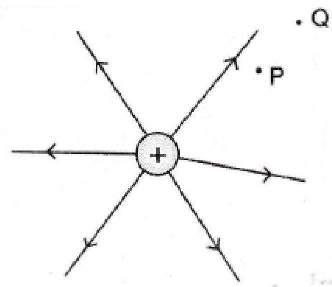
CLASS – 12

WORKSHEET- ELCTROSTAT POTENTIAL AND CAPACITANCE

A. ELECTROSTATIC POTENTIAL ENERGY

(1 Mark Questions)

1. Figure shows the field lines on a positive charge. Is the work done by the field in moving a small positive charge from Q to P positive or negative? Give reason.



Sol. Work done = q(Potential at Q – Potential at P), where q is the small positive charge.
The electric potential at a point distance r due to the field created by a positive charge Q is given by, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
$\because r_P < r_Q, \therefore V_P > V_Q$
Hence work done will be negative.

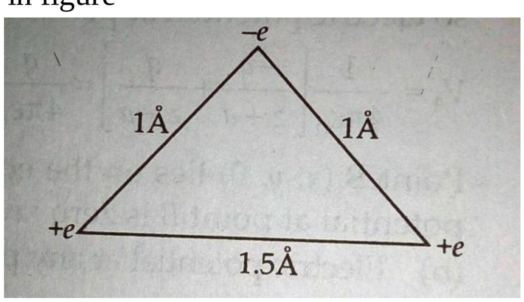
2. What is the amount of work done in moving a point charge around a circular arc of radius r at the centre of which another point charge is located?

Sol. Work one in carrying a charge on equipotential surface is always zero.

(2 Marks Questions)

3. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 A, and the electron is roughly 1 A from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Sol. The system is shown in figure



Charge in an electron, $q_1 = - e = - 1.6 \times 10^{-19}C$
Charge on a proton, $q_2 = q_3 + e = + 1.6 \times 10^{-19}C$
If the zero of the potential energy is taken at infinity, then potential energy of the sytem is

$$U = U_{12} + U_{23} + U_{13} = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_1} + \frac{q_2q_3}{r_2} + \frac{q_1q_3}{r_3}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{(-q)q}{1\times10^{-10}} + \frac{q.q}{1.5\times10^{-10}} + \frac{(-q)q}{1\times10^{-10}}\right]$$

$$= \frac{e^2}{4\pi\varepsilon_0\times10^{-10}}\left[-1 + \frac{1}{15} - 1\right]$$

$$= \frac{(1.6\times10^{-19})^2\times9\times10^9}{10^{-10}} \times \left(\frac{-4}{3}\right)J$$

$$= \frac{(1.6\times10^{-19})^2\times9\times10^9\times4}{1.6\times10^{-19}\times3}eV = - 19.2eV \; [\because 1eV = 1.6\times10^{-19}J]$$

(3 Marks Questions)

4. An electric dipole consists of two opposite charges each of magnitude 1μc separated by 2cm. The dipole is placed in an external electric field of 10^5 NC^{-1}. Find (i) the maximum torque exerted by the field on the dipole (ii) the work which the external agent will have to do in turning the dipole through 180° starting from the position θ = 0°.

Sol. Here $q = 1\mu C = 10^{-6}C$, $2a = 2cm = 0.02m$, $E = 10^5NC^{-1}$
∴ Dipole moment, $p = q \times 2a = 10^{-6} \times 0.02 = 2 \times 10^{-8}Cm$
(i) Maximum torque, $\tau_{max} = pE \sin 90° = 2 \times 10^{-8} \times 10^5 \times 1 = 2 \times 10^{-3}$ Nm
(ii) Here $\theta_1 = 0°$ and $\theta_2 = 180°$
∴ $W = pE(\cos \theta_1 - \cos \theta_2)$
$= 2 \times 10^{-8} \times 10^8 (\cos 0° - \cos 180°)$
$= 2 \times 10^{-3} (1 + 1) = 4 \times 10^{-3}J$

5. Two point charges A and B of values + 15μC and +9μC are kept 18cm apart in air. Calculate the work done when charge B is moved by 3cm towards A.

Sol. W = Final P.E. – Initial P.E.

$$\frac{q_1q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$= 9 \times 10^9 \times 15 \times 10^{-6} \times 9 \times 10^{-6}\left[\frac{100}{15} - \frac{100}{18}\right] = 1.35J.$$

6. Derive an expression for the potential energy of a system of two point charges and write its relation with electric potential; of a charge.

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7. In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 A: **(a)** Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton. **(b)** What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)? **(c)** What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 A separation?

Sol. (a) q_1 = - 1.6 × 10^{-19}C, q_2 = + 1.6 × 10^{-19}C, r = 0.53 Å = 0.53 × 10^{-10}m

PE of the electron proton system will be, U = $\frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r}$

= $9 \times 10^9 \times \frac{(-1.6\times10^{-19})\times1.6\times10^{-19}}{0.53\times10^{-10}}$

= $-\frac{9\times1.6\times1.6\times10^{-19}}{0.23}$J

= $-\frac{9\times1.6\times1.6\times10^{-19}}{0.53\times1.6\times10^{-19}}$eV = 27.2eV.

(b) KE of the electron in the orbit = ½ PE = ½ × 27.2 eV = 13.6 eV

∴ total energy of the electron = PE + KE

= (- 27.2 + 13.6)eV = - 13.6eV

As minimum energy of free electron is zero, so minimum work required to free electron = 0 – (- 13.6) = 13.6 eV

(c) When the two potential energy is not taken at infinity, the potential energy of the system is

U = $\frac{q_1q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$

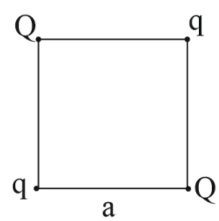
= $9 \times 10^9 \times (-1.6 \times 10^{-19}) \times 1.6 \times 10^{-19} \times \left[\frac{1}{0.53\times10^{-10}}-\frac{1}{1.06\times10^{-10}}\right]$J

= $-\frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{1.6\times10^{-19}\times0.53\times10^{-10}}\left[1-\frac{1}{2}\right]$eV

= $-\frac{9\times1.6}{0.53\times2}$eV = - 13.6eV

This indicates that the KE of 13.6 eV of case (a) is used up in increasing the PE from – 27.2eV to – 13.6eV as the electron is carried from 0.53 Å to 1.06 Å position. KE in the situation should be zero. As the total energy in this case is zero, therefore, minimum wirk required to free the electron = 0 – (- 13.6eV) = 13.6eV

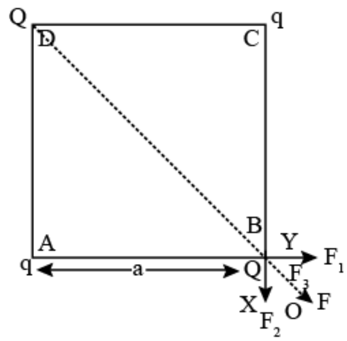
8. Four point charge Q, q, Q and q are placed at the corners of a square of side 'a' as shown in the figure.



Find the
(a) resultant electric force on a charge Q, and
(b) potential energy of this system

Sol. (a) Force on charge Q due to charge q.

$F_q = \frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{a^2}$



Force on charge Q due to another charge Q,

$F_Q = \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{(a\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}$

Now force on charge Q is $F_{net} = F_Q + \sqrt{F_q^2 + F_q^2} = F_Q + F_q\sqrt{2}$

$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{2a^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{a^2}\sqrt{2}$

$= \frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{Q}{2} + \sqrt{2}q\right]$ along diagonal

(b) Potential energy of the given system,

$U = U_{qQ} + U_{Qq} + U_{qQ} + U_{qq} + U_{QQ}$

$= 4U_{qQ} + U_{qq} + U_{QQ}$

$= \frac{4qQ}{4\pi\varepsilon_0} + \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}a)} + \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)}$

$= \frac{1}{4\pi\varepsilon_0}\left[4qQ + \frac{q^2}{\sqrt{2}a} + \frac{Q^2}{\sqrt{2}a}\right]$

(5 marks Questions)

9. Derive an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

Sol. Since net force on dipole in uniform electric field is zero, so no work is done in moving the electric dipole in uniform electric field, however some work is done in rotating the dipole against the torque on it. So, small work done in rotating the dipole by an angle dθ in uniform electric field E is dW = τ dq = pE sin θ dθ

Hence, net work done in rotating the dipole from angle θ_i to θ_f in uniform electric field is

$W = \int_{\theta_i}^{\theta_f} pE\sin\theta d\theta = pE[-\cos\theta]_{\theta_i}^{\theta_f}$

Or = pE [- cos θ_f + cos θ_i] = pE[cosθ_i – cosθ_f]

If initially, the dipole is placed at an angle θ_i = 90° to th direction of electric field, and is then rotated to the angle θ_f = θ, thennet work done s

W = pE[cos 90° - cosθ]

Or W = pE cosθ

This gives the work done in rotting the dipole through an angle q in uniform electric field, which gets stored in the form of potential energy i.e., U = - PE cosθ

This gives potential energy stored in electric dipole of moment p when placed in uniform electric field at an angle θ with its direction.

