CLASS – 11

WORKSHEET- MOTION IN PLANE

A. SCALAR AND VECTOR

(1 Mark Questions)

1. What are the magnitudes of $\hat{i}+\hat{j}$ and $\left(\hat{i}-\hat{j}\right)$?

Sol. $|\hat{i}+\hat{j}| = \sqrt{1^2+1^2} = \sqrt{2}$ and $|\hat{i} - \hat{j}| = \sqrt{1^2+(-1)^2} = \sqrt{2}$

2. State, for each of the following physical quantities, if it is a scalar or a vector: volume, mass, speed, acceleration, density, number of moles, velocity, angular frequency, displacement, angular velocity.

Sol. Volume, mass, speed, density, number of moles, and angular frequency are some of the scalar physical quantities. A vector quantity is specified by its magnitude as well as the direction associated with it. Acceleration, velocity, displacement, and angular velocity belong to this category.

3. Pick out the only vector quantity in the following list:
Temperature, pressure, impulse, time, power, total path length, energy, gravitational potential, coefficient of friction, charge.

Sol. Since, impulse = change in momentum = force `xx` time. As momentum and force are vector quantities hence impulse is a vector quantity.

4. Read each statement below carefully and state with reasons, if it is true or false:
(a)The magnitude of a vector is always a scalar.

Sol. True. The magnitude of a vector is a number. Hence, it is a scalar.
(b)Each component of a vector is always a scalar.

Sol. False. Each component of a vector is also a vector.
(c) The total path length is always equal to the magnitude of the displacement vector of a particle.

Sol. False. Total path length is a scalar quantity, whereas displacement is a vector quantity. Hence, the total path length is always greater than the magnitude of displacement. It becomes equal to the magnitude of displacement only when a particle is moving in a straight line.
(d) The average speed of a particle (defined as total path length divided by the time taken to cover the path) is either greater or equal to the magnitude of average velocity of the particle over the same interval of time.

Sol. True. It is because of the fact that the total path length is always greater than or equal to the magnitude of displacement of a particle.

(e) Three vectors not lying in a plane can never add up to give a null vector.

Sol. True. Three vectors, which do not lie in a plane, cannot be represented by the sides of a triangle taken in the same order.

5. Read each statement below carefully and state, with reasons and examples, if it is true or false: A scalar quantity is one that

(a) is conserved in a process

Sol. False:- For e.g. energy is a scalar quantity but is not conserved in inelastic collisions.

(b) can never take negative values

Sol. False:- For example temperature can take negative values in degree Celsius.

(c) must be dimensionless

Sol. False:- Since speed is a scalar quantity but has dimensions.

(d) does not vary from one point to another in space

Sol. False:- Gravitational potential varies in space from point to point.

(e) has the same value for observers with different orientations of axes.

Sol. True:- Since it doesn't have direction.

6. Consider the quantities, pressure, power, energy, impulse, gravitational potential, electrical charge, temperature, area. Out of these, the only vector quantities are

(a) Impulse, pressure and area (b) Impulse and area

(c) Area and gravitational potential (d) Impulse and pressure

Sol. (b)

As we know that Impulse I = FΔt = $\left(\frac{\Delta p}{\Delta t}\right)\Delta t = \Delta p$

Where F is force, Δt is time duration and Δp is change in momentum. As Δp is is a vector quantity hence impulse is also a vector quantity. Sometimes area can also be treated as vector In a two dimensional motion, instantaneous speed v_0 is a positive constant. Then which of the following are necessarily true?

(3 Marks Questions)

7. One of the rectangular components of a velocity of 80 km/h is 40 km/h. Find the other component.

Sol. Let v = 80 km/h, v_x = 40k,/h, then v_y = ?

$v = \sqrt{v_x^2 + v_y^2}$

Therefore $v_y = \sqrt{v^2 - v_x^2} = \sqrt{80^2 - 40^2} = \sqrt{6400 - 1600} = \sqrt{4800}$ = 69.28 km/h.

