CLASS – 11

WORKSHEET- UNITS AND MEASUREMENT

A. UNIT SYSTEM

(1 Mark Questions)

1. Which of the following system of units is not based on unit of mass, length and time?
(a) CGS (b) FPS (c) MKS (d) SI

Sol. (d)
SI is not based on units of mass, length and time alone.

2. How many light years are there in one metre?

Sol. 1.05×10^{-16} light years are there in one metre.

3. Why length, mass and time are chosen as base quantities in mechanics?

Sol. The length, mass and time cannot be derived from any other physical quantities and all physical quantities of mechanics. So, length, mass and time are chosen as the base quantities in mechanics.

(2 Marks Questions)

4. Express the average distance of the earth from the sun in (i) light year and (ii) parsec.

Sol. Sun is 8.3 Light-minutes away from the Earth. 8.3 light-minutes = 1.579×10^{-5} light-years. So, Distance from the Sun to the Earth in Parsecs = **4.84×10^{-6} Parsecs**.

(3 Marks Questions)

5. Define the following: (a) Light year (b) Parsecond (c) Astronomical unit.

Sol. (a) LY- Light year is the distance traveled by light in vaccum in one year. (b) Parsec - parsec is the distance from where the semi major axis of the earth subtends an angle of 1 second. (c) Astronomical unit -1 astronomical unit is equal to the mean distance between earth and sun.

(5 Marks Questions)

6. Explain this statement clearly :

"To call a dimensional quantity 'large' or 'small' is meaningless without specifying a standard for comparison". In view of this, reframe the following statements wherever necessary :

Sol. The given statement is true. A dimensional quantity may be small with respect to one reference and maybe large with respect to another reference. Hence, we require a standard reference to judge for comparison.

(a) atoms are very small objects.

Sol. An atom is a very small object with respect to a tennis ball. (but larger than an electron!)

(b) a jet plane moves with great speed

Sol. A jet plane moves with great speed with respect to a train.

(c) the mass of Jupiter is very large

Sol. The mass of Jupiter is very large as compared to an apple.

(d) the air inside this room contains a large number of molecules

Sol. The air inside this room contains a large number of molecules as compared to in your lungs.

(e) a proton is much more massive than an electron

Sol. A proton is much more massive than an electron

(f) the speed of sound is much smaller than the speed of light

Sol. The speed of sound is less than the speed of light

B. MEASUREMENT OF LENGTH, MASS, TIME

(1 Mark Questions)

1. Answer the following : (1 mark each)

(a)You are given a thread and a metre scale. How will you estimate the diameter of the thread?

Sol. The thread is wound on the metre scale so that it turns are as close as possible. Thickness 'l' of the thread coil is measured and the number of turns 'n' o fthe thread coil is mounted.

Therefore thickness of thread = l/n cm.

(b) A screw gauge has a pitch of 1.0 mm and 200 divisions on the circular scale. Do you think it is possible to increase the accuracy of the screw gauge arbitrarily by increasing the number of divisions on the circular scale?

Sol. Lest count of a screw gauge = $\frac{\textbf{Pitch}}{\textbf{Number of divisions on circular scale}}$

Theoretically, it appears that the least count can be decreased by increasing the number of divisions on the circular scale. Practically, it may not be possible to take the reading precisely due to low resolution of human eye.

(c) The mean diameter of a thin brass rod is to be measured by vernier callipers. Why is a set of 100 measurements of the diameter expected to yield a more reliable estimate than a set of 5 measurements only?

Sol. Larger the number of readings, closer is the arithmetic mean to the true value and hence smaller the random error. Hence result with a set of 100 measurements is more reliable than that with a set of 5 measurements.

2. The number of significant figures in 0.06900 is
(a) 5 (b) 4 (c) 2 (d) 3

Sol. (b)

3. The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is
(a) 663.821 (b) 664 (c) 663.8 (d) 663.82

Sol. (c)

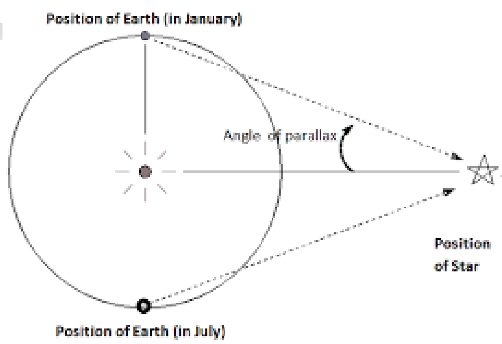
4. The mass and volume of a body are 4.237 g and 2.5 cm^3, respectively. The density of the material of the body in correct significant figures is
(a) 1.6048 g cm^{-3} (b) 1.69 g cm^{-3} (c) 1.7 g cm^{-3} (d) 1.695 g cm^{-3}

Sol. (c)

(2 Marks Questions)

5. Describe the parallax method to find the distance of an inferior planet from earth.

Sol. Copernicus method: Copernicus assumed circular orbits for the planets. The angel formed at the earth between the earth planet direction and earth sun direction is called the planet's elongation as shown in figure.



R_{ps} = distance of the planet from the sun, R_{pe} = distance of planet from the earth, R_{es} = distance of the earth from the sun, ε = planet's elongation.

When the elongation attaisn its maximum value and the planet appears farthest from the sun, the angle subtended by the sumn and the earth at the planet is 90°. Then from the right angle triangle shown in figure, we find that $R_{ps}/R_{es} = \sin \varepsilon$

Hence the distance of the planet from the sun is $R_{ps} = \sin \varepsilon$, $R_{es} = \sin \varepsilon$. AU

Where R_{es} is the average distance of the earth fromn the sun and is called astronomical unit (AU).

6. A new unit of length is chosen such that the speed of light in vacuum is unity. What is the distance between the Sun and the Earth in terms of the new unit if light takes 8 min and 20 s to cover this distance?

Sol. Speed of light = 1 new unit of length 1s
Time = 8 min 20s – 8×60+20 = 500s
Distance between the earth and the sun = speed of light × time = 1 × 500
= 500 new units of length.

7. The photograph of a house occupies an area of 1.75 cm^2 on a 35 mm slide. The slide is projected on to a screen, and the area of the house on the screen is 1.55 m^2. What is the linear magnification of the projector-screen arrangement?

Sol. Size of object = $1.75cm^2 = 1.75 \times 10^{-4}m^2$
Size of image = $1.55m^2$
Areal magnification= $\frac{\textbf{Size of image}}{\textbf{Since of object}} = \frac{\mathbf{1.55}}{\mathbf{1.75\times10^{-4}}} = \mathbf{8857}$
Linear magnification = $\sqrt{\textbf{Area magnification}} = \sqrt{\mathbf{8857}}$ = 94.1.

8. When the planet Jupiter is at a distance of 824.7 million kilometres from the Earth, its angular diameter is measured to be 35.72″ of arc. Calculate the diameter of Jupiter.

Sol. Distance of Jupiter from the earth, S = 824.7×10^6 km
Angular diameter of Jupiter, θ = 35.72° = $\left(\frac{\mathbf{35.72}}{\mathbf{60\times60}}\right)^{\circ} = \frac{\mathbf{35.72}}{\mathbf{3600}} \times \frac{\boldsymbol{\pi}}{\mathbf{180}}\,\textbf{rad}$
Diameter of Jupiter = S × θ = $824.7 \times 10^6 \times \frac{\mathbf{35.72}}{\mathbf{3600}} \times \frac{\boldsymbol{\pi}}{\mathbf{180}} = 1.426\times10^5$km.