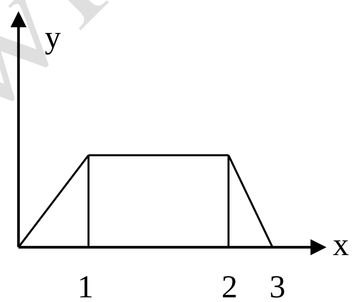
(i) When θ = 0°, then U_{min} = - pE

So, potential energy of an electric dipole is minimum p parallel to the direction of electric field E and so it is called its most stable equilibrium position.

(ii) When θ = 180°, then U_{max} = +pE

So, when energy of an electric dipole is maximum, when it is placed with its dipole moment p anti parallel to the direction of electric field E and so it is called its most unstable equilibrium position.

10. The electric potential as a function of distance x is shown in the figure. Draw a graph of the electric field E as a function of x



Sol. Electric field $E = -\frac{dV}{dt}$,..(i)

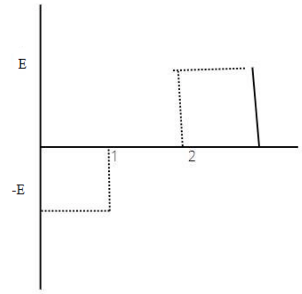
For x = 0to 1, V = kx

x = 1 to 2, V = k

x = 2 to 3, V =- - kx

where k is some constant

So, using (i) the variation of electric field is shown in figure.



B. POTENTIAL

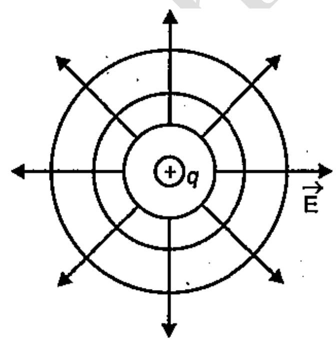
(1 Mark Questions)

1. The physical quantity having SI unit NC^{-1} m is _________.

Sol. The physical quantity having SI unit NC^{-1} m s electrostatic potential.

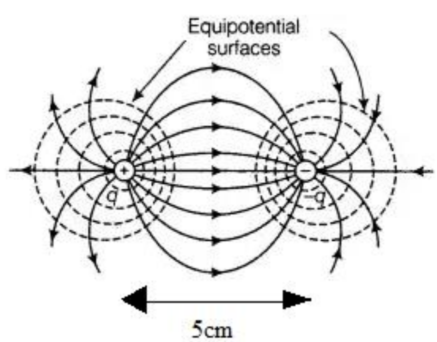
2. What is the geometrical shape of equipotential surface due to a single isolated point charge?

Sol. For an isolated charge the equipotential surfaces are concentric spherical shells and the separation between consecutive equipotential surfaces increases in the weaker electric field.



3. Two charges 2 μC and –2μC are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system.

Sol.



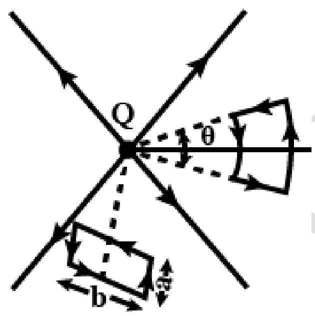
4. Can there be a potential difference between two adjacent conductors carrying the same charge?

Sol. Yes, if the sizes are different.

5. Can the potential function have a maximum or minimum in free space?

Sol. No, the potential function did not have a maximum or minimum in free space.

6. A test charge q is made to move in the electric field of a point charge Q along two different closed paths (Fig.). First path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?

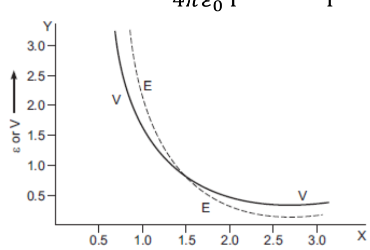


Sol. As electric field is conservative, work done will be zero in both the cases.

(2 Marks Questions)

7. Draw a plot showing variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q.

Sol. Electric field due to a point charge, $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, $E \propto \frac{1}{r^2}$



Potential due to a point charge, $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$; $V \propto \frac{1}{r}$

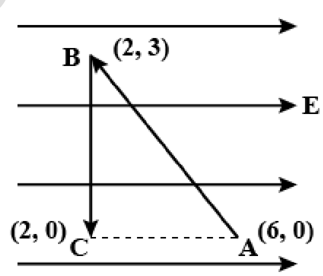
The variation of electric field E with distance r and also the variation of potential v with r as shown in the figure.

8. Define electric potential. Derive an expression for the electric potential at a distance r from a charge q.

9. Show that the electric field at any point is equal to the negative of the potential gradient at that point.

10. The point charge +Q is place at point O as shown in the figure. Is the potential difference $V_A - V_B$ positive, negative or zero?

11. A test charge 'q' is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure . (i) Calculate the potential difference between A and C. (ii) At which point (of the two) is the electric potential more and why?



Sol. (i) In the relation

$E = \frac{-dV}{dt} \Rightarrow E = -\left[\frac{V_C - V_A}{(2-6)}\right]$

$V_C - V_A = 4E$

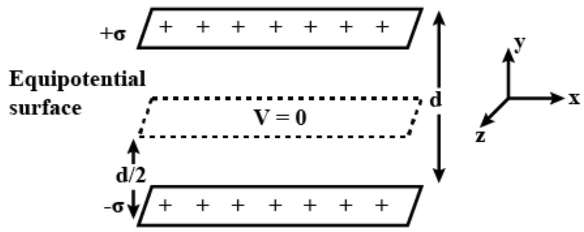
(ii) As $V_C - V_A = 4E$ is positive,

$\therefore V_C > V_A$

Potential is greater at point C than point A, as potential decreases along the direction fo electric field.

12. Two uniformly large parallel thin plates having densities +σ and –σ are kept in the X-Z plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge –q remains stationary between the plates, what is the magnitude and direction of this field?

Sol.



The equipotential surface is at a distance d/2 from either plate in XZ plane, - q charge experiences a force in a direction opposite tot eh direction of electric field.

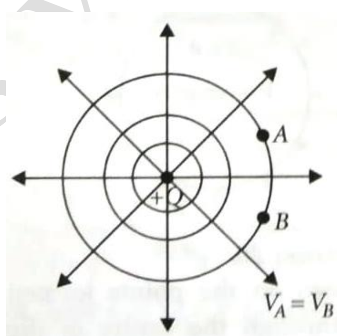
∴ - q charge balances when qE = mg

$E = \frac{mg}{q}$

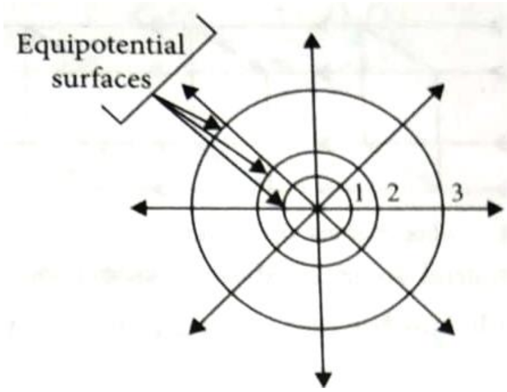
The direction of electric field along vertically downward direction. The XZ plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, Otherwise weight (mg) of chare particle could not be balanced.

13. (a) Draw equipotential surfaces due to point Q > 0.
(b) Are these surfaces equidistant from each other ? if no, explain why ?

Sol. (a)



(b) These surfaces are not equivalent from each other because electric field at a point, distance r from point charge is given by $E = +\frac{Q}{4\uparrow\pi\varepsilon_0 r^2}$. As electric field $E \propto \frac{1}{r^2}$, the field is non uniform. SO, distance between adjacent equipotential surfaces goes on increasing as shown in figure.



14. Two charges –q and +q are located at point A(0, 0, –a) and B(0, 0, +a) respectively. How much work is done in moving a test charge from point P(7,0,0) to Q (–3,0, 0)?

Sol. Potential at P(7, 0, 0) is

$V_1 = \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(7-0)^2+0+(-a-0)^2}} + \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(7-0)^2+0+(a-0)^2}}$

$= \frac{=q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{49+a^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{49+a^2}} = 0$

Potential at Q(- 3, 0, 0) is

$V_2 = \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(-3-0)^2+(-a)^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{(-3-0)^2+(-a)^2}}$

$= \frac{-q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{9+a^2}} + \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{\sqrt{9+a^2}} = 0$

∴ work done = $q(V_2 - V_1) = q(0 - 0) = 0$

Hence, W = 0

15. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the center of the hexagon. [Ans. 2.7×10^6V]

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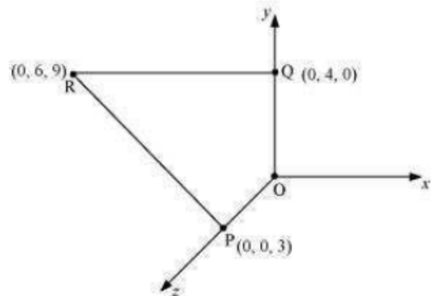
16. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P(0,0,3 cm) to a point Q (0,4 cm, 0), via a point R (0,6 cm, 9 cm).

