ .

- Ans. Assume a cylindrical Gaussian surface A cutting through a plane sheet of charge, such that point P lies on its surface, then net electric flux through surface S is



$$\phi = \oint_S \vec{E} \cdot d\vec{s} = \int_{\text{left plane face}} \vec{E} \cdot d\vec{s} + \int_{\text{curved surface}} \vec{E} \cdot d\vec{s} + \int_{\text{right plane face}} \vec{E} \cdot d\vec{s}$$

$$\text{Or } \phi = \int_{\text{left plane face}} \vec{E} \cdot d\vec{s} \cos 0^\circ + \int_{\text{curved surface}} \vec{E} \cdot d\vec{s} \cos 90^\circ + \int_{\text{right plane face}} \vec{E} \cdot d\vec{s} \cos 0^\circ$$

$$\text{Or } \phi = EA + 0 + EA = 2EA$$

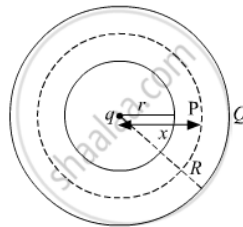
$$\text{But by Gauss's theorem } \phi = \frac{q}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

Where q is the charge in area A of sheet enclosed by cylindrical surface S and σ is uniform surface charge density of sheet

$\therefore 2EA = \frac{\sigma A}{\epsilon_0}$ or $E = \frac{\sigma}{2\epsilon_0}$ directed normal to surface of charged sheet (i) away from it, if it is positively charged and (ii) towards it, if it is negatively charged.

37. A small conducting sphere of radius 'r' carrying a charge +q is surrounded by a large concentric conducting shell of radius R on which a charge +Q is placed. Using Gauss's law derive the expressions for the electric field at a point 'x'.
- (i) between the sphere and the shell ($r < x < R$). (ii) outside the spherical shell.

- Sol. Consider a sphere of radius r with centre O surrounded by a large concentric conducting shell or radius R,



To calculate the electric field intensity at any point P, where $OP = x$, imagine a Gaussian surface with centre O and radius x, as shown in figure.

The total electric flux through the Gaussian surface is given by

$$\phi = \oint ds = E \oint ds$$

$$\text{Now, } \oint ds = 4\pi x^2$$

$$\therefore \phi = \dots(i)$$

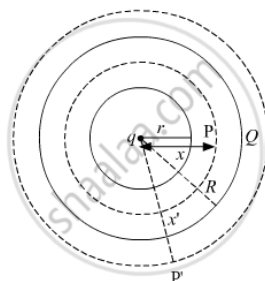
Since the charge enclosed by the Gaussian surface is q, according to Gauss's theorem,

$$\phi = \frac{q}{\epsilon_0} \dots(ii)$$

From (i) and (ii) we get

$$E \times 4\pi x^2 = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{4\pi\epsilon_0 x^2}$$

(ii) To calculate the electric field intensity at any point P', where P' lies outside the spherical shell, imaging a Gaussian surface with centre O and radius x'. as shown in figure.



According to Gauss's theorem,

$$E'(4\pi x'^2) = \frac{q+Q}{\epsilon_0} \Rightarrow E' = \frac{q+Q}{4\pi\epsilon_0 x'^2}$$

As the charge always resides only on the outer surface of a conduction shell, the charge flows essentially from the sphere to the shell when they are connected by a wire. It does not depend on the magnitude and sign of charge Q.

38. State and Prove Gauss's theorem.

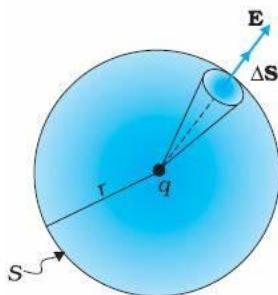
Sol. Gauss's theorem: This theorem gives a relation between the total flux passing through any closed surface and the net charge enclosed within the surface.

Gauss's theorem states that the total flux through a closed surface is $1/\epsilon_0$ times the net charge enclosed by the closed surface.

Mathematically, it can be expressed as

$$\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\epsilon_0}$$

Proof: For the sake of simplicity we prove Gauss's theorem for an isolated positive point charge q. As shown in figure suppose



the surface S is a sphere of radius r centred on q.. Then surface S is a Gaussian surface.

$$\text{Electric field at any point on S is } E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

This Field points radially outward at all points on S. Also, any area element points radially outwards, so it is parallel to \vec{E} , i.e. $\theta = 0^\circ$.