8. Determine a unit vector perpendicular to both $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$.

Sol. The perpendicular unit vector n is given by $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} = |\vec{A} \times \vec{B}|\hat{n}$

Therefore $\hat{n} = \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|}$

Now $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$

$= \hat{i}\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix}$

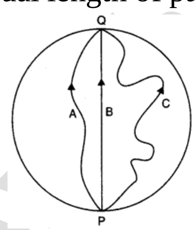
$= \hat{i}\,(2+1) - \hat{j}\,(4-1) + \hat{k}\,(-2-1)$

$= 3\hat{i} - 3\hat{j} - 3\hat{k}$

Therefore $|\vec{A} \times \vec{B}| = \sqrt{3^2 + (-3)^2 + (-3)^2} = \sqrt{27} = 3\sqrt{3}$

Hence $\hat{n} = \frac{3\hat{i} - 3\hat{j} - 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}(\hat{i} - \hat{j} - \hat{k})$

9. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in Fig. What is the magnitude of the displacement vector for each? For which girl is this equal to the actual length of path skate?



Sol. Displacement of each girl = $\overrightarrow{PQ}$

Magnitude of displacement vector for each girl = $|\overrightarrow{PQ}|$ = 2 × radius = 2×200 = 400m

For girl B, the magnitude of displacement vector = actual length of path.

10. A vector has magnitude and direction.

(i) Does it have a location in the space?

(ii) Can it vary with time?

(iii) Will two equal vectors **a** and **b** at different locations in space necessarily have identical physical effects? Give examples in support of your answer.

Sol. (i) In addition to magnitude and direction each vector has a definite location in space. For example a velocity vector has definite location at every point of uniform circular motion.

(ii) A vector can vary with time. For example increase in velocity produces acceleration.

(iii) Two vectors **a** and **b** having different locations may not produce identical physical effects. For example two equal forces (vectors) acting at two different points may not produce equal turning effects.

11. Can you associate vectors with (a) the length of a wire bent into a loop (b) a plane area (c) a sphere? Explain.

Sol. Out of these, only a plane area can be associated with a vector. The direction of this area vector is taken normal to the plane.

(5 Marks Questions)

12. State parallelogram law of vector addition. Find analytic the magnitude and direction of resultant vector. Apply it to find the resultant when,
(i) Two vectors are parallel to each other
(ii) Two vectors are perpendicular to each other.

Sol. Law of parallelogram of vectors can be defined as

"If two vectors of same physical quantity are represented in terms of magnitude and direction are represented by two adjacent sides of a parallelogram, taken in an order, then the diagonal drawn from common point of intersection represents the resultant in terms of magnitude and direction."

(i) Now of the vectors are parallel to each other,

so, resultant is so r is p+q.

As the vectors are parallel resultant will be parallel to each of them.

(ii) so replace cos beta in above formula by -1 to get the resultant.

Use formula

For finding the direction. alpha is angle made by p with r.

13. State triangle law of vector addition. Find analytically the magnitude and direction of resultant vector. Also discuss the special cases.

Sol. The Triangle Law of Vector Addition states that when two vectors are represented by two sides of a triangle in order of magnitude and direction, the magnitude and direction of the resultant vector are represented by the third side of the triangle.

Let's consider two vectors A and B. They have the same magnitude but they are not in the same direction.

Now to find the sum of A and B i.e.

A+BA+B

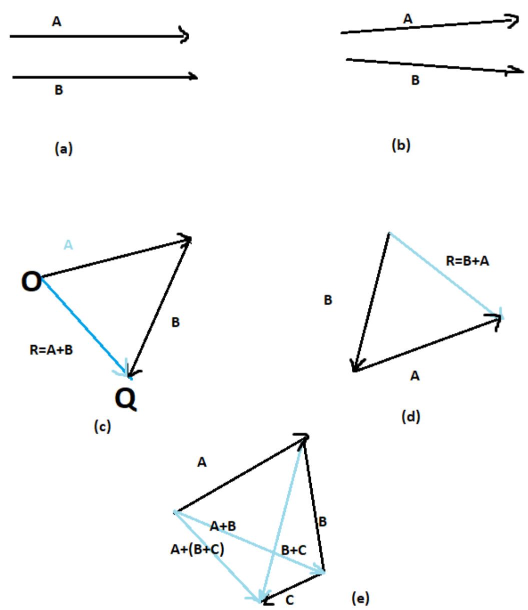
. We place vector B so that its tail is at the head of vector A as shown in (c). Then joining the tail of A to the head of B. OQ represents a vector R, that is, the sum of vectors A and B. Since, in this procedure of vector addition, vectors are arranged head to tail, this graphical method is called the head to tail method. The two vectors and their resultant form three Side. If we find the resultant of B+A as shown in figure (d), the same vector R is obtained. Thus vector addition is commutative which is given by

A+B=B+AA+B=B+A

The addition of vectors also obeys the associative law which is shown in figures (e). The result of adding vectors A and B first and then adding vector C is same as the result of adding B and C first and then adding vector A:

(A+B)+C=A+ (B+C)(A+B)+C=A+ (B+C)

of a triangle, so this method is also known as the triangle method of vector addition.