(3 Marks Questions)

9. Explain the method of measuring the size of oleic acid molecule.

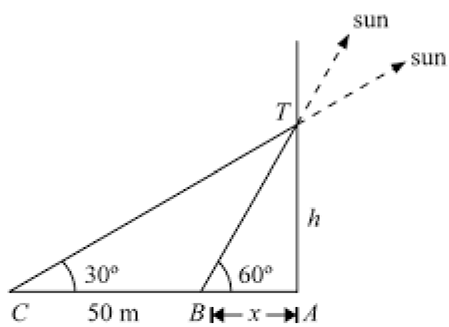
Sol. A simple method for estimating the molecular size of oleic acid is given below:
Dissolve 1cm^3 of oleic alcohol to make a solution of 20cm^3. Take 1cm^2 of this solution and dilute it to 20cm^3 using alcohol.
So concentration of the solution = $\frac{1}{20\times20}cm^3$. Now sprinkle some lycopodium powder on the surface of water in a large trough and put one drop of this solution in the water. This forms a thin, large and roughly circular film of molecular thickness on water surface.
Let volume of each drop = V cm^3
Suppose we dropped n drops then volume of n drops of solution = nV cm^3
Therefore amount of oleic acid in this solution = $nV\left(\frac{1}{20\times20}\right)cm^3$

If a solution of oleic acid spread to form a film of are A cm^2, then the thickness of the film, $t = \frac{\text{Volume of the film}}{\text{Area of the film}}$ or $t = \frac{nV}{20 \times 20 \times A}$ cm

Assuming that the film has one molecular thickness then t will be approximately the size or diameter of a molecule of oleic acid. The value of t is found to be or the order of 10^{-9}m.

10. The shadow of tower standing on a level plane is found to be 50m longer when sun's altitude is 30° than when it is 60°, Find the height of the tower.

Sol.

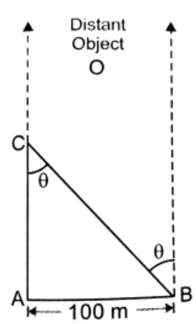


Here d = 50cm, $\theta_1 = 60°$, $\theta_2 = 30°$.

$h = \frac{d}{\cot\theta_2 - \cot\theta_1} = \frac{50}{\cot 30° - \cot 60°}$

So, $h = \frac{50}{\sqrt{3} - 1/\sqrt{3}} = \frac{50\sqrt{3}}{3-1} = 25\sqrt{3} = 25 \times 1.732 = 43.3$ m.

11. A man wishes to estimate the distance of a nearby tower from him. He stands at a point A in front of the tower C and spots a very distance object O in line with AC. He then walks perpendicular to AC upto B, a distance of 100m, and looks at O and C again. Since O is very distant, the direction BO is practically the same as AO; but he finds the line of sight of C shifted from the original line of sight by an angle θ = 40° (a is known as parallax). Estimate the distance of the tower C from his original position A.



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12. A drop of olive oil of radius 0.25mm spreads into a circular film of radius 10cm on the water surface. Estimate the molecular size of olive oil.

Sol. Let us suppose that olive oil spreads as a film which is one molecule thick on the water surface. then the thickness of the film would give an estimate of diameter of the molecule.

d= volume of the oil drop/ area of the film

$= 4/3 \times \pi \times (0.025\text{cm})^3/(\pi \times (10\text{ cm})^2$

$= 2.08\text{X}10^{-7}$ cm

13. If nth division of main scale coincides with (n+1)th divisions of vernier scale, find the least count of the vernier. Given one main scale division is equal to ‘a’ units.

Sol. Given data states that the nth division on main scale coincide with $(n+1)^{th}$ division of Vernier scale, least count has to be calculated.

Least count is the difference between the *main scale division (MSD)* and the *Vernier scale division (VSD)*.

Additional data is that 1 MSD equals **"a",** therefore,

n MSD = (n+1) VSD

=> $(\frac{n}{n+1})$ MSD = 1 VSD

=> 1 MSD = *a*

=> 1 VSD = $(\frac{n}{n+1} \times a)$

=> Least Count = a – $(\frac{na}{n+1}) = (\frac{a}{n+1})$

14. The nearest star to our solar system is 4.29 light-years away. How much is this distance in terms of parsecs? How much parallax would this star (named Alpha Centauri) show when viewed from two locations of the Earth six months apart in its orbit around the Sun?

Sol. As 1 light year = 9.45×10^{15} m, 1 parsec = 3.08×10^{16}m.

Therefore distance of Alpha Centauri from the earth, S = 4.29 light years = 4.29 × 9.46 × 10^{15}m = $\frac{\mathbf{4.29\times9.46\times10^{15}}}{\mathbf{3.08\times10^{16}}}$ $\mathbf{parsec = 1.32\ parsec}$

In an orbit around the sun, the distance between the two locations of the earth six months apart.

b = Diameter of the earth’s orbit = 2AU

Parallax of star, $\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{b}{S} = \frac{1\text{AU}}{1.32 \text{ parsec}}$ = 1.515 s of arc.

C. SIGNIFICANT FIGURES

(1 Mark Questions)

1. Solve the following and express the result to an appropriate number of significant figures: (1 mark each)

(i) Add 6.2g, 4.33g and 17.456 g

Sol. 6.2g + 4.33g + 17.456 = 27.986g ≅ 28.0 g

(ii) Subtract 63.54kg from 187.2 kg

Sol. (187.2 – 63.54)kg = 123.66kg = 123.7 kg

(iii) 75.5×125.5×0.51

Sol. 75.5 × 125.5 × 0.51 = 4820.826 = 4800

(iv) 2.13×24.78/458.2

Sol. $\frac{2.13\times24.78}{458.2} = \frac{52.7814}{458.2}$ = 0.1152 = 0.115

(v) $2.51\times10^{-4}\times1.81\times10^{7}/0.4463$

Sol. $\frac{2.51\times10^{-4}\times1.81\times10^{7}}{0.4463} = \frac{4.5431\times10^{3}}{0.4463} = 10.179\times10^{3} = 1.0179\times10^{4} = 1.02\times10^{4}$.

2. State the number of significant figures in the following measurements: (1 mark each)

(a) 0.009m^2 - 1

(b) 5.049Nm^{-2} - 4

(c) 0.1890g cm^{-3} - 4

(d) $1.90\times10^{11}\text{kg}$ - 3

(e) 0.20800m - 5

(f) 5.308J - 4

3. A cube has a side of length 1.2×10^{-2}m. Calculate its volume.

(a) $1.7\times10^{-6}\text{m}^3$ (b) $1.73\times10^{-6}\text{m}^3$ (c) $1.0\times10^{-6}\text{m}^3$ (d) $1.732\times10^{-6}\text{m}^3$

Sol. (a)

4. State the number of significant figures in the following : (1 mark each)

(a) 0.007 m^2 - 1; 7

(b) 2.64×10^{24} kg - 3; 2, 6, 4

(c) 0.2370 g cm^{-3} - 4; 2, 3, 7, 0

(d) 6.320 J - 4; 6, 3, 2, 0

(e) 6.032 N m^{-2} - 4; 6, 0, 3, 2

(f) 0.0006032 m^2 - 4; 6, 0, 3, 2

(2 Marks Questions)

5. The length, breadth and thickness of a rectangular sheet of metal are 4.234 m, 1.005 m, and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Sol. Here l = 4.234m, b = 1.005m, h = 2.01cm = 0.0201m

Area of sheet = 2(lb + bh + hl)

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)m^2$

$= 2\times 4.3604739\ m^2 = 8.7209478\ m^2$

$= 8.72\ m^2$ [Rounded off upto 3 significant figures]

Volume of sheet = lbh = $4.234 \times 1.005 \times 0.0201\ m^3$

$= 0.08555289\ m^3 = 0.0855m^3$ [Rounded upto three significant figures]

6. The mass of a box measured by a grocer's balance is 2.30 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is
(a) the total mass of the box,
(b) the difference in the masses of the pieces to correct significant figures?

