CLASS – 11

WORKSHEET- MOTION IN PLANE

A. SCALAR AND VECTOR

(1 Mark Questions)

1. What are the magnitudes of $\hat{i}+\hat{j}$ and $\left(\hat{i}-\hat{j}\right)$?

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2. State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

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3. Pick out the only vector quantity in the following list:
Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

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4. Read each statement below carefully and state with reasons, if it is true or false:
(1 mark each)

(a)The magnitude of a vector is always a scalar.

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(b)Each component of a vector is always a scalar.

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(c) The total path length is always equal to the magnitude of the displacement vector of a particle.

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(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.

(e) Three vectors not lying in a plane can never add up to give a null vector.

5. Read each statement below carefully and state, with reasons and examples, if it is true or false: A scalar quantity is one that (1 mark each)
(a) is conserved in a process

(b) can never take negative values

(c) must be dimensionless

(d) does not vary from one point to another in space

(e) has the same value for observers with different orientations of axes.

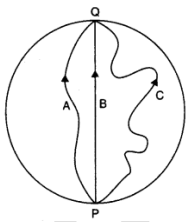
6. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are
(a) Impulse, pressure and area (b) Impulse and area
(c) Area and gravitational potential (d) Impulse and pressure

(3 Marks Questions)

7. One of the rectangular components of a velocity of 80 km/h is 40 km/h. Find the other component.

8. Determine a unit vector perpendicular to both $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

9. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate? [Ans. 400m]



10. A vector has magnitude and direction.
(i) Does it have a location in the space?
(ii) Can it vary with time?
(iii) Will two equal vectors a and b at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

11. Can you associate vectors with (a) the length of a wire bent into a loop (b) a plane area (c) a sphere? Explain.

(5 Marks Questions)

12. State parallelogram law of vector addition. Find analytic the magnitude and direction of resultant vector. Apply it to find the resultant when,
(i) Two vectors are parallel to each other
(ii) Two vectors are perpendicular to each other

13. State triangle law of vector addition. Find analytically the magnitude and direction of resultant vector. Also discuss the special cases.

B. ADDITION AND SUBTRACTION OF VECTOR

(1 Mark Questions)

1. Which of the following is not a property of a null vector?
   (a) $\vec{A}+\vec{0}=\vec{A}$ (b) $\lambda\vec{0}=\vec{0}$ where λ is scalar (c) $0\vec{A}=\vec{A}$ (d) $\vec{A}-\vec{A}=0$

2. Which of the following quantities dependent of the choice of orientation of the coordinate axes?
   (a) $\vec{A}+\vec{B}$ (b) $A_x + B_y$ (c) $\left|\vec{A}+\vec{B}\right|$ (d) Angle between $\vec{A}$ and $\vec{B}$

3. If $\left|\vec{A}+\vec{B}\right|=\left|\vec{A}-\vec{B}\right|$, then angle between $\vec{A}$ and $\vec{B}$ will be
   (a) 30° (b) 45° (c) 60° (d) 90°

4. Fifteen vectors, each of magnitude 5 units, are represented by the sides of a closed polygon, all taken in same order. What will be their resultant?
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   _____

5. At what angle the two forces A + B and A – B act so that their resultant is $\sqrt{3A^2+B^2}$?
   _____
   _____

6. Give an example of a zero vector.
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   _____

(2 Marks Questions)

7. Can we apply the commutative and associative laws to vector subtraction also?
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8. ABCDE is a pentagon. Prove that: $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EA}=0$.
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   _____
   _____
   _____

9. What would be the angle θ between two vectors $\vec{A}$ and $\vec{B}$ for their resultant $\vec{R}$ to be maximum?

10. Establish the following inequalities geometrically or otherwise:

(a) $|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$ (b) $|\vec{A}+\vec{B}| \ge |\vec{A}|-|\vec{B}|$

(c) $|\vec{A}-\vec{B}| \le |\vec{A}|+|\vec{B}|$ (d) $|\vec{A}-\vec{B}| \ge |\vec{A}|-|\vec{B}|$

When does the equality sign above apply?

(3 Marks Questions)

11. Prove that $|\vec{a}-\vec{b}| \ge ||\vec{a}|-|\vec{b}||$. When does the equality holds?

12. If vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ have magnitude 5, 12 and 13 units and $\vec{P}+\vec{Q}=\vec{R}$. Find the angle between $\vec{Q}$ and $\vec{P}$.

13. State polygon law of vectors and prove it with the help of triangle law of vectors.

14. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant becomes perpendicular to $\vec{P}$, then find the magnitude of $\vec{R}$.

15. Find a unit vector parallel to the resultant of the vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$.

16. Two forces 5N and 7N act on a particle with an angle of 60° between them. Find the resultant force.

17. Two forces equal to P and 2P act on a particle. If the first be doubled and the second be increased by 20 Newton, the direction of the resultant is unaltered. Find the value of P.