Note: Vector quantity is the quantity which has both magnitude and direction. Along with triangle law of addition, subtraction and multiplication of vectors are also possible. The vector addition must follow commutative and associative law. If two vectors represented as sides are parallel to each other or if they have an angle of 180 degree then you cannot use a triangle of vector addition.

B. ADDITION AND SUBTRACTION OF VECTOR
(1 Mark Questions)

1. Which of the following is not a property of a null vector?

(a) $\vec{A}+\vec{0}=\vec{A}$ (b) $\lambda\vec{0}=\vec{0}$ where λ is scalar (c) $0\vec{A}=\vec{A}$ (d) $\vec{A}-\vec{A}=0$

Ans. (c)

2. Which of the following quantities dependent of the choice of orientation of the coordinate axes?

(a) $\vec{A}+\vec{B}$ (b) $A_x + B_y$ (c) $|\vec{A}+\vec{B}|$ (d) Angle between $\vec{A}$ and $\vec{B}$

Ans. (b)

3. If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be

(a) 30° (b) 45° (c) 60° (d) 90°

Ans. (b)

4. Fifteen vectors, each of magnitude 5 units, are represented by the sides of a closed polygon, all taken in same order. What will be their resultant?

Sol. Their resultant will be zero. This is because the vector sum of all the vectors represented by the sides of a closed polygon taken in the same order is zero.

5. At what angle the two forces A + B and A – B act so that their resultant is $\sqrt{3A^2+B^2}$?

Sol. Applying the triangle law of vector addition

The Resultant of forces A and B acting at angle θ to each other is the root of

$A^2+B^2+2AB\cos\theta$

hence; $3A^2+B^2=(A+B)^2+(A-B)^2+2(A+B)(A-B)\cos\theta$

$= 3A^2+B^2=2A^2+2B^2+2(A^2-B^2)\cos\theta$

$A^2- B^2 =2(A^2 – B^2)\cos\theta$

$\cos\theta=1/2$

θ=60 degrees

6. Give an example of a zero vector.

Sol. Suppose two people are pulling a rope from its two ends with equal force but in opposite directions. So, the net force applied to the rope will be a zero vector (null vector) as the two equal forces balance each other out because they are in opposite directions.

(2 Marks Questions)

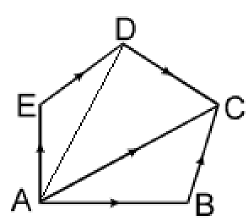
7. Can we apply the commutative and associative laws to vector subtraction also?

Sol. (i) No we cannot apply commutative law to vector subtraction because $\vec{a} – \vec{b} \neq \vec{b} – \vec{a}$.

(ii) Yes, associative law can be applied to vector subtraction because $(\vec{a} + \vec{b}) – \vec{c} = \vec{a} + (\vec{b} – \vec{c})$.

8. ABCDE is a pentagon. Prove that: $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EA}=0$.

Sol.



LHS = $(\overrightarrow{AB} + \overrightarrow{BC}) + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$

$= (\overrightarrow{AC} + \overrightarrow{CD}) + \overrightarrow{DE} + \overrightarrow{EA}$

$= \overrightarrow{AD} + \overrightarrow{DE} + \overrightarrow{EA} = (\overrightarrow{AD} + \overrightarrow{DE}) + \overrightarrow{EA}$

$= \overrightarrow{AE} + \overrightarrow{EA} = - \overrightarrow{EA} + \overrightarrow{EA} = 0$ = RHS.

9. What would be the angle θ between two vectors $\vec{A}$ and $\vec{B}$ for their resultant $\vec{R}$ to be maximum?

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10. Establish the following inequalities geometrically or otherwise:

(a) $|\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$ (b) $|\vec{A}+\vec{B}| \ge |\vec{A}|-|\vec{B}|$

(c) $|\vec{A}-\vec{B}| \le |\vec{A}|+|\vec{B}|$ (d) $|\vec{A}-\vec{B}| \ge |\vec{A}|-|\vec{B}|$

When does the equality sign above apply?