Sol. (a) Total mass of the box = 2.3kg + 0.02015kg + 0.02017kg = 2.34032kg = 2.3kg

The result has been rounded off to first place of decimal because mass (2.3kg) of box has digits upto this place of decimal.

(b) Difference in masses of 2 gold pieces = 20.17g – 20.15g = 0.02g.

(3 Marks Questions)

7. The radius of a muonic hydrogen atom is 2.5×10^{-13}m. What is the total atomic volume in m^3 of a mole of such hydrogen atoms?

Sol. We know that the shape of atom is spherical.

Now, One mole of Hydrogen consist of 12.044×10^{23} atoms.

$\therefore$ Volume of one mole of atoms of hydrogen = $12.044 \times 10^{23} \times$ Volume of 1 atom of Hydrogen.

Hence, the volume of 1 mole of hydrogen atoms is $7.89 \times 10^{-14}\ m^3$.

D. ERRORS AND PROPAGATION OF ERROR

(1 Mark Questions)

1. Which of the following measurements is most precise?
(a) 5.00km (b) 5.00 m (c) 5.00 cm (d) 5.00 mm

Sol. (d)

2. You measure two quantities as A = 1.0 m ± 0.2 m, B = 2.0 m ± 0.2 m. We should report correct value for $\sqrt{AB}$ as:
(a) 1.4 m ± 0.4 m (b) 1.41m ± 0.15 m (c) 1.4m ± 0.3 m (d) 1.4m ± 0.2 m

Sol. (d)

3. Which of the following measurements is most precise?
(a) 5.00 mm (b) 5.00 cm (c) 5.00 m (d) 5.00 km.

Sol. (a)

(2 Marks Questions)

4. Mention the various sources of occurrence of errors, while taking measurements.

Sol. While taking measurements, sources of occurrence of errors may be as follows:
(i) Instrumental errors due to faulty graduations, zero error etc.
(ii) Personal errors due to personal peculiarities of the experiment.
(iii) Error due to imperfection in experimental arrangement, such as loss of heat due to radiation in calorimetry.
(iv) Errors due to external causes such as expansion of scale due to rise in temperature.

(3 Marks Questions)

5. Distinguish between 'accuracy' and 'precision' of a measurement.

Sol. Precision: is the degree of exactness or refinement of a measurement. It defines the limitation of measuring instrument. It is not the result of human error or lack of calibration. It depends upon the least count of measuring instrument. The smaller the least count, the more precise will be the measurement.
Accuracy: is the extent to which a reported measurement approaches the true value. IT depends upon the number of significant figures in it. The larger the significant digits, the higher the accuracy. Problem with accuracy are due to errors. For example personal error, method error, instrumental error. As we reduce the errors, measurement accuracy increases.

6. If two resistors of resistances $R_1 = (4 \pm 0.5)\Omega$ and $R_2 = (16 \pm 0.5)\,\Omega$ are connected in series and (ii) in parallel; find the equivalent resistance in each case with limits of percentage error

Sol. resistance of first resistor $R_1 = 4 \pm 0.5$ ohm.
resistance of second resistor $R_2 = 16 \pm 0.5$ ohm.
total change in resistance for circuit is
$\Delta R = 0.5 + 0.5$
$\Delta R = 1 ohm$
<u>case 1):</u> resistors in series
equivalent resistance in series connection is given by

$R_{es} = R_1 + R_2$
$R_{es} = 4 + 16$
$R_{es} = 20ohm$

Here R_{es} is series equivalent resistance.

Now, percentage error in series connection is

$\%e_s = \frac{\Delta R}{R_{es}} * 100$

From the given values we know that change in resistance of two resistors will be

Then,

$\%e_s = \frac{1}{20} * 100$

$\%e_s = 0.05 * 100$
$\%e_s = 5\%$

case 2): resistors in parallel

equivalent resistance in parallel connection is given by

percentage error of parallel connection is

∴ **case (1):**

equivalent resistance in series $R_{es} = 20ohm$

percentage error in series $\%e_s = 5\%$

case (2):

equivalent resistance in parallel $R_{ep} = \frac{16}{5}ohms$

percentage error in parallel $\%e_p = 31.25\%$

7. The length and breadth of a rectangle are (5.7±0.1)cm and (3.4±0.2)cm. Calculate area of the rectangle with error limits.

Sol. Given:

Dimensions of a rectangle,

Length,l= 5.7 ± 0.1 cm , breadth,b = 3.4 ± 0.2 cm

To find area of rectangle without the error limits,

We know that area of a square, A = l×b

On substituting the value,

A = (5.7 × 3.4) = 19.38 cm^2

To find the area of rectangle with error limits,

$\Delta A = A \times (\Delta L/L + \Delta b/b) = 19.38 \times (0.1/5.7 + 0.2/3.4) = 1.48\ cm^2$

Finally the area of rectangle with the error limits = (19.38 ± 1.48) cm

8. The measure of the diameter of a cylinder is (1.60±0.01)cm and its length is (5.0±0.1)cm. Calculate the percentage error in its volume.

Sol. Volume of cylinder , $V = \pi r^2 L = \pi(d/2)^2 L = \pi d^2 L/4$

Here, r is the radius of base of cylinder,
L is the length of cylinder, and d is diameter.
For finding error in volume, we have to use the formula,
$\Delta V/V = 2\Delta d/d + \Delta L/L$
$\Rightarrow \Delta V/V \times 100 = 2 \times \{\Delta d/d \times 100\} + \Delta L/L \times 100$
$\Rightarrow$ % error in volume = $200 \times \Delta d/d + 100 \times \Delta L/L$
Here, $\Delta d = 0.01$, $d = 1.6$, $\Delta L = 0.1$ and $L = 5$
$\therefore$ % error in volume = $200 \times 0.01/1.6 + 100 \times 0.1/5$
= 20/16 + 2 = 1.25 + 2 = 3.25 %
Hence, percentage error in volume is 3.25 %

9. Show that the maximum error in the sum of two quantities is equal to the sum of the absolute errors in the two individual quantities.

Sol. Let ΔA and ΔB be the absolute errors in the two quantities A and B respectively. Then
Measured value of A = $A \pm \Delta A$ and measured value of B = $B \pm \Delta B$
Consider the sum, Z = A + B
The error in ΔZ is the given by
$Z \pm \Delta Z = (A \pm \Delta A) + (B \pm \Delta B)$
$= (A + B) \pm (\Delta A + \Delta B) = Z \pm (\Delta A + \Delta B)$ or $\Delta Z = \Delta A + \Delta B$
Hence the rule : The maximum possible error in the sum of two quantities is equal to the sum of the absolute errors in the individual quantities.