\therefore Flux through area \vec{dS} is

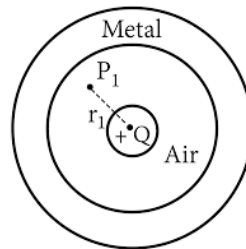
$$d\phi_E = \vec{E} \cdot \vec{dS} = E \cdot ds \cos 0^\circ = EdS$$

Total flux through surface S is

$$\begin{aligned} \phi_E &= \oint_S d\phi_E = \oint_S E dS = E \oint_S dS \\ &= E \times \text{Total area of sphere} \\ &= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2 \\ \text{Or } \phi_E &= \frac{q}{\epsilon_0} \end{aligned}$$

This proves Gauss' theorem.

39. (i) Define electric flux. Write its SI unit.
 (ii) A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2 .



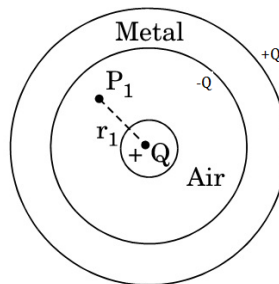
- (iii) Draw the pattern of electric field lines in this arrangement.

Sol. (i) Electric flux linked with a surface is the number of electric lines of force cutting through the surface normally. Its SI unit is $\text{N m}^2 \text{C}^{-1}$ or Vm .

(ii) Using Gauss's theorem, electric field at P_1 , $E_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1^2} = 0$

Again at field P_2 , $E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{r_2^2} = 0$

Because electric field inside a conductor is zero.



- (a) The dipole will not experience any force.
 (b) The dipole will experience a force towards right.
 (c) The dipole will experience a force towards left.
 (d) The dipole will experience a force upwards.

Sol.

(c)

The electric field lines, are directed away from positively charged source and directed toward negatively charged source. In electric field force are directly proportional to the electric field strength hence, higher the electric field strength greater the force and vice-versa.

The space between the electric field lines is increasing, from left to right so strength of electric field decreases with the increase in the space between electric field lines. Then the force on charges also decreases from left to right. Thus, the force on charge $-q$ is greater than force on charge $+q$ in turn dipole will experience a force towards left.

2. Define the term electric dipole moment of a dipole. State its SI unit.

Sol. The dipole moment of an electric dipole is a vector whose magnitude is either charge times the separation between the two opposite charges and the direction is along the dipole axis from the negative to the positive charge. The SI unit of dipole moment is coulomb metre (Cm).

3. An electric dipole placed in a non-uniform electric field can experience

- (a) a force but not a torque (b) a torque but not a force
 (c) always a force and a torque (d) neither a force nor a torque

Sol.

(c)

If an electric dipole is placed in a nonuniform electric field, then the positive and the negative charges of the dipole will experience a net force. And as one end of the dipole is experiencing a force in one direction and the other end in the opposite direction, so the dipole will have a net torque also.

4. Write the expression for a torque $\vec{\tau}$ acting on a dipole moment \vec{p} placed in an electric field \vec{E} .

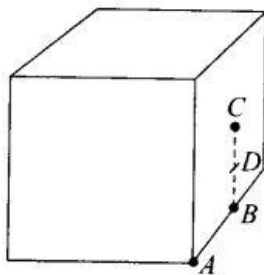
Sol. $\vec{\tau} = \vec{p} \times \vec{E}$

5. What is the electric flux through a cube of side 1cm which encloses an electric dipole?

Sol. The net charge enclosed within the cube is 0 . i.e. $Q = 0$. Hence, the electric flux through the cube is 0.

(2 Marks Questions)

6. What will be the total flux through the faces of the cube as given in the figure with side of length a if a charge q is placed at

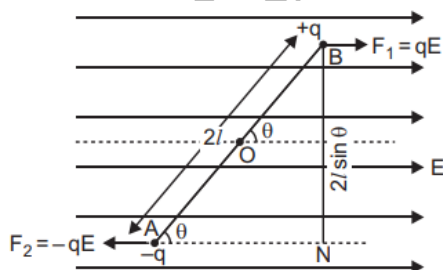


- (a) A corner of the cube (b) B mid-point of an edge of the cube
 (c) C centre of a face of the cube (d) D mid-point of B and C

Ans. (a) $\frac{q}{8\epsilon_0}$ (b) $\frac{q}{4\epsilon_0}$ (c) $\frac{q}{2\epsilon_0}$ (d) $\frac{q}{2\epsilon_0}$

7. Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. Identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium.

Sol. Torque on a dipole in uniform field. When electric dipole is placed in a uniform electric field, its two charges experience equal and opposite forces, which experience equal and opposite forces, which cancel each other and hence net force on an electric dipole in a uniform electric field is zero.