Sol. (a) If θ be the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, then

$|\vec{\mathbf{A}} + \vec{\mathbf{B}}| = \sqrt{|\vec{\mathbf{A}}|^2 + |\vec{\mathbf{B}}|^2 + +2|\vec{\mathbf{A}}||\vec{\mathbf{B}}|\cos\theta}$

Now, $|\vec{\mathbf{A}} + \vec{\mathbf{B}}|$ will be maximum when, cos θ = 1 or θ = 0°

Therefore $|\vec{\mathbf{A}} + \vec{\mathbf{B}}|_{max} = \sqrt{|\vec{\mathbf{A}}|^2 + |\vec{\mathbf{B}}|^2 + 2|\vec{\mathbf{A}}||\vec{\mathbf{B}}|\cos 0^\circ} = \sqrt{|\vec{\mathbf{A}}|^2 + |\vec{\mathbf{B}}|^2} = |\vec{\mathbf{A}}| + |\vec{\mathbf{B}}|$

Here $|\vec{\mathbf{A}} + \vec{\mathbf{B}}| \le |\vec{\mathbf{A}}| + |\vec{\mathbf{B}}|$

The equality sign is applicable when θ = 0° i.e. when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in same direction.

(b) Again $|\vec{\mathbf{A}} + \vec{\mathbf{B}}| = \sqrt{|\vec{\mathbf{A}}|^2 + |\vec{\mathbf{B}}|^2 + +2|\vec{\mathbf{A}}||\vec{\mathbf{B}}|\cos\theta}$

The value of $|\vec{\mathbf{A}} + \vec{\mathbf{B}}|$ will be minimum when, cos θ = -1 or θ = 180°

$= \sqrt{|\vec{\mathbf{A}}|^2 + |\vec{\mathbf{B}}|^2 + +2|\vec{\mathbf{A}}||\vec{\mathbf{B}}|\cos 180^\circ} = \sqrt{(|\vec{\mathbf{A}}| - |\vec{\mathbf{B}}|)^2} = |\vec{\mathbf{A}}| - |\vec{\mathbf{B}}|$

Hence $|\vec{\mathbf{A}} - \vec{\mathbf{B}}| \ge |\vec{\mathbf{A}}| - |\vec{\mathbf{B}}|$

The equality sign is applicable when θ = 180° i.e., when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions.

(c) If θ is the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, then the angles between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ will be (180 – θ).

Therefore $|\vec{\mathbf{A}} - \vec{\mathbf{B}}+ = |\vec{\mathbf{A}} + (-\vec{\mathbf{B}})|$

$= \sqrt{|\vec{\mathbf{A}}|^2 + |-\vec{\mathbf{B}}|^2 + 2|\vec{\mathbf{A}}||-\vec{\mathbf{B}}|\cos(180-\theta)} = \sqrt{|\vec{\mathbf{A}}|^2 + |-\vec{\mathbf{B}}|^2 + 2|\vec{\mathbf{A}}||-\vec{\mathbf{B}}|\cos\theta}$

[since $|-\vec{\mathbf{B}}| = |\vec{\mathbf{B}}|$, cos (180 – θ) = - cos θ]

$|\vec{\mathbf{A}} - \vec{\mathbf{B}}|$ will be maximum between cos θ = - 1 or θ = 180°

Therefore $|A - \vec{\mathbf{B}}|_{max} = \sqrt{|\vec{\mathbf{A}}|^2 + |-\vec{\mathbf{B}}|^2 + 2|\vec{\mathbf{A}}||-\vec{\mathbf{B}}|\cos(180-\theta)} = \sqrt{(|\vec{\mathbf{A}}| + |\vec{\mathbf{B}}|)^2} = |\vec{\mathbf{A}}| + |\vec{\mathbf{B}}|$

Hence $|\vec{\mathbf{A}} - \vec{\mathbf{B}}| \le |\vec{\mathbf{A}}| + |\vec{\mathbf{B}}|$

The equality sign is applicable when θ = 180°.