10. Show that the maximum error in the quotient of two quantities is equal to the sum of their individual fractional errors.

Sol. Consider the quotient, Z = A/B
The error ΔZ in Z is given by $Z \pm \Delta Z = \frac{A \pm \Delta A}{B \pm \Delta B} = \frac{A\left(1 \pm \frac{\Delta A}{A}\right)}{B\left(1 \pm \frac{\Delta B}{B}\right)} = \frac{A}{B}\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)^{-1}$
Or $Z \pm \Delta Z = Z\left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right)$ [Since $(1+x)^n = 1 + nx$, when $x \ll 1$]
Dividing both sides by Z we get
$1 \pm \frac{\Delta Z}{Z} = \left(1 \pm \frac{\Delta A}{A}\right)\left(1 \pm \frac{\Delta B}{B}\right) = 1 \pm \frac{\Delta A}{A} \mp \frac{\Delta B}{B} \pm \frac{\Delta A}{A} . \frac{\Delta B}{B}$
As the terms $\frac{\Delta A}{A}$ and $\frac{\Delta B}{B}$ are small, their product term can be neglected. The maximum fractional error in Z is given by $\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
Hence the rule: The maximum fractional error in the quotient fo two quantities is equal to the sum of their individual fractional errors.

11. A student measures the thickness of a human hair by looking at it through a microscope of magnification 100. He makes 20 observations and finds that the

average width of the hair in the field of view of the microscope is 3.5 mm. What is the estimate on the thickness of the hair?

Sol. Average thickness of hair as observed through microscope = 3.5mm

Magnification produced by the microscope = 100

Actual thickness of hair = $\frac{\textbf{Observed thickness}}{\textbf{Magnification}} = \frac{\textbf{3.5}}{\textbf{100}} = \mathbf{0.035mm}$.

12. A physical quantity P is related to four observables a, b, c and d as follows:

$P = a^3b^2 / \sqrt{c}\, d$

The percentage errors of measurement in a, b, c and d are 1%, 3%, 4% and 2%, respectively. What is the percentage error in the quantity P? If the value of P calculated using the above relation turns out to be 3.763, to what value should you round off the result?

Sol. Given P = $\frac{\mathbf{a^3b^2}}{\sqrt{\mathbf{cd}}}$

The percentage error in the quantity P is given by

$100 \times \frac{\Delta P}{P} = \mathbf{3} \times \mathbf{100}.\frac{\Delta a}{a} + \mathbf{2} \times \mathbf{100}.\frac{\Delta b}{b} + \frac{1}{2} \times \mathbf{100}.\frac{\Delta c}{c} + \mathbf{100} \times \frac{\Delta d}{d}$

= 3 × 1% + 2 × 3% + ½ × 4% + 2% = 13%

Since 13% = 0.13, so there are two significant figures in the percentage error. Hence P should also be rounded off to 2 significant figures.

Therefore P = 3.763 = 3.8.

(5 Marks Questions)

13. The diameter of a wire as measured by a screw gauge was found to be 0.026cm, 0.028cm, 0.029cm, 0.027cm, 0.024cm, 0.027cm. Calculate (i) mean value of the diameter (ii) mean absolute error (iii) relative error (iv) percentage error. Also express the result in terms of absolute error and percentage error.

Sol. Given observations are- 0.026,0.028, 0.027, 0.029, 0.024, 0.027

No. of Observations = 6

(i)∵ Mean = Sum of all observations/No. of observations.

= (0.026 + 0.028 + 0.027 + 0.029 + 0.024 + 0.027)/6 = 0.161/6

= 0.0268 cm.

For the Approximate Calculations, Taking Mean = 0.027 cm.

By taking in approx, answer can be easily calculated.

(ii) For the Relative Error

x_1 = |0.026 - 0.027| = 0.001

x_2 = |0.028 - 0.027| = 0.001

x_3 = |0.027 - 0.027| = 0.000

$x_4 = |0.029 - 0.027| = 0.002$

$x_5 = |0.024 - 0.027| = |-0.003|$ (

MODULUS OF ANYTHNG IS ALWAYS POSITIVE)

= 0.003

$x_6 = |0.027 - 0.027| = 0.000$

(iii)∵ Absolute Error(Δx) = $(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)/6$

= (0.001 + 0.001 + 0.000 + 0.002 + 0.003 + 0.000)/6 = 0.007/6

= 0.00116 cm.

(iv)∴ Relative Error = Absolute Error/Mean Value

= 0.00116/0.027 = 0.04296 cm.

14. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0cm known to 1mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. What is the accuracy in the determination of g?

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E. DIMENSIONS

(1 Mark Questions)

1. Name any three physical quantities having the same dimensions and also give their dimensions.

Sol. The three physical quantities which are having same dimensions are Work, Torque, and Energy. Their dimensions are ML^2T^{-2}.

2. Name the physical quantities whose dimensional formulae are as follows: (1 mark each)

(i) $ML^2 T^{-2}$ – Work, energy, torque (ii) $ML^2 T^{-3}$ - Power

(iii) MT^{-2} – Force constant (iv) $ML^{-1} T^{-1}$ – Coefficient of viscosity

(v) $ML^{-1} T^{-2}$ – Young's modulus

3. Deduce the dimensional formulae for the following physical quantities: (1 mark each)

(i) Gravitational constant = $G = \frac{\text{Force}\times(\text{distance})^2}{\text{Mass}\times\text{ mass}} = \frac{MLT^{-2}L^2}{M\times M} = [M^{-1}L^3T^{-2}]$

(ii) Power = $\frac{\text{Work}}{\text{Time}} = \frac{ML^2T^{-2}}{T}$=$[ML^2T^{-3}]$

(iii) Young's modulus = $N/m^2 = kgm/s^2m^2 = \frac{kg}{ms^2} = [ML^{-1}T^{-2}]$

(iv) Coefficient of viscosity: $\frac{\text{Force}\times\text{distance}}{\text{Area}\times\text{velocity}} = \frac{MLT^{-2}\times L}{L^2\times LT^{-1}} = [ML^{-1}T^{-1}]$

(v) Surface tension: $\frac{\text{Force}}{\text{Length}} = \frac{MLT^{-2}}{L} = [MT^{-2}]$ or $[ML^0T^{-2}]$

(vi) Planck's constant: $\frac{E}{v} = \frac{\text{Energy}}{\text{Frequency}} = \frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$

4. The equation of state for real gas is given by $(P+a/V^2)(V – b) = RT$. The dimensions of the constant a are:

(a) $[ML^5T^{-2}]$ (b) $[M^{-1} L^5T^2]$ (c) $[ML^{-5}T^{-1}]$ (d) $[ML^5T^{-1}]$

Sol. (a)

5. Which of the following has the dimensions of pressure?

(a)) $[MLT^{-2}]$ (b)) $[ML^{-1}T^{-2}]$ (c)) $[ML^{-2}T^{-2}]$ (d)) $[M^{-1} L^{-1}]$

Sol. (b)

6. Fill in the blanks (1 mark each)

(a) The volume of a cube of side 1 cm is equal to …..m^3

(b) The surface area of a solid cylinder of radius 2.0 cm and height 10.0 cm is equal to…$(mm)^2$

(c) A vehicle moving with a speed of 18 km h^{-1} covers….m in 1 s

(d) The relative density of lead is 11.3. Its density is ….g cm^{-3} or ….kg m^{-3}

Sol. (a) $V = t^3 = (1cm)^3 = (10^{-2}m)^3 = 10^{-6}m^{-3}$.