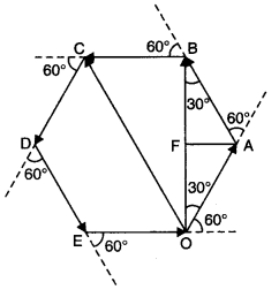
18. Find unit vector parallel to the resultant of the vectors $\vec{A} = \hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$.

19. If $\vec{A} = 3\hat{i} + 5\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{B}$.

20. A train is moving with a velocity of 30 km/h due east and a car is moving with a velocity of 40km/h due north. What is the velocity of car as appears to a passenger in the train?

21. A plane is travelling eastward at a speed of 500 km/h. But a 90km/h wind is blowing southward. What is the direction and speed of the plane relative to the ground?

22. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case. [Ans. 1.5 km, 3 km, 4km]



C. MULTIPLICATION OF VECTOR

(1 Mark Questions)

1. What is the maximum of components into which a vector can be resolved?

2. What is the dot product of two similar unit vectors?

3. What is the value of $\hat{i}.\left(\hat{j}\times 2\hat{k}\right)$?

4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful: (1 mark each)
   (a) adding any two scalars,

   (b) adding a scalar to a vector of the same dimensions,

   (c) multiplying any vector by any scalar,

(d) multiplying any two scalars,

(e) adding any two vectors,

(f) adding a component of a vector to the same vector.

5. The angle between $\vec{A}= \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j}$, is
(a) 45° (b) 90° (c) –45° (d) 180°

6. It is found that $|\vec{A}+\vec{B}|=|\vec{A}|$. This necessarily implies,
(a) $\vec{B} = 0$ (b) $\vec{A},\vec{B}$ are antiparallel
(c) $\vec{A},\vec{B}$ are perpendicular (d) $\vec{A}.\vec{B} \leq 0$

(2 Marks Questions)

7. Find the angles between the vectors $\vec{A} = 2\hat{i} - 4\hat{j} + 6\hat{k}$ and $\vec{B} = 3\hat{i} + \hat{j} + 2\hat{k}$.

8. For what value of a are the vectors $\vec{A} = a\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = 2a\hat{i} + a\hat{j} - 4\hat{k}$ and perpendicular to each other.

9. Find the angles between the following pairs of vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = -2\hat{i} - 2\hat{j} - 2\hat{k}$.

10. (a) If $\hat{i}$ and $\hat{j}$ are unit vectors along X- and Y-axis respectively, then what is the magnitude and direction of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

(b) Find the components of along the directions of vectors and $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$.

[Ans. (a)45°,45°, (b) $5/2(\hat{i}+\hat{j})$, $-\frac{1}{2}(\hat{i}-\hat{j})$.]

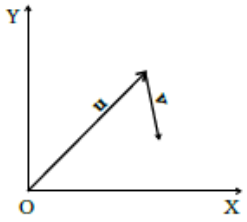
(3 Marks Questions)

11. Two vectors both equal in magnitude, have their resultant equal in magnitude of the either. Find the angle between the two vectors.

D. INTRODUCTION OF MOTION IN PLANE

(1 Mark Questions)

1. Figure shows the orientation of two vectors u and v in the XY plane. If $u=a\hat{i}+b\hat{j}$ and $v=p\hat{i}+q\hat{j}$



which of the following is correct?

(a) a and p are positive while b and q are negative.

(b) a, p and b are positive while q is negative.

(c) a, q and b are positive while p is negative.

(d) a, b, p and q are all positive.

2. For any arbitrary motion in space, which of the following relations are true: (1 mark each)

(a) $\vec{v}_{average} = (1/2)[\vec{v}(t_1) + \vec{v}(t_2)]$

(b) $\vec{v}_{average} = [\vec{r}(t_2) - \vec{r}(t_1)]/(t_2 - t_1)$

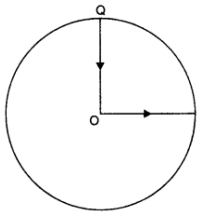
(c) $\vec{v}\ (t) = \vec{v}\ (0) + \vec{a}\ t$

(d) $\vec{r}\ (t) = \vec{r}\ (0) + \vec{v}\ (0)t + (1/2)\ \vec{a}\ t^2$

(e) $\vec{a}_{\ average} = [\vec{v}\ (t_2) - \vec{v}\ (t_1)]/(t_2 - t_1)$
(The average stands for average of the quantity over the time interval t_1 to t_2)

(3 Marks Questions)

3. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist? [Ans. (iii) 21.43 kmh^{-1}]



4. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cab man takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

[Ans. 49.3 kmh^{-1}, 21.4 kmh^{-1}]

5. The position of a particle is given by

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k}\ m$$

Where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the value of $\vec{v}$ and $\vec{a}$ of the particle,

(b) What is the magnitude and direction of velocity of the particle at t = 2.0s?

[Ans. 70° with x-axis]

(5 Marks Questions)

6. The position vector of a particle is given by: $\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k}$

Where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle.