However these forces are not collinear, so they give rise to some torque on the dipole given by, Torque = Magnitude of either force \times Perpendicular distance between them

$$\tau = Fr_1 = qE \cdot 2a \sin\theta = q2a \cdot E \cdot \sin\theta$$

or $\tau = pE \sin\theta$ where θ is the angle between the directions of \vec{p} and \vec{E} .

In vectorial form, $\tau = \vec{p} \times \vec{E}$

(a) When $\theta = 0^\circ$ or 180° the $\tau_{\text{net}} = 0$

(b) When $\theta = 90^\circ$ then $\tau_{\text{max}} = pE$

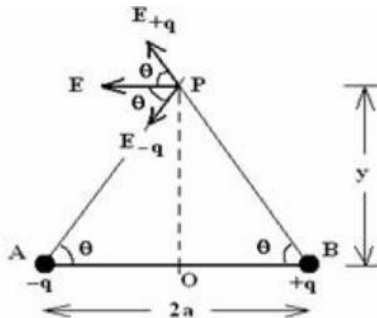
Thus, torque on a dipole tends to align it in the direction of uniform electric field. If the field is not uniform in that condition the net force on electric force is not zero.

When $\theta = 0$; $\tau = 0$ and \vec{p} and \vec{E} are parallel and the dipole is in a position of stable equilibrium.

(3 Marks questions)

8. Derive the expression for the electric field due to dipole of dipole moment \vec{p} at a point on its perpendicular bisector.

Sol Electric field on the equatorial line of an electric dipole: Electric field at any point on the perpendicular bisector of an electric dipole at distance r from its centre is



$$E_{\text{net}} = E_x = E_{PA} \cos \theta + E_{PB} \cos \theta \quad (\text{Vertical component cancel each other})$$

$$\text{Or } E_{\text{net}} = 2E_{PA} \cos \theta \quad (E_{PA} = E_{PB})$$

$$E_{\text{net}} = 2 \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)} \cdot \frac{a}{(r^2+a^2)^{1/2}}$$

$$E_{\text{net}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \cdot 2a}{(r^2+a^2)^{3/2}}$$

$$\text{Or } E_{\text{net}} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{(r^2+a^2)^{3/2}}$$

Directed antiparallel to dipole moment \vec{p} . For short dipole, when $r \gg a$, the electric field at point P is

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$$

In vectorial form, the electric field intensity at point P on the perpendicular bisector of short electric dipole is then given by $\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{-\vec{p}}{r^3} \cdot \hat{r}$

9. Derive the expression for electric field at a point on the equatorial line of an electric dipole.

Sol. Same as 8

10. Depict the orientation of the dipole in (a) stable, (b) unstable equilibrium in a uniform electric field.

Ans. (a) Same as 7

(b) When $\theta = 180^\circ$, $\tau = 0$ and \vec{p} and \vec{E} are antiparallel and the dipole is in a position of unstable equilibrium.

11. A system has two charges $q_A = 2.5 \times 10^{-7} \text{C}$ and $q_B = -2.5 \times 10^{-7} \text{C}$ located at points A : (0, 0, -15 cm) and B (0, 0, +15 cm), respectively. What is the total charge and electric dipole moment of the system?

Sol. At A, amount of charge, $q_A = 2.5 \times 10^{-7} \text{C}$ (Given)

At B, amount of charge, $q_B = -2.5 \times 10^{-7}$

Total charge of the system,

$$q = q_A + q_B = 2.5 \times 10^{-7} \text{C} = 0$$

Distance between two charges at points A and B,

$$d = 15 + 15 = 30 \text{cm} = 0.3 \text{m}$$

Electric dipole moment of the system is given by,

$$p = q_A \times d = 2.5 \times 10^{-7} \times 0.3 = 7.5 \times 10^{-8} \text{Cm along positive z-axis.}$$

Therefore, the electric dipole moment of the system is 7.5×10^{-8} along the positive z-axis.

12. **An electric dipole with dipole moment $4 \times 10^{-9} \text{Cm}$ is aligned at 30° with the direction of a uniform electric field of magnitude $5 \times 10^4 \text{NC}^{-1}$. Calculate the magnitude of the torque acting on the dipole.**

Sol. Electric dipole moment, $p = 4 \times 10^{-9} \text{C m}$

Angle made by p with a uniform electric field, $\theta = 30^\circ$

Electric field, $E = 5 \times 10^4 \text{N C}^{-1}$

Torque acting on the dipole is given by the relation, $\tau = pE \sin\theta$

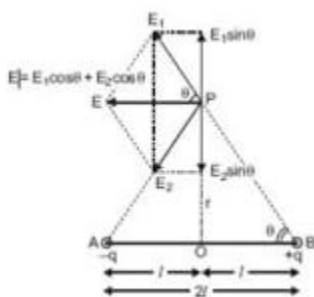
$$= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30 = 20 \times 10^{-5} \times 1/2 = 10^{-4} \text{Nm}$$

Therefore, the magnitude of the torque acting on the dipole is 10^{-4}N m .