(b) r = 2cm = 20mm, h = 10cm = 100mm

$5 = 2\pi r(r + h) = 2 \times 3.14 \times 20(20+100) = 15072mm^2$.

(c) $v = 18\ km/h = \frac{18\times1000m}{60\times60s} = 5m/s$

(d) Density = Relative density ×density of water at 4°C

$= 11.3 \times 1gcm^{-3} = 11.3\ gcm^{-3}$.

$= 11.3 \times 10^3\ kg\ m^{-3} = 11300\ kgm^{-3}$

7. Fill in the blanks by suitable conversion of units

(a) 1 kg m^2 s^{-2} = ….g cm^2 s^{-2}
(b) 1 m = ….. 1ly
(c) 3.0 m s^{-2} = …. km h^{-2}
(d) G = 6.67 × 10^{-11} N m^2 $(kg)^{-2}$ = …. $(cm)^3 s^{-2}$ g^{-1}

Sol. (a) $1kgm^2s^{-2} = 1(10^3g)(10^2cm)^2s^{-2} = 10^7cm^2s^{-2}$
(b) As 1 light year = 9.46×10^{15}m
Therefore 1m = $\frac{1}{9.46\times10^{15}}$ light year = 1.053 × 10^{-16} light year = 10^{-16} light year
(c) $3ms^{-2} = 3(10^{-3}km)\left(\frac{1}{60\times60}h\right)^2 = 3\times10^{-3}\times3600\times3600kmh^{-2}$
$= 3.888\times10^4$ kmh^{-2} = 3.9×10^4 kmh^{-2}.
(d) G = 6.67 × $10^{11}Nm^2kg^{-2}$ = 6.67 × 10^{11} kg $ms^{-2}.m^2kg^{-2}$
$= 6.67\times10^{-11}m^2s^{-2}kg^{-1} = 6.67\times10^{-11}(10^2cm)s^{-2}(1000g)^{-1} = 6.67\times10^{-8}s^2g^{-1}$.

8. Which of the following pairs of physical quantities does not have same dimensional formula?
(a) Work and torque. (b) Angular momentum and Planck's constant.
(c) Tension and surface tension. (d) Impulse and linear momentum

Sol. (c)

9. If momentum (P), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula
(a) $(P^1 A^{-1} T^1)$ (b) $(P^2 A^1 T^1)$ (c) $(P^1 A^{-1/2} T^1)$ (d) $(P^1 A^{1/2} T^{-1})$

Sol. (d)

(2 Marks Questions)

10. Convert 1 dyne into newton.

Sol. 1 dyne= x newton
The dimensional form of the above expression
$H_1^1 L_1^1 T_{21} = xH_1^2 L_1^2 T_{22}$
To find the value of x
$x = 1 \times \left[\frac{H_1}{H_2}\right]^1 \left[\frac{L_1}{L_2}\right]^1 \left[\frac{T_1}{T_2}\right]^2$
$X = 1 \times [gm/kg][cm/m][S/S]$
Convert the value of kg into gm and m into cm and cancel the common terms
$\mathrm{X} = 1 \times [\mathrm{gm}/1000\mathrm{gm}][\mathrm{cm}/100\mathrm{cm}]1$
$X = 1 \times \left[\frac{1}{10^3}\right]\left[\frac{1}{10^2}\right] \times 1$
$X = \frac{1}{10^5}$
$= 10^{-5}$

1 dyne = 10^{-5}N

11. If the value of universal gravitational constant in SI is 6.6×10^{-11} Nm^2 kg^{-2}, then find its value in CGS system.

Sol. Given : G = 6.67 x 10^{-11} N m^{-2}/kg^2

To determine the value of G is (G.S. system formula used)

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

The dimensional formula of $G = [M^{-1}L^3T^{-2}]$

$\therefore \quad a = -1, b = 3, c = -2$

Suppose,

n_2 dyne cm^2.g^{-2} = 6.67×10^{-11} N.m^2 kg^{-2}

$\therefore \quad n_1 = 6.67\times10^{-11}$

Therefore $\quad n_2 = 6.67\times10^{-11}\left[\frac{kg}{g}\right]^{-1}\left[\frac{m}{cm}\right]^3\left[\frac{s}{s}\right]^{-2}$

$= 6.67\times10^{-11}\left[\frac{1000\,g}{g}\right]^{-1}\left[\frac{100\,cm}{cm}\right]^3.[1]^{-2}$

$= 6.67\times10^{-11}[1000]^{-1}\times[100]^3\times1$

$= 6.67\times10^{-11}\times\frac{1}{1000}\times100\times100\times100$

$= 6.67\times10^{-8}$

$\therefore \quad \mathbf{G = 6.67\times10^{-8}}$ dyne cm^{2}g^{-2}

12. Name any three physical quantities having the same dimensions and also give their dimensions.

Sol. Work, energy and torque. Each quantity has the dimensions [ML^{2}T^{-2}].

(3 Marks Questions)

13. (a) Explain the principle of homogeneity of dimensions.

(b) Pressure is defined as momentum per unit volume. Is it true?

Sol. (a) According to this principle, a physical relation is dimensionally correct if the dimensions of fundamental quantities (mass, length and time etc.) are the same in each and every term on either side of the equation. This principle is based on the fact that only quantities of the same kind (or dimensions) can be added or subtracted. For example, consider the equation, A = B +C. Here the quantities A, B and C must have the same dimensions.

(b) No because this is mot dimensionally true.

$$\frac{\text{Measurement}}{\text{Volume}} = \frac{\text{Mass}\times\text{velocity}}{\text{Volume}} = \frac{[MLT^{-1}]}{[L^3]} = [ML^{-2}T^{-1}]$$

Pressure = $\frac{\text{Force}}{\text{Area}} = \frac{[MLT^{-2}]}{[L^2]} == [ML^{-1}T^{-2}]$

14. The frequency depends upon (a) tension of the string. (b) length of the string. (c) linear mass density. Using dimensional analysis, derive an expression of frequency.

Sol. Let the relation be: Frequency $\upsilon = kL^a f^b m^c$ where k is a dimensionless constant, L is the length, f is the tension and m is the linear mass density. The dimensions of each quantity are:

$[L] = [M^0L^1T^0]$; $[f] = [M^1L^1T^{-2}]$; $[m] = [M^1L^{-1}T^0]$, $[\upsilon] = [M^0L^0T^{-1}]$

$[M^0L^0T\text{-}^1] = [M^0L^1T^0]^a\ [M^1L^1T^{-2}]^b[M^1L^{-1}T^0]^c$. This now equating the dimensions on both sides of the equation.

So, b +c = 0; a + b – c = 0; -2b = -1

Thus b = ½, c = - ½, a = -1

Using the formula for the frequency is $\upsilon = \frac{k}{L}\sqrt{\frac{f}{m}}$

15. The velocity 'v' of water waves depends on the wavelength 'λ', density of water 'ρ' and the acceleration due to gravity 'g'. Deduce by the method of dimensions the relationship between these quantities.