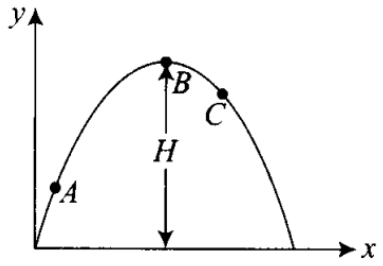
(b) What is the magnitude and direction of velocity of the particle at t = 2s?

E. PROJECTILE MOTION

(1 Mark Questions)

1. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be
(a) 60 m (b) 71 m (c) 100 m (d) 141 m

2. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Figure, where x and y indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points A, B and C.



3. A football is kicked into the air vertically upwards. What is its (a) acceleration, and (b) velocity at the highest point?

4. In case of a projectile motion, what is the angle between the velocity and acceleration at the highest point?
(a) 0° (b) 45° (c) 90° (d) 180°

5. From a certain height above the ground a stone A is dropped gently. Simultaneously another stone B is fired horizontally. Which of the two stone will arrive on the ground earlier?

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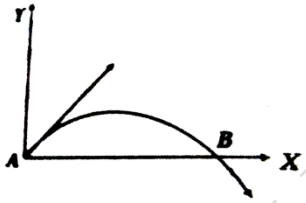
(2 Marks Questions)

6. The velocity of a projectile at the initial point A is $(2\,\hat{i} + 3\,\hat{j})$ m/s. What will be its velocity (in m/s) at point B?



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7. Prove that the maximum horizontal range is four times the maximum height attained by a projectile which is fired along the required oblique direction.

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(3 Marks Questions)

8. Show that motion of one projectile as seen from another projectile will be a straight line.

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9. The equations of motion of a projectile are given by x = 36t m and $2y = 96t - 9.8t^2$ m. Find the angle of projection.

10. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground.

11. A projectile has a range of 50m and reaches a maximum height of 10m. Calculate the angle at which the projectile is fired.

12. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range.

13. A projectile is fired with a velocity 'u' making an angle θ with the horizontal. Show that its trajectory is a parabola.

14. Find the angle of projection at which the horizontal range and maximum height of a projectile are equal.

15. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall? [Ans. 150.7 m]

16. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball? [Ans. 50 m]

17. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft? Time taken by aircraft from A to B is 10 s. [Ans. 182.2 ms^{-1}]

18. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away? Assume the muzzle speed to the fixed, and neglect air resistance. [Ans. 3.46 km]

19. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km h^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 m s^{-2})? [Ans. 16 km]

(5 Marks Questions)

20. A body is projected with velocity v at an angle θ with the horizontal. Find the (a) Time of flight (b) Maximum height attained (c) Maximum range for the body.

F. RELATIVE VELOCITY IN TWO DIMENSIONS

(3 Marks Questions)

1. A motorboat is racing towards north at 25 km/h and the water current in that region is 10km/h in the direction of 60° east of south. Find the resultant velocity of the boat.

2. A boatman can row with a speed of 10km/h in still water. If the river flows steadily at 5km/h, in which direction should the boatman row in order to reach a point on the other bank directly opposite to the point from where he started? The width of the river is 2km.

3. Rain is falling vertically with a speed of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella? [Ans. 18°26']

4. A man can swim with a speed of 4.0 km h^{-1} in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km h^{-1} and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank? [Ans. 15 min, 0.75 km]

G. UNIFORM CIRCULAR MOTION

(1 Mark Questions)

1. A stone tied to the end of a string 100cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 22s, then the acceleration of the stone is

(a) 16 ms^{-2} (b) 4 ms^{-2} (c) 12 ms^{-2} (d) 8 ms^{-2}

2. Velocity vector and acceleration vector in a uniform circular motion are related as
(a) both in same direction (b) perpendicular to each other
(c) both in opposite direction (d) not related to each other

3. Can an object be accelerated without speeding up or slowing down? Explain.

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4. What the angular frequency of the object if it complete 100 revolutions in 50 seconds?

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5. Read each statement below carefully and state, with reasons, if it is true or false:
(1 mark each)

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

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(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that point.

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(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector

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(2 Marks Questions)

6. A car is moving along a circular road at speed of 20m/s. The radius of the circular road is 10m. If the speed is increased at the rate of $30m/s^2$, what is the resultant acceleration?

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(3 Marks Questions)

7. Assuming that the moon completes one revolution in a circular orbit around the earth in 27.3 days, calculate the acceleration of the moon towards the earth. The radius of the circular orbit can be taken as 3.85×10^5 km.

8. **A** stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone? [Ans. 9.90 ms^{-2}]

9. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity. [Ans. 6.38]

(5 Marks Questions)

10. (i) Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r.

(ii) A stone tied to the end of the string 80cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone?

H. CHALLENGING PROBLEMS

1. In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat? [Ans. 0.01°]

2. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn? [Ans. 0.86 ms^{-1}, 54°28']

3. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

(b) Shows that the projection angle θ_0 for a projectile launched from the origin is given by: $\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$

Where the symbols have their usual meaning.

4. A vector has both magnitude and direction. Does that mean anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation and the angle of rotation about the axis. Does that make any rotation a vector?

SPACE FOR ROUGH WORK

SPACE FOR NOTES