(5 Marks Questions)

13. Define electric dipole moment. Is it a scalar or a vector quantity? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

Sol. Strength of an electric dipole is measured by its electric dipole moment, whose magnitude is equal to product of magnitude of either charge and separation between the two charges i.e., $\vec{P} = q \cdot (2\vec{a})$ and is directed from negative to positive charge, along the line joining the two charges. Its SI unit is C m. It is a vector quantity



Electric field on the equatorial line of an electric dipole: Electric field of any point on the perpendicular bisector of an electric dipole of distance r from its centre is

$$E_{\text{net}} = E_x = E_{PA} \cos\theta + E_{PB} \cos\theta \text{ (vertical component cancel each other)}$$

$$\text{Or } E_{\text{net}} = 2E_{PA} \cos\theta \text{ (} E_{PA} = E_{PB} \text{)}$$

$$E_{\text{net}} = 2 \cdot \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)} \cdot \frac{a}{(r^2 + a^2)^{1/2}}$$

$$E_{\text{net}} = \frac{1}{4\pi\epsilon_0} \frac{q \cdot 2a}{(r^2 + a^2)^{3/2}}$$

$$\text{Or } E_{\text{net}} = \frac{1}{4\pi\epsilon_0} \frac{P}{r^2 + a^2)^{3/2}}$$

Directed antiparallel to dipole moment p . For short dipole, when $r \gg a$, then electric field at point P is $E = \frac{1}{4\pi\epsilon_0} \frac{P}{r^3}$

In vectorial form, the electric field intensity at point P on the perpendicular bisector of short electric dipole is then given by $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{-\vec{P}}{r^3} \cdot \hat{r}$.

F. CASE STUDY

1. Electric charge has three basic properties: quantisation, additivity and conservation. Quantisation of electric charge means that total charge (q) of a body is always an integral multiple of basic quantum of charge (e) i.e., $q = ne$, where $n = 0, \pm 1, \pm 2, \pm 3, \dots$. Proton and electron have charges i.e., $-e$ respectively. For macroscopic charges for which n is a very large number, quantization of charge can be ignored.

Additivity of electric charge means that the total charge of a system is the algebraic sum (i.e., the sum taking into account proper signs) of all individual charges in the system.

Conservation of electric charge means that the total charge of an isolated system remains unchanged with time. This means that when bodies are charged through friction, there is a transfer of electric charge from one body to another, but no creation or destruction of charge.

- (i) Choose the correct statement
- Charge is not a conserve quantity
 - Charge can be created in an isolated system
 - Total charge of the universe is constant
 - Charge of an object change with its motion

Ans.

(c)

- (ii) Total number of electrons in $8.0 \times 10^{-19} \text{C}$ is

(a) 4 (b) 8 (c) 9 (d) 5

Ans.

(d)

- (iii) Total positive charge in 250g of water will be

(a) $1.44 \times 10^9 \text{C}$ (b) $1.33 \times 10^7 \text{C}$ (c) $1.2 \times 10^{11} \text{C}$ (d) $1.56 \times 10^{12} \text{C}$

Ans.

(b)

2. A charge q in an electric field \vec{E} experiences a force of magnitude $q\vec{E}$. Direction of force could be along the field opposite to it depending on the nature of charge. If a charge is placed at rest in a uniform electric field, it would move along a straight line either along or opposite to the field. However, if the charge enters the field moving at same speed, its path inside the field will depend on the inclination of its velocity at the time of entering

the field with the field direction. In general, if it enters moving along a direction that makes an angle θ with the field direction, the path is parabolic for $\theta \neq 0^\circ, 180^\circ$.