Sol. The velocity of the water wave (v) may depend on their wavelength 'λ', the density of water 'ρ', and acceleration due to gravity 'g'. The method of dimensions gives the relation $v^2 \propto \lambda g \rho$ between these quantities as. (A) $\mathbf{v^2 \propto \lambda g \rho}$

16. Derive dimensionally the relation: $S = ut + ½ at^2$.

Sol. We have `$S = ut + 1/2\ at^2$

Checking the dimensions on both sides,

LHS = $[M^0\ L^1\ T^0]$ and

RHS = $[LT^{-1}][\ T] + [LT^{-2}]\ [T^2] = [M^0\ L^1\ T^0\ ] + [M^0\ L^1\ T^0] = [\ M^0\ L^1\ T^0]$

Comparing the `LHS and RHS` , we get `LHS = RHS`

17. Define dimensional formula. Give uses of dimensional analysis. Write down the limitations of dimensional analysis.

Sol. The method of studying a physical phenomenon on the basis of dimensions is called dimensional analysis. Following are the three main uses of dimensional analysis: (i) To convert a physical quantity from one system to another (ii) To check the correctness of a given physical relation (iii) To derive a relationship between different physical quantities.

Limitations:

(i) The method does not give any information about the dimensionless constant K.

(ii) It fails when a physical quantity depends on more than three physical quantities

(iii) It fails when a physical quantity (e.g. $s = ut + ½ at^2$) is the sum or difference fo two or more quantities.
(iv) It fails to derive relationships which involve trigonometric logarithmic or exponential functions
(v Sometimes, it is difficult to identify the factors on which th ephysiocal quantity depends. The method becomes more complicated when dimensional constants like G, h, etc are involved.

18. State the principle of homogeneity of dimensions. Test the dimensional homogeneity of the following equation: $h = h_0 + v_0t + ½ gt^2$.

Sol. The principle of homogeneity of dimensions states that a physical equation will be dimensionally correct if the dimensions fo all the terms occurring on both sides fo the equation are the same.
$[h] = L$, $[h_0] = L$, $[v_0t] = LT^{-1}. T = L$, $[1/2gt^2] = LT^{-2}.T^2 = L$
As all the terms have the same dimensions, so the given equation is dimensionally homogenous.

19. A famous relation in physics relates 'moving mass' m to the 'rest mass' mo of a particle in terms of its speed v and the speed of light, c. (This relation first arose as a consequence of special relativity due to Albert Einstein). A boy recalls the relation almost correctly but forgets where to put the constant c. He writes :

$$m = \frac{m_0}{(1-v^2)^{1/2}}$$

Guess where to put the missing c.

Sol. Since quantities of similar nature can only be added or subtracted, v^2 cannot be subtracted from dimensionless constant 1. If should be divided by c^2 so as to make it dimensionless. Hence the corrected relation is $m = \frac{\mathbf{m_0}}{\left(\mathbf{1-\frac{v^2}{c^2}}\right)^{\mathbf{1/2}}}$.

20. In van der Waal's equation $(P + a/V^2)(V – b) = RT$. Determine the dimensions of a and b?

Sol. From van der Waal's equation, $\left(P + \frac{a}{V^2}\right)(V - b) = RT$
Dimension of P = Dimensions of $\frac{a}{V^2}$
Dimensions of a = Dimension of P × Dimension of V^2
$\Rightarrow a = \frac{[MLT^{-2}]}{[L^2]} \times [L^6] = \{ML^5T^{-2}]$
$\Rightarrow a = [ML^5T^{-2}]$
Dimension of V = Dimension of b ⇒ $b = [L^2]$.

21. Write the dimension of a in the relation $F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

Sol. $F = a\sqrt{x} \propto bt^2$

For dimension of a, $F = a\sqrt{x} = a(x)^{1/2}$

$a = \frac{F}{(x)^{1/2}} = \frac{[MLT^{-2}]}{[L]^{1/2}} = [ML^{\frac{1}{2}}T^{-2}]$

22. Give an example of (a) a physical quantity which has a unit but no dimensions. (b) a physical quantity which has neither unit nor dimensions. (c) a constant which has a unit. (d) a constant which has no unit

Sol. (a) Angle in a plane in measured in radian, though it has no dimensions. (b) Strain has neither units nor dimensions. (c) Universal gravitational constant (G) has a constant value : `$6.67xx10^{-11}$ Nm^2 kg^{-2}. (d) Reynold number is a constant which has no units.

23. The displacement of a progressive wave is represented by y = A sin(wt – k x), where x is distance and t is time. Write the dimensional formula of (i) ω and (ii) k.

Sol. According to the principle of homogeneity dimensional formula on L.H.S. & R.H.S. is equal.

Thus, the dimension of $A \ \sin(\omega t - kx)$ = Dimension of y

$\omega t - kx$ has no dimensions because it is an angle of sin.

$$\frac{2\pi}{T}t = Kx$$

$$[M^0L^0T^0] = k[L]$$

Thus, ω has no dimension & dimension of

$k = [M^0L^{-1}T^0]$.

24. Time for 20 oscillations of a pendulum is measured as t_1= 39.6 s; t_2= 39.9 s; t_3= 39.5 s. What is the precision in the measurements? What is the accuracy of the measurement?

Sol. (a) Precision is given by the least count of the instrument.

For 20 oscillations, precision = 0.1 s

For 1 oscillation, precision = 0.005 s.

(b) Average time t = (39.6 + 39.9 + 39.5/3)s = 39.6s

period = 39.6/20 = 1.98 s

Max. observed error = (1.995 - 1.980)s = 0.

(5 Marks Questions)

25. If the velocity of light c, the constant of gravitation G and Planck's constant h be chosen as fundamental units, find the dimensions of mass, length and time in terms of c, G and h.

Sol. Let $m \propto c^x h^y G^z$

$m = Kc^x h^y G^z$ ....(A)

$h = [ML^2T^{-1}]$, $c = [LT^{-1}]$, $G = [M^{-1}L^3T^{-2}]$ (k = dimensionless)

Or, $[ML^0T^0] = [LT^{-1}]^x [ML^2T^{-1}]^y [M^{-1}L^3T^{-2}]^z$
$=[M^{y-z}L^{x+2y+3z}T^{x-y-2z}]$
Comparing powers $- y - z = 1$ .... (1)
$x + 2y + 3z = 0$ ....(2)
$-x - y - 2z = 0$ ....(3)
Adding above all three equations- $2y = 1 \Rightarrow y = 1/2$, $z = -1/2$, $x = 1/2$
Putting in eq.n (A) $= kc^{1/2} h^{1/2}G^{-1/2}$ m = $k \sqrt{(ch/G)}$
(ii) Let $L \propto c^x h^y G^z$ L..........(B)
Substituting in B $[M^0LT^0] = [LT^{-1}]^x \times [ML^2T^{-1}]^y \times [M^{-1}L^3T^{-2}]^z = [M^{y-z}L^{x+2y+3z}T^{-x-y-2z}]$
Comparing powers $- y - z = 0$ ....(a)
$x + 2y + 3z = 1$ ....(b)
$-x - y - 2z = 0$ ....(c)
Adding (a), (b), (c), we get $y = 1/2$, $z = 1/2$ 12 , $x = -3/2$
Putting in (B), $L = kc^{-3/2} h^{1/2}B^{1/2}$
$L = k \sqrt{(hG/c^3)}$
(iii) Let $L \propto c^x h^y G^z$
$T = kc^x h^y G^z$ ....(C)
Substituting in B $[M^0L^0T] = [LT^{-1}]^x \times [ML^2T^{-1}]^y \times [M^{-1}L^3T^{-2}]^z$
$= [M^{y-z}L^{x+2y+3z}T^{-x-y-2z}]$
Comparing powers $y - z = 0$ ....(1)
$x + 2y + 3z = 1$ ....(2)
$-x - y - 2z = 0$ ....(3)
Adding (1), (2), (3), we get $y = 1/2$, $z = 1/2$, $x = -5/2$
Putting in (B), $T = kc^{-5/2} h^{1/2} B^{1/2}$
$T = k \sqrt{h G/c^5}$