Sol. As the work done in taking charge from one point to another is independent of the path followed, therefore

$W = q_0[V_Q - V_P] = q_0\left[\frac{q}{4\pi\varepsilon_0 r_2} - \frac{q}{4\pi\varepsilon_0 r_1}\right]$

$= \frac{q_0 q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$



Here, $q = 8mC = 8 \times 10^{-3}C$, $q_0 = -2 \times 10^{-9}C$, $r_1 = 3cm = 3 \times 10^{-2}$ m, $r_2 = 4cm = 4 \times 10^{-2}m$

$\therefore W = -2 \times 10^{-9} \times 8 \times 10^{-3} \times 9 \times 10^{9} \times \left[\frac{1}{4\times10^{-2}} - \frac{1}{3\times10^{-2}}\right] = 1.2J$

17. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Sol Length of longest diagonal of the cube = $\sqrt{b^2 + b^2 + b^2} = \sqrt{3}b$

Distance of each charge (placed at vertex) from the centre of the cube is

$r = \frac{\sqrt{3}}{2}b$

$\therefore$ Potential at the centre of the cube is

$V = 8.\frac{1}{4\pi\varepsilon_0}.\frac{q}{r} = 8 \times \frac{1}{4\pi\varepsilon_0}.\frac{2q}{\sqrt{3}b} = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}.$

Electric fields at the centre due to any pair of charges at the opposite corners wil be equal and opposite and thus cancelling out in pairs. Hence resultant electric field at the centre will be zero.

(3 Marks Questions)

18. Write a relation between potential energy of a charge and electric potential. Hence define electric potential.

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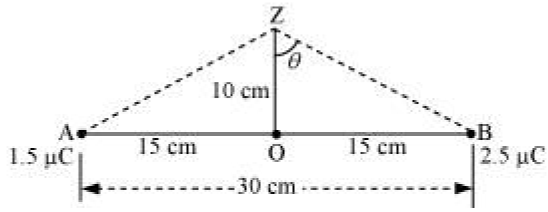
19. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

(a) at the mid-point of the line joining the two charges, and

(b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Sol. The two situations are shown in figure.



(a) Here, $r_1 = r_2 = 15cm = 0.15m$

∴ Potential at the mid-point O of the line joining the two charges is

$V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$= 9 \times 10^9\left[\frac{1.5\times10^{-6}}{0.15} + \frac{2.5\times10^{-6}}{0.15}\right]V$

$= 9 \times 10^9 \times 10^{-6}\left[10 + \frac{50}{3}\right]V$

$= 9 \times 10^9 \times \frac{80}{3}V = 2.4 \times 10^5 V$

(b) Here $r_1 = r_2 = \sqrt{10^2 + 15^2} = \sqrt{325} = 18cm = 0.18m$

∴ Potential at point C due to the two charges is

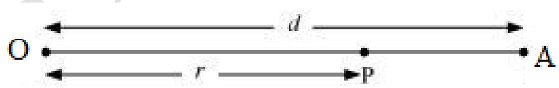
$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$= 9 \times 10^9\left[\frac{1.5\times10^{-6}}{0.18} + \frac{2.5\times10^{-6}}{0.18}\right]$

$= \frac{9\times10^9 + 4\times10^{-6}}{0.18} = 2 \times 10^5 V$

20. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Sol. As shown in figure, suppose the two point charges are placed on X axis with the positive charge located on the origin O.



Le the potential be zero at the point P and OP = x. From x < 0 (i.e., to the left of O), the potentials of the two charges cannot add up to zero. Clearly, x must be positive. If x lies between O and A, then

$$V_1 + V_2 = 0$$

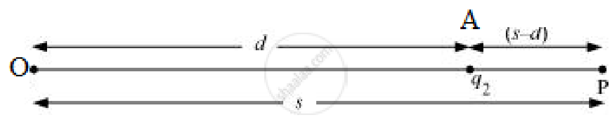
$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{x} + \frac{q_2}{0.16-x}\right] = 0$$

$$\text{Or } \frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{0.16-x} = 0$$

$$\text{Or } \frac{5}{x} - \frac{3}{0.16-x} = 0$$

Which gives x = 0.10m = 10cm

The other possibility is that x many also lie on OA produced as shown in figure.



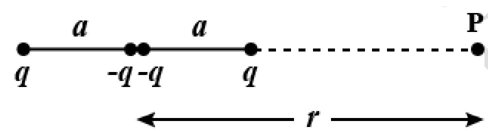
As $V_1 + V_2 = 0$

$\therefore \frac{1}{4\pi\varepsilon_0}\left[\frac{5\times10^{-8}}{x} - \frac{3\times10^{-8}}{x-0.16}\right] = 0$

Which gives x = 0.40m = 40cm

Thus the electric potential is zero at 10 cm and 40 cm away from the positive charge on the side of the negative charge.

21. Figure shows a charge array known as an electric quadrupole. For a point on the axis of quadrupole, obtain the dependence of potential on r for r/a >> 1, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



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(5 marks Questions)

22. Derive the potential due to a dipole at angular position.

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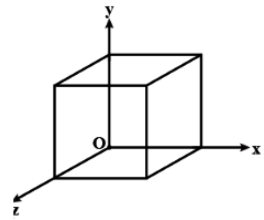
23. A cube of side 20cm is kept in a region as shown in the figure. An electric field $\vec{E}$ exists in the region such that the potential at a point is given by V = 10x + 5, where V is in volt and x is in m.

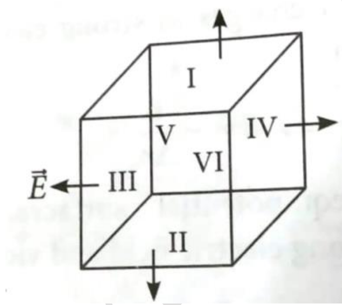


(i) electric field $\vec{E}$, and

(ii) total electric flux through the cube.

Sol. (i) Now electric field, $\vec{E} = \frac{\partial V}{\partial r} = \frac{-dV}{dx} = \frac{-d}{dx}(10x + 5) = -10\hat{\imath}$

(ii) Now the total electric flux through the cube, $\phi = \int E.ds.$



$\phi = \int_{I} E.ds + \int_{II} E.ds + \int_{III} E.ds + \int_{IV} E.ds + \int_{V} E.ds + \int_{VI} E.ds$

$= 0 + 0 + (+10)(20 \times 10^{-2})^2 + (-10)(20 \times 10^{-2})^2 + 0 + 0 = 0$

24. The magnitude of electric field (in NC^{-1}) in a region varies with the distance r (in m) as

$$E = 10r + 5$$

By how much does the electric potential increase in moving from point at r = 1 m to a point at r = 10 m.

Sol. Given E = 10r + 5

Now the electric potential, $V = -\int E.dr$

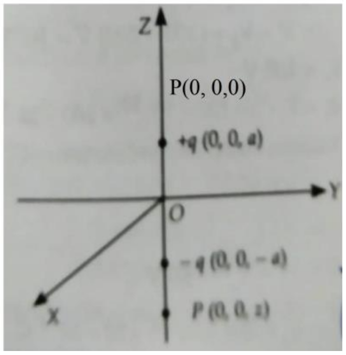
$= -\int_{1}^{10}((10r + 5)dr = -\left[\frac{10r^2}{2} + 5r\right]_{1}^{10}$

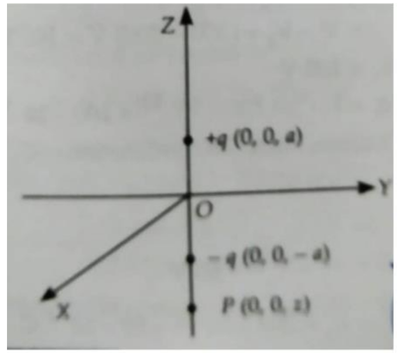
$= -1[5r^2 + 5d]_{1}^{10}$

$= -1(5 \times 100 + 50) - (5 + 5)] = -540$ V

25. Two charges -q and +q are located at points (0,0, -a) and (0,0, a), respectively.
(a) What is the electrostatic potential at the points (0,0,z) and (x,y,0)?
(b) Obtain the dependence of potential on the distance r of a point from the origin when r/a»1.

(c) How much work is done in moving a small test charge from the point (5, 0, 0) to (-7, 0, 0) along the x-axis?
Does the answer change if the path of the test charge between the some points is not along the x-axis? [Ans. 0, $1/r^2$. 0]





Sol.