- (i). A neutron enter inside uniform electric field \vec{E} , then the force on the neutron due to electric field will be
 (a) $2.6 \times 10^{-19} \text{N}$ (b) $1.6 \times 10^{-19} \text{N}$ (c) $8.0 \times 10^{-19} \text{N}$ (d) zero

Ans. (d)

- (ii). A uniform electric field is lying \vec{E} is directed towards east. An electron is inside this field. Then the electric force on this electron will be (e – charge of electron)

(a) $+e\vec{E}$ towards east (b) $+e\vec{E}$ towards north (c) $-e\vec{E}$ towards west (d) $+e\vec{E}$ towards north

Ans. (c)

- (iii). Acceleration of the electron in above question will be (m = mass of electron)

(a) $e\vec{E}/m$ (b) $-e\vec{E}/m$ (c) $-em\vec{E}$ (d) $em\vec{E}$

Ans. (b)

- (iv). A proton is projected with speed 'u' at angle 45° with uniform electric field then path of proton will be

(a) straight line (b) parabolic (c) circular (d) elliptical

Ans. (b)

G. ASSERTION-REASON TYPE QUESTIONS

- (a) If both assertion and reason are true and reason is the correct explanation of assertion.
 (b) If both assertion and reason are true but reason is not the correct explanation of assertion.
 (c) If assertion is true but reason is false (d) If both assertion and reason are false
 (e) If assertion is false but reason is true.

1. Assertion: No two lines of force can intersect each other.

Reason: Tangent at any point of electric line of force gives the direction of electric field.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.

If the two electric lines of force can intersect each other than at the point of intersection we can draw two tangents to the two lines of force. This would mean two directions of electric field intensity at the point of intersection, which is not possible.

2. Assertion: Sharper is the curvature of spot on a charged body lesser will be the surface density of charge at that point.

Reason: Electric field is zero inside a charged conductor.

Ans. (e) Assertion is false but reason is true.

Surface of a charged conductor is always an equipotential surface, whatever may be its shape. Hence $\sigma R = \text{constant}$, at every point on the surface of charged conductor i.e. at the sharpest point ($R \rightarrow 0$) of the surface, charge density will be maximum. A uniformly charged conductor exerts no electrostatic force on a point charge located anywhere inside the conductor of electric field is zero.

3. Assertion: A charged particle free to move in an electric field always move along an electric line of force.

Reason: The electric line of force diverge from a positive charge and converge at a negative charge.

Ans. (e) Assertion is false but reason is true

If the charged particle is initially at rest in an electric field, it will move along the electric line of force. But when the initial velocity of charged particle makes some angle with the line of force then the resultant path is not along the line of force. Because electric line of force may not coincide with the line of velocity of the charge.

4. Assertion: Mass of a body decreases slightly when it is negatively charged.

Reason: Charging is due to transfer of electrons.

Ans. (d) Both assertion and reason are false.

The whole charge of a conductor can be transferred to another isolated conductor, if it is placed inside the hollow insulated conductor and connected with it.

5. Assertion: Electric current will not flow between two charged bodies when connected if their charges are same.

Reason: Current is the rate of flow of charge.

Ans. (e) Assertion is false but reason is true

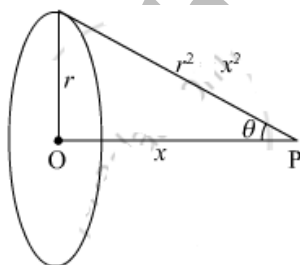
Current will not flow only when two bodies are at the same potential. When their charges are same, their potential may be different. Hence current may flow in this case.

H. CHALLENGING PROBLEMS

1. In a certain region of space, electric field is along the Z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive Z-direction at the rate of $10^5 \text{ NC}^{-1} \text{ m}^{-1}$. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} C m in the negative Z-direction? [Ans. - $10^{-2} \text{ N}, 0$]

2. A thin circular ring of radius r is charged uniformly so that its linear charge density becomes λ . Derive an expression for the electric field at a point P at a distance x from it along the axis of the ring. Hence, prove that at large distances ($x \gg r$), the ring behaves as a point charge.

Sol. Give radius = r , so circumference = $2\pi r$ and charge = $dq = \lambda dl$.



$$dE_x = dE \cos \theta = \frac{\lambda x dl}{4\pi\epsilon_0 (r^2 + x^2)^{3/2}}$$

$$\therefore E_x = \int dE \cos \theta = \int \frac{\lambda x dl}{4\pi\epsilon_0 (r^2 + x^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{x\lambda l}{(r^2 + x^2)^{3/2}}$$

When $x \gg r$, then $E = \frac{1}{4\pi\epsilon_0} \frac{x\lambda l}{(x^2)^{3/2}} = \frac{1}{4\pi\epsilon_0} \frac{\lambda l}{x^2}$

3. Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [Hint: Use Coulomb's law directly and evaluate the necessary integral].

4. A particle of mass m and charge $(-q)$ enters the region between two charged plates initially moving along x -axis with speed v_x . The length of plate is L and a uniform electric field E is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/2mv_x^2$.

SPACE FOR ROUGH WORK

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SPACE FOR NOTES

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