F. CASE STUDY

1. The dimensional method is a very conventional way of finding the dependence of a physical quantity on other physical quantities of a given system. This method has its own limitations. In a complicated situation, it is often not easy to guess the factors on which a physical quantity will depend. Secondly, this method gives no information about the dimensionless, proportionality constant. Thirdly this method is used only if physical quantity depends on the product of other physical quantities. Fourthly this method will not work if a physical quantity depends only on another quantity as a trigonometric or exponential function. Finally, this method does not give complete information in cases where a physical quantity depends on more than three quantities in problems in mechanics.

(i) The dimensionless method cannot be obtain dependence of
(a) the height to which a liquid rises in a capillary tube on the angle of contact.
(b) speed of sound in an elastic medium on the modulus of electricity.

(c) height to which a body, projected upwards with a certain velocity, will rise on time t.
(d) the decrease in energy of a damped oscillator on time t.

Sol. The correct choices are a, c and d. The height of a liquid in a capillary tube depends on cos θ, where θ is the angel of contact. The height S to which a body rises is given by $S = ut + ½ at^2$, which us sum of two terms ut and $½ at^2$. The energy of a damped oscillator decreases exponentially time time.

(ii) In dimensionally method, the dimensionless proportionality constant is to be determined
(a) experimentally (b) by a detailed mathematical derivation
(c) by using the principle of dimensional homogeneity
(d) by equating the powers of M. L and T.

Sol. The correct choices are a and b.

G. ASSERTION REASON TYPE QUESTIONS:

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true.

1. Assertion: When we change the unit of measurement of a quality, its numerical value changes.
Reason: Smaller the unit of measurement smaller is its numerical value.

Ans. (c) Assertion is true but reason is false
If u_1 and u_2 be the units to measure a quantity Q and n_1, n_2 be the numerical values respectively then we know that $Q = n_1u_1 = n_2u_2$. Since the quantity Q does not change irrespective of the units used to measure it Q = constant. So nu = constant $\Rightarrow n \propto \frac{1}{u}$ i.e. smaller the unit of measurement, greater is the corresponding numerical value.

2. Assertion: The equation y = x + t cannot be true, where x, y are distance and t is time.
Reason: Quantities with different dimensions cannot be added.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
The given equation (y = x +f) cannot be true, because time cannot be added to distance.

3. Assertion: Energy cannot be divided by volume.
Reason: Dimensions of energy and volume are different.

Ans. (e) Assertion is false but reason is true.
Since in division and multiplication there is no bar that quantities involved must have the same dimensions.

4. Assertion: Number of significant figure is 0.005 and is one and that in 0.500 is three.
Reason: Zeroes are not significant figures.

Ans. (c) Assertion is true but reason is false
In a number less than one, zeroes the decimal point and first non zero digit are not significant. But zeroes to the right of last non zero digit are significant.

5. Assertion: When percentage errors in the measurement of mass and velocity are 1% and 2% respectively, the percentage error in KE is 5%.
Reason: $\frac{\Delta E}{E} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
Kinetic energy, $E = \frac{1}{2} mv^2$
Taking log on both sides, ln E = ½ ln m + 2 ln v i.e. E = ln ½ + ln m + 2 ln v
Differentiating both sides we get, $\frac{\Delta E}{E} = \frac{\Delta m}{m} + 2\frac{\Delta v}{v}$
Or $\frac{\Delta E}{E} \times 100 = \frac{\Delta m}{m} \times 100 + 2 \times \frac{\Delta v}{v} \times 100$ = 1% + (2 × 2%) = 5%.

H. CHALLENGING PROBLEMS

1. A calorie is a unit of heat (energy in transit) and it equals about 4.2 J where 1J =1 kg m^2 s^{-2}. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length equals β m, the unit of time is γ s. Show that a calorie has a magnitude of $4.2\ \alpha^{-1} \beta^{-2} \gamma^2$ in terms of the new units.

Sol. A 1 calorie = 4.2J, where 1J = 1kg m^2s^{-1}, Therefore a = 1, b = 2, c = - 2.

SI	New ssytem
$n_1 = 4.2$	$n_2 = ?$
$M_1 = 2kg$	$M_2 – \alpha$ kg
$L_1 = 1m$	$L_2 = \beta$ m
$T_1 = 1s$	$T_2 = \gamma$ s

$\therefore n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c = 4.2 \left[\frac{1kg}{\alpha kg}\right]^1 \left[\frac{1m}{\beta m}\right]^2 \left[\frac{1s}{\gamma s}\right]^{-2} = 4.2\alpha^{-1}\beta^{-2}\gamma^2.$

2. The unit of length convenient on the atomic scale is known as an angstrom and is denoted by Å: 1 Å = 10^{-10} m. The size of a hydrogen atom is about 0.5 Å. What is the total atomic volume in m^3 of a mole of hydrogen atoms?

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3. One mole of an ideal gas at standard temperature and pressure occupies 22.4 L (molar volume). What is the ratio of molar volume to the atomic volume of a mole of hydrogen? (Take the size of hydrogen molecule to be about 1 Å). Why is this ratio so large?

Sol. Radius of a hydrogen molecule = 1 Å = 10^{-10}m

Atomic volume of a mole of hydrogen = Avogadro's no. × Volume of a hydrogen molecule = $6.023 \times 10^{23} \times \frac{4}{3} \times 3.14 \times (10^{-10})^3 = 25.2 \times 10^{-7} m^3$.

Molar volume = 22.4L = $22.4 \times 10^{-3} m^3$

$\therefore \frac{\text{Molar volume}}{\text{Atomic volume}} = \frac{22.4\times10^{-3}}{25.2\times10^{-7}} = 0.89\times10^4 = 10^4$.

This ratio is large because the actual size of the gas molecules is negligibly small in comparison with the intermolecular separation.

4. Explain this common observation clearly: If you look out of the window of a fast-moving train, the nearby trees, houses etc. seem to move rapidly in a direction opposite to the train's motion, but the distant objects (hilltops, the Moon, the stars etc.) seem to be stationary. (In fact, since you are aware that you are moving, these distant objects seem to move with you).

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5. The principle of 'parallax' is used in the determination of distances of very distant stars. The baseline AB is the line joining the Earth's two locations six months apart in its orbit around the Sun. That is, the baseline is about the diameter of the Earth's orbit ≈ 3×10^{11}m. However, even the nearest stars are so distant that with such a long baseline, they show parallax only of the order of 1" (second) of arc or so. A parsec is a convenient unit of length on the astronomical scale. It is the distance of an object that will show a parallax of 1" (second of arc) from opposite ends of a baseline equal to the distance from the Earth to the Sun. How much is a parsec in terms of metres?