(a) When the point P lies closer to the charge +q as shown in figure (a), the potentials at this point P will be

$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{z-a} - \frac{q}{z-(-a)}\right]$

$= \frac{q}{4\pi\varepsilon_0}.\frac{2a}{z^2-a^2}$

Or $V = \frac{1}{4\pi\varepsilon_0}.\frac{p}{z^2-a^2}$ [$\because$ p = q × 1a]

When the point P lies closer to the charge – q as shown in figure (b), it can be easily seen as $-\frac{1}{4\pi\varepsilon_0}.\frac{p}{z^2-a^2}$

Again any point (x, y, 0) lies in XY plane which is perpendicular bisector of Z axis, Such a point will be at equal distances from the charges – q and +q. Hence potential at point (x, y, 0) will be zero.

(b) If the distance of point P from the origin O is r, then from the results of part a) we get

$V = \pm\frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2-a^2}$ [Put z = r]

As r >>a, we can neglect a^2 compared to r^2, so $V = \pm\frac{1}{4\pi\varepsilon_0}.\frac{p}{r^2}$

$\therefore$ For r >>a, the dependence of potential V on r is $1/r^2$ type.

(c) (5, 0, 0) an d(-7, 0, 0) are the points on the X axis i.e. these points lie on the perpendicular bisector of the dipole. Each point is at the same distance from the two charges. Hence electric potential at each of these points is zero.

Work done in moving the test charge q_0 from the point (5, 0, 0) to (-7, 0, 0) is

$W = q(V_1 - V_2) = q(0 - 0) = 0$

No, answer will not change if the path of the net charge between the same two points is not along X axis. This is because the work done by the electrostatic field between two points is independent of the path connecting the two points.

26. Describe schematically the equipotential surfaces corresponding to

- A constant electric field in the z-direction.
- a field that uniformly increases in magnitude but remains in a constant (say, z) direction.
- a single positive charge at the origin, and
- a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Sol. (i) For a constant electric field in Z direction equipotential surfaces will be planes parallel to XY planes.
(ii) In this case also, the equipotential surfaces will be planes parallel to XY plane. However, as field increases, such planes will get closer.
(iii) For a single positive charge at the origin, the equipotential surfaces will be concentric spheres having origin as their common center. The separation between the equipotentials differing by a constant potential increases with increase in distance from the origin.
(iv) Near the grid the equipotential surfaces will have varying shapes. At far off distances, the equipotential surfaces will be planes to the grid.

27. Draw an expression for the potential energy of an electric dipole in a uniform electric field. Explain conditions for stable and unstable equilibrium.

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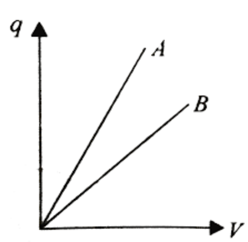
C. CAPACITOR

(1 Mark Questions)

1. The given graph shows variation of charge 'q' versus potential difference 'V for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than the of C_1. Which line (A or B) corresponds of C_1 and why?

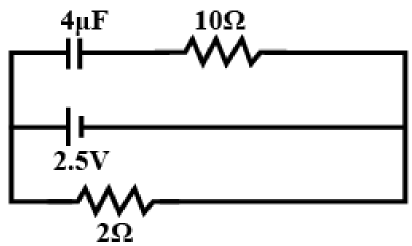


Sol. The plate area of C_2 is greater than that of C_1. Since capacitor is directly proportional to the area of the plates.
$\therefore C_2 > C_1$

Now, $C = \frac{q}{V}$

Therefore, slope of a line (=q/V) is directly proportional to the capacitance of the capacitor, it represents. Since the slope of line A is more than that of B, line A represents C_2 and the line B represent C_1.

2. A capacitor of 4 μ F is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω . The amount of charge on the capacitor plates will be



(a) 0 (b) 4 μ C (c) 16 μ C (d) 8 μ C

Ans. (d)

Current in lower arm of the circuit, $I = \frac{2.5V}{2\Omega+0.5\Omega} = 1A$

Potential difference across the internal resistance of the cell = (0.5Ω)(1A) = 0.5V

And potential difference across 4μF capacitor = 2.5V – 0.5V = 2V

Charge on capacitor plates, Q = CV = (4μF(2V)) = 8μC

(2 Marks Questions)

3. What is law of capacitance?

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4. What is the area of the plats of 2F parallel plate capacitor having separation between the plates is 0.5 cm?

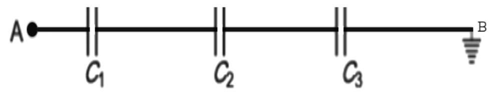
Sol. Here C = 2F, d = 0.5cm = 0.5×10^{-2}m. $e_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1}m^{-2}$

$\therefore C = \frac{\varepsilon_0 A}{d}$

$A = \frac{Cd}{\varepsilon_0} = \frac{2\times0.5\times10^{-2}}{8.854\times10^{-12}}$

$A = 1.13 \times 10^9\ m^2$

5. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure . Given potential at A is 90V. C_1 = 20 μF, C_2 = 30 μF and C_3 = 15 μF.



Sol. The equivalent capacitance (C_{eq}) of the circuit is given by

$\frac{1}{C_{eq}} = \frac{3+2+4}{60}$

$C_{eq} = \frac{60}{9}\mu F$

Charge on equivalent capacitor, $Q = C_{eq}V = \frac{60}{9} \times 10^{-6} \times 90$

$Q = 600\ \mu C$

Charge on each capacitor is same as they arein series

Now, potential drop across C_2

$V_2 = \frac{Q}{C_2} = \frac{600\times10^{-6}}{30\times10^{-6}} = 20$ volt

Energy, $U = \frac{1}{2} C_2V_2^2$

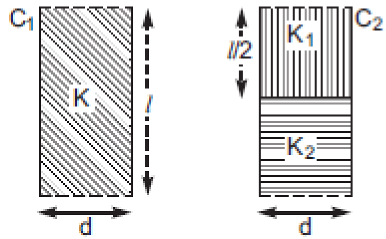
$U = \frac{1}{2} \times 30 \times 10^{-6} \times (20)^2 = 6 \times 10^{-3}$ joule

6. A 12 μF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? [Ans. 1.5×10^{-8} J]

7. A 600 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 μF capacitor. How much electrostatic energy is lost in the process? [Ans. 6×10^{-6}J]

8. What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [Ans. 1130 km^2]

9. Two identical parallel plate (air) capacitor C_1 and C_2 have capacitances C each. The area between their plates is now filled with dielectrics as shown.



If the two capacitors still have equal capacitance, obtained the relation between dielectric constants K, K_1 and K_2.

Sol. Let A → area of each plate and C_1 and C_2 are capacitance of each slab.

Let initially $C_1 = C = \frac{\varepsilon_0 A}{d} = C_2$

After inserting respective dielectric slabs; $C_1' = KC$ …(i)

And $C_2' = K_1\frac{\varepsilon_0(\frac{A}{2})}{d} + K_2\frac{\varepsilon_0(\frac{A}{2})}{d}$

$= \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$; $C_2' = \frac{C}{2}(K_1 + K_2)$ ….(ii)

From (i) and (ii)

$C_1' = C_2'$

$KC = \frac{C}{2}(K_1 + K_2)$

$K = \frac{1}{2}(K_1 + K_2)$

(3 Marks Questions)

10. Derive the capacitance of a parallel plate capacitor.

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11. Three capacitors of capacitances C_1, C_2 and C_3 are connected in series. Find their equivalent capacitance.

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12. In a parallel plate capacitor with air between the plates, each plate has an area of 6 ×

10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this, capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor? [Ans. 15pF, 1.8×10^{-9}C]

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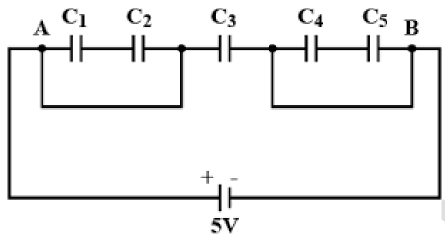
13. In the figure given below, find the

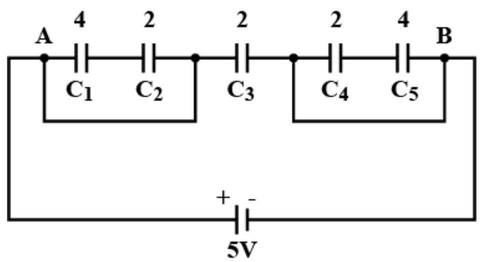


(a) equivalent capacitance of the network between points A and B.
Given : $C_1 = C_5 = 4$ μF, $C_2 = C_3 = C_4 = 2$ μF

(b) Maximum charge supplied by the battery and

(c) total energy stored in the network

Sol. (a)



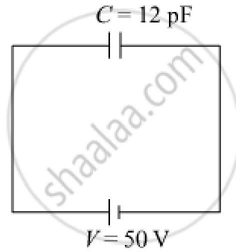
So, the equivalent capacitance between A and B is $C_3 = 2\mu F$

(b) Charge, $Q = CV = 2\mu F \times 5V = 10\ \mu C$

(c) Total energy stored = $\frac{1}{2} CV^2 = \frac{1}{2} \times 2\mu F \times (5V)^2 = 6\mu F$

14. A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor ? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor.

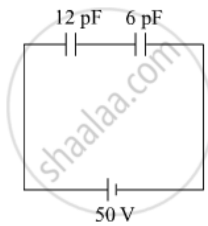
Sol. Electrostatic energy stored in the capacitor, $U = \frac{1}{2} CV^2 = \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$



(As C = 12pF, V = 5-V)

$U = 1.5\times10^{-8}J$

When 6pF is connected in series with 13 pF, charge stored across each capacitor.