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6. Precise measurements of physical quantities are a need for science. For example, to ascertain the speed of an aircraft, one must have an accurate method to find its positions at closely separated instants of time. This was the actual motivation behind the discovery of radar in World War II. Think of different examples in modern science where precise measurements of length, time, mass etc. are needed. Also, wherever you can, give a quantitative idea of the precision needed.

Sol. Some of the examples of modern science, where precise measurements play an important role are as follows:

1. Electron microscope uses an electron beam of wavelength 0.2 Å to study very minute objects like viruses, microbes and the crystal structure of solids.
2. The successful launching of artificial satellite has been made possible only due to the precise technique available for accurate measurement of time intervals.
3. The precision with which the distances are measured in Michelson-Morley Interferometer helped in discarding the idea of hypothetical medium ether and in developing the Theory of Relativity by Einstein.

7. The Sun is a hot plasma (ionized matter) with its inner core at a temperature exceeding 107 K, and its outer surface at a temperature of about 6000 K. At these high temperatures, no substance remains in a solid or liquid phase. In what range do you expect the mass density of the Sun to be, in the range of densities of solids and liquids or gases? Check if your guess is correct from the following data: a mass of the Sun = 2.0×10^{30} kg, radius of the Sun = 7.0×10^{8} m.

Sol. Mass of the sun, M = 2.0×10^{30}kg, Radius of the sun, R = 7.0×10^{8}m,

Volume of the sun, $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times (7.0 \times 10^8)^3 = 1.437 \times 10^{27} m^3$.

Density of sun, $\rho = \frac{M}{V} = \frac{2.0\times10^{30}}{1.437\times10^{27}}$ = 1391.8 kg m^{-3} = 1.4×10^3 kg m^{-3}.

The density of the sun is in the range of the densities of the solids and liquids but not gases. The high density due to the inward gravitational attraction on the outer layer due to the inward layers of the sun.

8. A man walking briskly in rain with speed v must slant his umbrella forward making an angle θ with the vertical. A student derives the following relation between θ and v: tan θ = v and checks that the relation has a correct limit: as v →0, θ →0, as expected. (We are assuming there is no strong wind and that the rain falls vertically for a stationary man). Do you think this relation can be correct? If not, guess the correct relation.

Sol. Since trigonometric functions are dimensionless, so, [tan θ] = 1

But [v] = LT^{-1}

Therefore dimensions of LHS × Dimensions of RHS
Hence the give relation in dimensionally wrong.
This relation can be corrected by dividing EHS by the speed 'u' of the rainfall. So the corrected relation is tan θ = v/u.

9. Estimate the average mass density of a sodium atom assuming its size to be about 2.5 Å. (Use the known values of Avogadro's number and the atomic mass of sodium). Compare it with the mass density of sodium in its crystalline phase: 970 kg m^{-3}. Are the two densities of the same order of magnitude? If so, why?

Sol. Radius of sodium atom, r = 2.5 Å = 2.5×10^{-10}m

Volume of sodium atom, V = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (2.5 \times 10^{-10})^3 = 65.42 \times 10^{-30}$kg.

Mass of sodium atom = $\frac{\text{mass number}}{\text{Avogadro's number}} = \frac{23}{6.02\times10^{23}}$ g

= 3.82×10^{-23}g = 3.82×10^{-26}kg

Average density of sodium atom = $\frac{\text{Mass}}{\text{Volume}} = \frac{3.82\times10^{-26}}{65.42\times10^{-30}}$ = 0.58×10^3kg m^{-3}

Density of sodium in crystalline phase = 970 kg m^{-3} = 0.970×10^3 kg m^{-3}

Hence the average mass density of sodium atom and the density of crystalline sodium are of the same order of magnitude (10^3). This is because sodium atoms in crystalline phase are closely packed.

10. The unit of length convenient on the nuclear scale is a fermi: 1 f = 10–15 m. Nuclear sizes obey roughly the following empirical relation: r = r_0 $A^{1/3}$ where r is the radius of the nucleus, A its mass number, and r_0 is a constant equal to about, 1.2 f. Show that the rule implies that nuclear mass density is nearly constant for different nuclei. Estimate the mass density of the sodium nucleus. Compare it with the average mass density of a sodium atom obtained in previous Question.

Sol. Radius of a nucleus, r = $r_{0=A}{}^{1/3}$

Mass of a nucleus = $\frac{\text{Mass number}}{\text{Avogadro's number}} = \frac{A}{N_A}$

Nuclear mass density = $\frac{\text{Mass of a nucleus}}{\text{Volume of a nucleus}} = \frac{3}{4\pi N_A r_0^3}$

As ρ is the independent of A, so nuclear mass density is same for different nuclei.

For a kilomole, $N_A = 6.02 \times 10^{26}$, $r_0 = 1.2$f $= 1.2 \times 10^{-15}$m

Therefore ρ = $\frac{3}{4\pi\times6.02\times10^{26}\times(1.2\times10^{-15})^3}$ = 2.3×10^{17} kg m^{-3}

Density of sodium nucleus should also be = 2.3×10^{17} kg m^{-3}

From previous qs, density of sodium atom = $.58\times10^3$kg m^{-3}

Therefore, $\frac{\text{Nuclear mass density}}{\text{Atomic mass density}} = \frac{2.3\times10^{17}}{0.58\times10^3}$ = 3.96×10^{14}.

Nuclear density is typically 10^{15} times atomic density of matter.

11. A LASER is a source of very intense, monochromatic, and the unidirectional beam of light. These properties of a laser light can be exploited to measure long distances. The distance of the Moon from the Earth has been already determined very precisely using a laser as a source of light. A laser light beamed at the Moon takes 2.56 s to return after

reflection at the Moon's surface. How much is the radius of the lunar orbit around the Earth?

Sol. Here t = 2.56s, c = 3×10^8 ms^{-1}

Radius of the lunar orbit around the earth = Distance of the moon from the earth

$= \frac{c\times t}{2} = \frac{3\times10^8\times2.25}{2} = 3.84 \times 10^8$m.

12. A SONAR (sound navigation and ranging) uses ultrasonic waves to detect and locate objects underwater. In a submarine equipped with a SONAR, the time delay between generation of a probe wave and the reception of its echo after reflection from an enemy submarine is found to be 77.0 s. What is the distance of the enemy submarine? (Speed of sound in water = 1450 m s^{-1}).

Sol. Here t = 77s, c = 1450 ms^{-1}

Distance of enemy submarine $= \frac{c\times t}{2} = \frac{1450\times77}{2} = 55825$m.

13. The farthest objects in our Universe discovered by modern astronomers are so distant that light emitted by them takes billions of years to reach the Earth. These objects (known as quasars) have many puzzling features, which have not yet been satisfactorily explained. What is the distance in km of a quasar from which light takes 3.0 billion years to reach us?

Sol. Here t = 30 billion years = $3.0\times 10^9 \times 365.25 \times 24 \times 60 \times 60$s

Speed of light, c = 3×10^5 kms^{-1}.

Distance of quaser = ct = $3\times10^5 \times 3.0\times10^9\times365.25 \times24\times60\times60 = 2.84\times10^{22}$km.

SPACE FOR ROUGH WORK

SPACE FOR NOTES