$Q = \frac{C_1C_2}{C_1+C_2}V$

$= \frac{12\times6\times10^{-24}}{(12+6)\times10^{-12}} \times 50 = 200pC$

Now potential difference across 12 pF is $= \frac{Q}{C_1} = \frac{200\times10^{-12}}{12\times10^{-12}} = 16.67V$

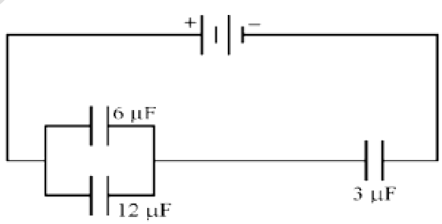
Potential difference across 6 pF is $= \frac{Q}{C_2} = \frac{200\times10^{-12}}{6\times10^{-12}} = 33.33V$

15. In the following arrangement of capacitors, the energy stored in the 6 μF capacitor is E. Find the value of the following.

(i) Energy stored in 12μF capacitor

(ii) Energy stored in 3 μF capacitor

(iii) Total energy drawn from the battery



Sol. (i) Given that energy of the 6μF capacitor is E. Let be the potential difference along the capacitor of capacitance 6μF.

Since $\frac{1}{2}CV^2 = E$

$\therefore \frac{1}{2} \times 6 \times 10^{-6} \times V^2 = E$

$\Rightarrow V^2 = \frac{E}{3} \times 10^6$

Since potential is same for parallel connection, the potential through 12μF capacitor is also V. Hence energy of 12μF capacitor is

$E_{12} = ½ \times 12 \times 10^{-6} \times V^2 = ½ \times 12 \times 10^{-6} \times \frac{E}{3} \times 10^6 = 2E$

(ii) Since charge remains constant in series, the charge on 6μF and 12 μF capacitors combined will be equal to the charge on 3μF capacitor, Using the formula, Q = CV, we can write, $(6 + 12) \times 10^{-6} \times 12E \times 10^6 = 18E$

(iii) Total energy drawn from the battery is $E_{total} = E + E_{12} + E_3 = E + 2E + 18E = 21E$

16. Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitors if the combination is connected to a 120 V supply?

Sol. (a) If C is the equivalent capacitance of the series combination, then

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{9} = \frac{1}{3}$

Or C = 3 pF.

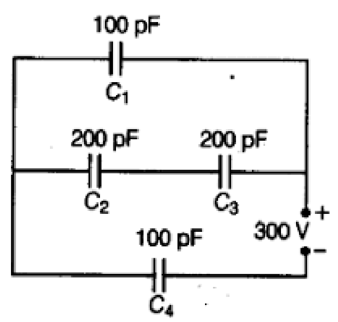
(b) As all the capacitors have equal capacitance, so potential drop ΔV

$\therefore V = \Delta V_1 + \Delta V_2 + \Delta V_3$

$= \Delta V + \Delta V + \Delta V = 3\Delta V$

Or $\Delta V = \frac{\mathbf{V}}{\mathbf{3}} = \frac{\mathbf{120}}{\mathbf{3}} = 40V.$

17. Obtain the equivalent capacitance of the network in figure. For a 300 V supply, determine the charge and voltage across each capacitor. [Ans. 10^{-8}C]



Sol. As C_2 and C_3 are in series, their equivalent capacitance $= \frac{C_2C_3}{C_2+C_3} = \frac{200\times200}{200+200} = 100pF$

Series combination of C_2 and C_3 is in parallel with C_1 their equivalent capacitance = 100pF + 100pF = 200pF

The combination of C_1, C_2 and C_3 is in series with C_4 equivalent capacitance of the network $= \frac{200\times100}{200+100}pF = \frac{200}{3}pF$

Total charge on the network is $q = CV = \frac{200}{3} \times 10^{-12} \times 300 = 2 \times 10^{-8}C$

This must be equal to charge on C_4 and also to the sum of the charges on the combination of C_1, C_2 and C_3

$\therefore q_4 = q - 2\times 10^{-8}C$

$V_4 = \frac{q_4}{C_4} = \frac{2\times10^{-8}}{100\times10^{-12}}V = 200V$

PD between A and B = $V - V_A = (300 - 200)V = 100V$
$\therefore V_1 = 100V$
$q_1 = C_1V_1 = 100 \times 100^{-12} \times 100 = 10^{-8}C$
Also PD across the series combination of C_2 and C_3 = 100V
Now since $C_2 = C_3$
$\therefore V_2 = V_3 - \frac{100}{2} = 50V$ and $q_2 = q_3 = 200 \times 10^{-12} \times 50 = 10^{-8}C$

18. Show that the force on each plate of a parallel plate capacitor has magnitude equal to ½ QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor 1/2.

Sol. Let A be the plate area and σ, the surface charge density of the capacitor. Then,
$1 = \sigma A$
$E = \sigma/\varepsilon_0$
Suppose we increase the separation of the capacitor plate by small distance Δx against the force F. Then work done by the external agency = F.Δx
If u be the energy stored per unit volume or the energy density of the capacitor, then increase in potential energy of the capacitor = u × increase in volume = u.A. Δx
$\therefore F.\Delta x = u.A.\Delta x$
Or $F = uA - \frac{1}{2} \varepsilon_0 E^2. A = \frac{1}{2} (\varepsilon_0 E)A.E$
$= \frac{1}{2} .\sigma A.E = \frac{1}{2} qE$
The physical origin of the factor ½ in the force formula lies in the fact that just inside the capacitor, field is E, and outside it is zero. SO, the average value E/2 contributes to the force.

19. In a Van-de-Graff type generator, a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is $5 \times 10^7 \text{ V m}^1$. What is the minimum radius of the spherical shell required?

Sol. Maximum permissible potential, $V = 1.5 \times 10^6 V$
For safety the maximum permissible electric field is E = 10% of dielectric strength
$= 10\%$ of $5 \times 10^7 \text{ Vm}^{-1} = 5 \times 10^6 \text{ Vm}^{-1}$
Now for a spherical shell, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$
$E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2} = \frac{V}{r}$
∴ Minimum radius required is
$r = \frac{V}{E} = \frac{1.5 \times 10^6 V}{5 \times 10^6 Vm^{-1}} = 3 \times 10^{-1} m = 30cm.$

(5 marks Questions)

20. Three capacitors of capacitances 2 μF, 3 μF and 4 μF are connected in parallel.
(a) What is the total capacitance of the combination.
(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Sol. (a) For a parallel combination, total capacitance is given by
$C = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9pF$
(b) When the combination is connected to 100V supply, charges on the capacitors will be
$q_1 = C_1V = 2 \times 10^{-12} \times 100 = 2 \times 10^{-10}C$
$q_2 = C_2V = 3 \times 10^{-12} \times 100 = 3 \times 10^{-10}C$
$q_3 = C_1V = 4 \times 10^{-12} \times 100 = 4 \times 10^{-10}C$

21. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Sol. The charges will flow between the two spheres till their potentials are equal, Then the charges on the two spheres would be
$\frac{Q_1}{Q_2} = \frac{C_1V}{C_2V} = \frac{C_1}{C_2}$
But $\frac{C_1}{C_2} = \frac{a}{b}$
$\therefore \frac{Q_1}{Q_2} = \frac{a}{b}$
The ratio of the electric fields at the surface of the two spheres will be
$\frac{E_1}{E_2} = \frac{1/4\pi\varepsilon_0}{1/4\pi\varepsilon_0}.\frac{Q^2/a^2}{Q^2/b} = \frac{Q_1}{Q_2}.\frac{b^2}{a^2} = \frac{a}{b}.\frac{b^2}{a^2} = \frac{b}{a}$
Also, $\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2}$
$\therefore \frac{\sigma_1}{\sigma_2} = \frac{b}{a}$
Thus the surface charge densities are inversely proportional to the radii of the spheres. Since the flat portion may be considered as a spherical surface of large radius and a pointed portion as that of small radius, that is why, the surface charge density on the sharp and pointed ends of a conductor is much higher than that on its flatter portion.

D. EFFECT OF DIELECTRIC ON CAPACITOR

(3 Marks Questions)

1. Explain why the polarization of a dielectric reduces the electric field inside the dielectric. Hence define dielectric constant.

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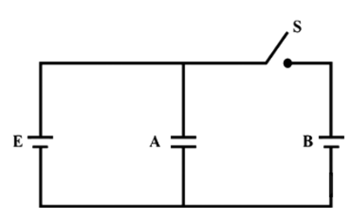
2. Define ‘dielectric constant’ of a medium. Briefly why the capacitance of a parallel plate capacitor increases, on introducing a dielectric medium between the plates?

3. Explain what would happen if in the capacitor of capacitance 18pF and charge 1.8×10^{-9}C, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates.
(a) While the voltage supply remained connected.
(b) After the supply was disconnected.

Sol. We have, $C_0 = 1.8 \times 10^{-11}$F = 18 pF, $q_0 = 1.8 \times 10^{-9}$C
Also, $\kappa = 6$
(i) When the voltage supply remains connected, the potential difference between capacitor plates remains same, i.e. 100V
The capacitance increases κ times
$\therefore C = \kappa C_0 = 6 \times 18 = 108$ pF.
The charge on the capacitor plats will be
$q = CV = 108 \times 10^{-12} \times 100 = 1.08 \times 10^{-8}$C
(ii) After the supply was disconnected, the charge on the capacitor plates remains same, i.e., $q_0 = 18 \times 10^{-19}$C
The capacitance increase κ times
$C = \kappa C_0 = 108$ pF
The potential difference between the capacitor plats becomes, $V = \frac{V_0}{\kappa} = \frac{100}{6} = 16.6$V

4. Two identical parallel plate capacitors A and B are connected to a battery of V volt with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



Sol. Initially when the switch is closed, both the capacitors A and B aer in parallel and therefore the energy stored in the capacitors is
$U_i = 2 \times ½\, CV^2 = CV^2$ …(i)

When switch S is opened, B gets disconnected from the battery. The capacitor B is now isolated and the charge on an isolated capacitor remains constant, often referred to as bound charge. ON the other hand, A remains connected to the battery.

However, potential V remains constant on it.

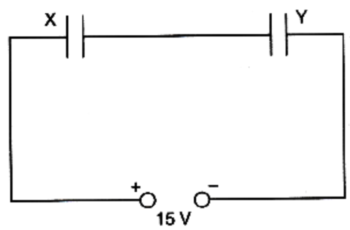
When the capacitors are filled with dielectric, their capacitors are filled with dielectric, their capacitance increases to KC. Therefore, energy stored in B changes which remains constant, and energy stored in A changes to ½ KCV^2, where V is the potential on A, which remains constant. Thus, the final total energy stored in the capacitors is

$U_f = \frac{1}{2}\frac{(CV)^2}{KC} + \frac{1}{2}KCV^2 = \frac{1}{2}CV^2\left(K + \frac{1}{K}\right)$ …(ii)

From eqs. (i) and (ii), we find

$\frac{U_i}{U_f} = \frac{2K}{K^2+1}$

5. Two parallel plate capacitors X and Y have the same area of separation between them. X has air between the plates while Y contains a dielectric of $\varepsilon_r = 4$.

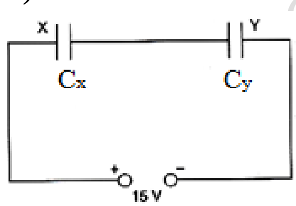


(i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4μF.

(ii) Calculate the potential difference between the plates of X and Y.

(iii) Estimate the ratio of electrostatic energy stored in X and Y.

Sol.



Here $C_x = \frac{\varepsilon_0 A}{d}$ and $C_y = \frac{\varepsilon_0 \varepsilon_r A}{d} = \varepsilon_r C_x = 4C_x$

(i) C_x and C_y are in series, so equivalent capacitance is given by, $C = \frac{C_x \times C_y}{C_x + C_y}$

$\Rightarrow 4 = \frac{C_x \times 4C_x}{C_x + 4C_x}$

$\Rightarrow 4 = \frac{4C_x}{5} \therefore C_x = 5\mu F$ and $C_y = 20\mu F$

(ii) Charge on each capacitor, Q = CV

$Q = 4 \times 10^{-6} \times 15 = 60 \times 10^{-6} C$

Potential difference between plates of X, $V_x = \frac{Q}{C_x} = \frac{60\times10^{-6}}{5\times10^{-6}} = 12V$

Potential difference between the plaes of Y, $V_y = V - V_x = 15 - 12 = 3V$

(iii) Ratio of electrostatic energy stored,

$$\frac{U_x}{U_y} = \frac{\frac{Q^2}{2C_x}}{\frac{Q^2}{2C_y}} = \frac{C_y}{C_x} = \frac{4C_x}{C_x} = 4$$

6. A parallel plate capacitor of capacitance 'C' is charged to 'V' volt by a battery. After sometime the battery is disconnected and the distance between the plates is doubled. Now a slab of dielectric constant 1 < K < 2 is introduced to fill the space between the plates. How will the following be affected?
(i) the electric field between the plate of the capacitor?
(ii) The energy stored in the capacitor.
Justify your answer in each case.

Sol. (i) The electric field between the plates is $E = \frac{V}{d}$
The distance between plates is doubled, d = 2d

$\therefore E' = \frac{V'}{d'} = \left(\frac{V}{K}\right) \times \frac{1}{d} = \frac{1}{2}\left(\frac{E}{K}\right)$

Therefore if the distance between the plates is double, the electric field will reduce to one half.

(ii) As the capacitance of the capacitor, $C' = \frac{\varepsilon_0 KA}{2d} = \frac{1}{2}C$

Energy stored in the capacitor is $U = \frac{Q^2}{2C'} = \frac{Q2^2}{2\left(\frac{1}{2}\right)C} = 2\left(\frac{Q^2}{2C}\right) = 2U$

Therefore, when the distance between the plates is doubled, the capacitance reduces to half and the energy stored in the capacitor becomes double.

7. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant, κ = 6?

Sol. Capacitance of the capacitor with air between ties plates = $\frac{\varepsilon_0 A}{d}$ = 8pF

When the capacitor is filled with dielectric (κ = 6) between its plates and the distance between the plates is reduced to half, capacitance becomes

$C' = \frac{\varepsilon_0 \kappa A}{d'} = \frac{\varepsilon_0 \times 6 \times A}{d/2} = 12\frac{\varepsilon_0 A}{d}$

Or C' = 12 × 8 = 96 pF.

8. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 V m^{-1}. For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 μF?

Sol. Maximum permissible voltage = 1kV = 10^3V
Maximum permissible electric field = 10% of 10^7 Vm^{-1} = 10^6 Vm^{-1}

∴ Minimum separation d required between the plates is given by

$E = \frac{V}{d}$ or $d = \frac{V}{E} = \frac{10^3}{10^6} = 10^{-3}$m

Capacitance of a parallel plate capacitor is

$C = \frac{\kappa\varepsilon_0 A}{d}$

$\therefore A = \frac{Cd}{\kappa\varepsilon_0} = \frac{50\times10^{-12}\times10^{-3}}{3\times8.85\times10^{-12}}\, m^2$

$= 18.8 \times 10^{-4} m^2 = 19 cm^2$

(5 Marks Questions)

9. Two capacitors with capacity C_1 and C_2 are charged to potential V_1 and V_2 respectively and then connected in parallel. Calculate the common potential across the combination, the charge on each capacitor, the electrostatic energy stored in the system and the change in the electrostatic energy from its initial value.

10. Derive the expression for the capacitance of a parallel plate capacitor of area A and plate separation d if a dielectric slab of thickness t (where t<d) is introduced one by one between the plates of the capacitor.

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11. State the working of principle of Van-de-graft generator with the help of neat and clean diagram.

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E. CASE STUDY

1. The substances such as metals which allow the charge to flow freely through them are called conductors. In metals conduction evolves the movement of free electrons. In case of conductors in electrostatics, it is worth noting that:

1. In charging a conductor electrons are either added or removed from it. If electrons are removed, conductor becomes positively charged and its potential increases and if added, it becomes negatively charged and its potential decreases.
2. When a conductor is charged by induction, induced charge (which is free to move) is equal and opposite to the inducing charge, i.e. q' = - q.
3. Charge resides on the outer surface of a conductor, However, distribution of charge on the surface is generally not uniform and surface density of charge varies inversely as the radius of curvature of that part of the conductor, i.e., $\sigma \propto (1/R)$.
4. The dielectric constant of conductors in electrostatics is infinite, i.e., $K = \infty$.
5. Electric intensity inside a conductor is zero while outside (near its surface) is (σ/ε_0), i.e., $E_{in} = 0$ and $E_{out} = (\sigma/\varepsilon_0)$.
6. Conductor is an equipotential surface, i.e., potential at its surface or inside everywhere is same, i.e., for a conductor V = constant.
7. Electric field and hence lines of force are normal to the surface of a conductor.
8. The field in a cavity inside a conductor is zero resulting in 'electrostatic shielding.'

(i) A charged glass rod having charge 20μC placed near a metallic isolated sphere. Then, the total charge induced on the sphere will be

(a) – 20 μC (b) 40 μC (c) 80 μC (d) zero

Ans. (d)

(ii) A metallic cube have a positive charge on it. Then,

(a) potential at the corner of cube will be greater than the potential at flat surface
(b) potential at the corner of cube will be less than the potential at flat surface
(c) potential at the corner of cube will be equal to the potential at flat surface
(d) potential inside the cube will be zero

Ans. (c)

(iii) In Question (ii),

(a) electric field near the corner of cube will be maximum
(b) electric field near the corner of cube will be minimum
(c) electric field inside the cube will be non-zero
(d) electric potential inside the cube will be zero

Ans. (a)

2. Any charge is an electric field experiences the action of a force. Consequently, a certain work is done when a charge moves in the field. This work depends on the field strength at different points and on the charge displacement. However, if a charge describes curve, that is, returns to the original position, the work done by the field is equal to zero irrespective of the field configuration and the shape of the path along which the charge has moved.

(i) A charged particle move from a point A to a different point B in an uniform electric field E. Now, the strength of electric field increased to twice, then work done by the field will be

(a) increase (b) decrease (c) remains same (d) none

Ans. (a)

(ii) A charged particle start moving inside a uniform electric field along a circular path and return to its initial position, then the work done by the electric field will be

(a) positive and non zero (b) negative
(c) zero (d) none of these

Ans. (c)

(iii) In above question, consider that there is a non uniform electric field, then work done will be

(a) positive and non zero (b) negative
(c) zero (d) none of these

Ans. (c)

F. ASSERTION REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false **(d) If both assertion and reason are false**

(e) If assertion is false but reason is true.

1. Assertion: A capacitor can be given only a limited quantity of charge.
Reason: A capacitor in an arrangement which can store sufficient quantity of charge.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.
The quantity of charge that can be given to the capacitor is limited by the fact that every dielectric is limited by the dielectric strength of medium between the plates. The dielectric strength o is a maximum value of electric field that it can tolerate without breakdown. Beyond this field the dielectric no more behaves as dielectric but it conduct electricity.

2. Assertion: When a charged capacitor is filed completely with a metallic slab, its capacity becomes very large.
Reason: The dielectric constant for metals is infinity.

Ans. (e) Assertion is false but reason is true.
A capacitance of a capacitor filed partially with a dielectric of thickness t is given by: $C = \frac{\varepsilon_0 A}{d - t[1 - \frac{1}{K}]}$.
For metals, K = ∞, Therefore $C = \frac{\varepsilon_0 A}{d - t}$
Now if the capacitor is filled completely with a metallic slab, then t = d. Therefore C = ∞,i.e., when a charge capacitor filled fully with a metallic slab, then capacitor is short circuited i.e., it will no more work as a capacitor.

3. Assertion: The lightening conductor at the top of high building has sharp pointed ends.
Reason: The surface density of charge at sharp points is very high resulting in setting up of electric wind.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
A charged cloud induces opposite charge on point conductors. At sharp points of the conductor surface density of charge is very high and charge begins to leak from the pointed ends setting up of oppositely charged electric wind. This wind, when it comes in contact with the charged cloud, neutralizes some of its charge lowering the potential difference between the cloud and the building. This reduces the chances of lightening striking the building [if the lightning strikes the building, the charge is conducted to the earth and the building remains safe].

4. Assertion: Increasing the charge on the plates of a capacitor means increasing the capacitance.
Reason: Capacitance is directly proportional to charge.

Ans. (d) Both assertion and reason are false.
On increasing the charge, potential increases. But capacity of a capacitor is fixed by geometry of condenser ($C = \varepsilon_0 A/d$). Capacitance is independent of charge.

5. Assertion: A bird perches on a high power line and nothing happens to the bird.
Reason: The level of bird is very high from the ground.

Ans. (c) Assertion is true but reason is false.
Electric shock is due to the electric current flowing through a living body. When the bird perches on a single high power line, no current passes though its body because its body is

at equipotential surface i.e., there is no potential difference. While when man touches the same line, standing bare feet on the ground the electric circuit is completed through the ground. The hands of man are at high potential and his feet are at low potential. Hence large amount of current flow through the body of the man and the person therefore gets a fatal shock.

E. CHALLENGING PROBLEMS

1. Answer the following:

(i). The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 V m^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage, so there is no field inside).

(ii) A man fixes outside house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2. Will he get an electric shock if he touches the metal sheet next morning?

(iii) The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

(iv) What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning?

Sol. (i) Normally the equipotential surfaces are parallel to the surface of the earth. Now our body is a good conductor. So, as we step out into the open, the original equipotential surfaces of open air get modified, but keeping our head and the ground at the same potential and we do not get any electric shock.

(ii) Yes, the aluminium sheet and ground form a capacitor with insulating slab as dielectric. The discharging current in the atmosphere will charge the capacitor steadily and raise its voltage. Next morning, if the man touches the metal sheet, he will receive shock to the extent depending upon the capacitance of the capacitor formed.

(iii) The atmosphere is continuously being charged by thunder storms and lightning bolts all over the globe and maintains an equilibrium with the discharge of the atmosphere in ordinary weather conditions.

(iv) The electrical energy is lost as (i) light energy involved in lightning (ii) heat and sound energy in the accompanying thunder.

2. A long charged cylinder of linear charge density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Sol. $\phi_E = \frac{q}{\varepsilon_0}$

Or E. 2πrL = $\frac{\lambda L}{\varepsilon_0}$

E = $\frac{\lambda}{2\pi\varepsilon_0 r}$

∴ Potential difference between two cylinder is

V = $-\int_a^b \vec{E}.\overrightarrow{dr} = \int_a^b E.dr$ [∵ $\vec{E}$ and $\overrightarrow{dr}$ are in opposite directions]

= $\int_a^b \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{1}{r} dr$

= $\frac{\lambda}{2\pi\varepsilon_0}[\ln r]_a^b = \frac{\lambda}{2\pi\varepsilon_0}[\ln b - \ln a]$

Or V = $\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}$

Total charge on each cylinder is Q = Lλ.

∴ Capacitance of cylinder capacitor is

C = $\frac{Q}{V} = \frac{L\lambda}{\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{b}{a}}$ or C = $\frac{2\pi\varepsilon_0 L}{\ln\frac{b}{a}}$

3. Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by $\left(\vec{E}_2 - \vec{E}_1\right).\hat{n} = \frac{\sigma}{\varepsilon_0}$ where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of
n is from side 1 to side 2). Hence show that just outside a conductor, the electric field is $\sigma\frac{\hat{n}}{\varepsilon_0}$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another.

Sol. (a) Electric field near a plane sheet of charge is given by, E = $\frac{\sigma}{2\varepsilon_0}$

If n̂ is a unit vector normal to the sheet from side 1 to side 2, then electric field on side 2,

$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{n}$

In the direction of the outward normal to the side 2. Similarly, electric field on side 1 is

$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0}\hat{n}$

In the direction of the outward normal to the side 1

∴ $(\vec{E}_2 - \vec{E}_1).\hat{n} = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$

As $\vec{E}_1$ and $\vec{E}_2$ act in opposite directions, there must be discontinuity at the sheet of charge.

Now electric filed vanishes inside a conductor, Therefore $\vec{E}_1 = 0$

Hence outside the conductor, the electric field is $\vec{E} = \vec{E}_2 = \frac{\sigma}{\varepsilon_0}\hat{n}$

(b) Let XY be the charged surface on the two sides of the charged surface. Consider a rectangular loop ABCD with length l and negligibly small breadth. Line integral along the closed path ABCD will be

$\int \vec{E}.\vec{dl} = \vec{E}_1.\vec{l} = \vec{E}_2.\vec{l} = 0$

Or $E_1 l \cos\theta_1 - E_2 l \cos\theta_2 = 0$

$= (E_1 \cos\theta_1 - E_2 \cos\theta_2) = 0$

$(E_1' - E_2') = 0$

Where E_1' and E_2' are tangential components of $\vec{E}_1$ and $\vec{E}_2$ respectively. Thus

$E_1' = E_2'$ ($\because l \neq 0$)

Hence the tangential component of the electrostatic field is continuous across the surface.

4. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Sol. Let this arrangement require n capacitors of 1 μF each in series and m such series combinations to be connected in parallel.

P.D. across each capacitor of a series combustion = 1000/n = 400 or n = 1000/400 = 2.5

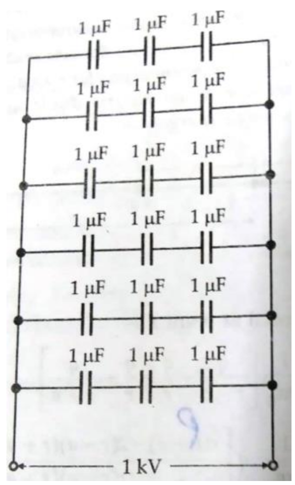
But number of capacitors cannot be a fraction,

$\therefore$ n = 3

Equivalent capacitance of the combination is 1/n.m = 2 or m = 2n = 6

$\therefore$ Total number of capacitors required = 3 × 6 = 18

So six series combinations each of 1μF capacitors should be connected n parallel as shown in figure



5. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e. bending of the field lines at the ends). [Ans. 2.9×10^4V]

6. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessary flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Sol. Collecting action of a hollow sphere: Consider a small surface of radius r placed inside a large spherical shell of radius R. Let the sphere carry charges q and Q respectively.

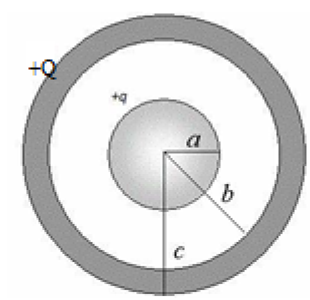
Total potential on the outer sphere,

V_A = Potential due to its own charge Q + potential due to the charge q on the inner sphere

$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R} + \frac{q}{R}\right]$

Potential on the inner sphere due to its own charge is

$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$



As the potential at every point a charged sphere is the same as that on its surface, so potential on the inner sphere due to charge Q on outer sphere is

$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

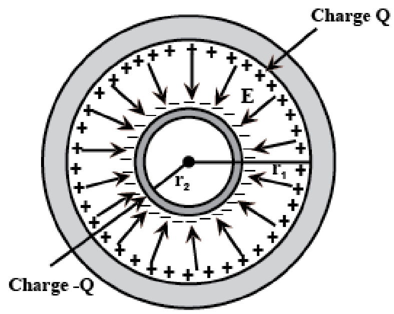
∴ Total potential on inner sphere, $V_r = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{Q}{R}\right]$

Hence the potential difference is $V_r - V_R = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$

So, if q si positive the potential of the inner sphere will always be higher than that of the outer sphere. Now if the two spheres are connected by a conducting wire, the charge q will flow entirely to the outer sphere, irrespective of the charge Q already present on the outer sphere. In fact this is true for conduction of any shape.

7. A spherical capacitor consists of two concentric spherical conductors held in position by suitable insulating supports. Show that the capacitance of a spherical capacitor is given by $C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$ where r_1 and r_2 are the radii of outer and inner spheres, respectively.

Sol.



A spherical capacitor consists of two concentric spherical shells of inner and outer radii a and b. The two shells carry charges – Q and +Q receptively. Since the electric field inside a hollow conductor is zero, $\vec{E} = 0$ for r < a for r > b. A radial field $\vec{E}$ exists in the region between the two shells due to the charge on the inner shell only.

$\phi_E = E.4\pi r^2 = Q/\varepsilon_0$ or $E = Q/4\pi\varepsilon_0 r^2$

The potential difference (caused by the inner sphere alone) between the two shells will be

$V = -\int_a^b \vec{E}.\overrightarrow{dr} = \int_a^b E dr = \int_a^b \frac{Q}{4\pi\varepsilon_0 r^2} dr$

$= \frac{Q}{4\pi\varepsilon_0 r^2}\int_a^b r^{-2} dr = \frac{Q}{4\pi\varepsilon_0 r^2}\left[-\frac{1}{r}\right]_a^b = \frac{Q}{4\pi\varepsilon_0 r^2}\left[\frac{1}{a} - \frac{1}{b}\right]$ [$\because \vec{E}$ points radially inward and $\overrightarrow{dr}$points outward so $\vec{E}.\overrightarrow{dr}$ = Edr 180° = - Edr]

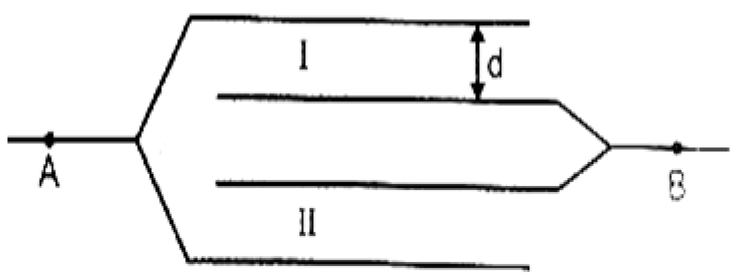
The capacitance of the spherical capacitor is $C = \frac{Q}{V} = \frac{Q}{\frac{Q}{4\pi\varepsilon_0 r^2}\left[\frac{1}{a}-\frac{1}{b}\right]}$ or $C = \frac{4\pi\varepsilon_0 ab}{b-a}$

8. Answer carefully:

1. Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other, is the magnitude of electrostatic force between them exactly given by $\frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$ r is the distance between their centres?
2. If coulomb's law involved 1 /r^3 dependence (instead of 1/r^2), would Gauss's law be still true?
3. A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?
4. What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?
5. We know that electric field is discontinuous across the surface of a charged conductor. `Is electric potential also discontinuous there?
6. What meaning would you give to the capacitance of a single conductor?
7. Guess a possible reason why water has a much greater dielectric constant {- 80) then say, mica (= 6).

Sol. 1. No when two spheres are brought close to each other, their charge distribution do not remain uniform and they will not act as point charges.
2. No, Gauss's law will not hold if Coulomb's law involved $1/r^3$ or any other power of r (except 2). In that case, the electric flux will depend upon r also.
3. Not necessarily. The small test charge will move along the line of force only if it is a straight line. The line of force gives the direction of acceleration and not that of velocity.
4. Zero. But when the orbit is elliptical, work is done in moving the electron from one pint to the other. However, net work done over a complete cycle is zero.
5. No, potential is everywhere constant as it is a scalar quantity.
6. A single conductor is a capacitor with one plate at infinity. It is also capacitance.
7. Because of this bent shape and the presence of two highly polar O – H bonds, a water molecule possesses a large permanent dipole moment about (1.6×10^{-29}Cm. Hence water has a large dielectric constant.

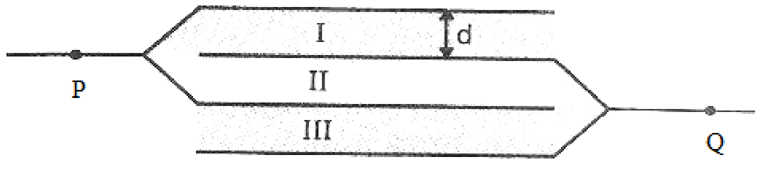
9. What is the capacitance of arrangement of a plates of area A at distance d in air in figure?



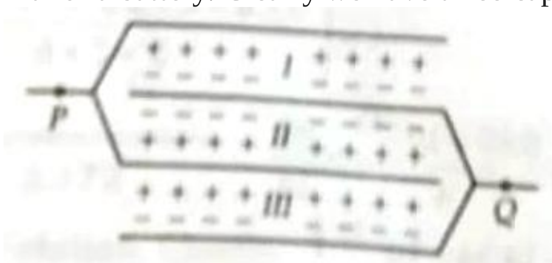
Sol. As shown in figure, suppose the point P is connected to the positive terminal and point Q to the negative terminal of a battery. Clearly, we have two capacitors I and II with their positive plates connected together and their negative plats connected together. So the two capacitors are in parallel.

∴ Equivalent capacitance, $C_p = C_1 + C_2 = C + C = 2C = \frac{2\varepsilon_0 A}{d}$

10. What is the capacitance of arrangement of a 4 plates of area A at distance d in air in figure?



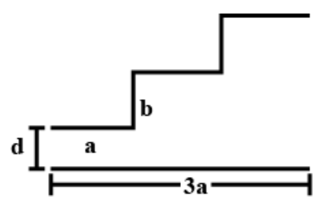
Sol. As shown in figure, suppose the point P is connected to the positive terminal and point Q to the negative terminal of a battery. Clearly we have three capacitors, II and III.



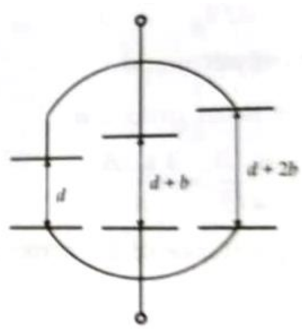
Their positive plats are connected to the same point p while the negative plates are connected to the same point Q. So the three capacitors are in parallel.

$C_p = C_1 + C_2 + C_3 = 3C = \frac{3\varepsilon_0 A}{d}$

11. A capacitor is made of flat plate area A and a second plate having a stair like structure, as shown in figure. The width of each stair is a and height is b. Find the capacitance of the assembly.



Sol. A shown in figure, the given arrangement is equivalent to a parallel combination of three capacitors C_1, C_2 and C_3.



Here, $C_1 = \frac{\varepsilon_0 A/3}{d}$; $C_2 = \frac{\varepsilon_0 A/3}{d+b}$; $C_3 = \frac{\varepsilon_0 A/3}{d+2b}$

$\therefore C = C_1 + C_2 + C_3 = \frac{\varepsilon_0 A}{3}\left[\frac{1}{d} + \frac{1}{d+b} + \frac{1}{d+2b}\right]$

$= \frac{\varepsilon_0 A}{3}\left[\frac{(d+b)(d+2b)+d(d+2b)+d(d+b)}{d(d+b)(d+2b)}\right]$

$= \frac{\varepsilon_0 A(3d^2+6bd+2b^2)}{3d(d+b)(d+2b)}$

SPACE FOR ROUGH WORK

SPACE FOR NOTES