(d) $|\vec{\mathbf{A}} - \vec{\mathbf{B}}|$ will be minimum when cos θ = 1 or θ = 0°

Therefore $|\vec{\mathbf{A}} - \vec{\mathbf{B}}|_{min} = \sqrt{|\vec{\mathbf{A}}|^2 + |-\vec{\mathbf{B}}|^2 + 2|\vec{\mathbf{A}}||-\vec{\mathbf{B}}|\cos 0°} = \sqrt{(|\vec{\mathbf{A}}| - |\vec{\mathbf{B}}|)^2} = |\vec{\mathbf{A}}| - |\vec{\mathbf{B}}|$

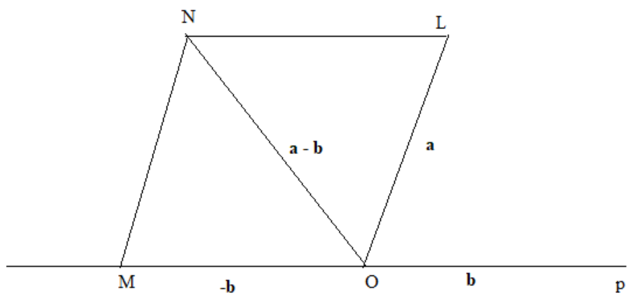
Hence $|\vec{\mathbf{A}} - \vec{\mathbf{B}}| \ge |\vec{\mathbf{A}}| - |\vec{\mathbf{B}}|$

The equality sign is applicable when θ = 0°.

(3 Marks Questions)

11. Prove that $|\vec{a} - \vec{b}| \ge ||\vec{a}| - |\vec{b}||$ When does the equality holds?

Sol.



In the figure the ΔOMN. IT follows that ON + OM > MN or ON > | MN – OM|.

The modulus of MN – OM has been taken for the reason that whereas LHS is positive, RHS may be negative, in case MN is smaller than OM. Since MN = OL, we have

ON > |OL – OM|

$|\vec{a} - \vec{b}| > ||\vec{a}| = |\vec{b}||$

Or $|\vec{a} - \vec{b}| > ||\vec{a}| - |\vec{b}||$ …(i)

In case the vectors a and b are along the same straight line and point in the same direction, then $|\vec{a} - \vec{b}| = ||\vec{a}| - |\vec{b}||$ …(ii)

Combining the conditions stated in equations (i) and (ii) we have

$|\vec{a} - \vec{b}| = ||\vec{a}| - |\vec{b}||$.

12. If vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ have magnitude 5, 12 and 13 units and $\vec{P} + \vec{Q} = \vec{R}$. Find the angle between $\vec{Q}$ and $\vec{P}$.

Sol. Given $|\vec{P}|$ = 5units, $|\vec{Q}|$ = 12 units, $|\vec{R}|$ = 13 units and $\vec{R} = \vec{P} + \vec{Q}$

Using parallelogram of vector addition

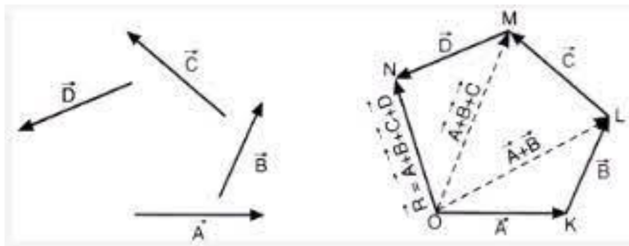
$|\vec{R}|^2 = |\vec{P}|^2 + |\vec{Q}|^2 + 2|\vec{P}||\vec{Q}|\cos\theta$

$13^2 = 5^2 + 12^2 + 2 \times 5 \times 12 \cos\theta$

0 = cosθ or θ = 90°.

13. State polygon law of vectors and prove it with the help of triangle law of vectors.

Sol.



Polygon law of vectors is the generalized form of triangle law of vectors and is used to add more than two vectors. According to this law: If a number of vectors can be represented, both in magnitude and direction, by the sides of an open convex polygon taken in the same order, then their resultant is represented completely in magnitude and direction by the closing side of the polygon, taken in the opposite order.

Let $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ be four vectors acting at the point O as shown in figure. These vectors can be represented both in magnitude and direction by the sides $\overrightarrow{ab}$, $\overrightarrow{bc}$, $\overrightarrow{cd}$ and $\overrightarrow{de}$ respectively of a polygon abcde taken in the same order.

If $\vec{R}$ is the resultant of all these vectors, then

$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} = \overrightarrow{ab} + \overrightarrow{bc} + \overrightarrow{cd} + \overrightarrow{de}$

$= \overrightarrow{ab} + \overrightarrow{cd} + \overrightarrow{de}$ (as $\overrightarrow{ab} + \overrightarrow{bc} = \overrightarrow{ac}$)

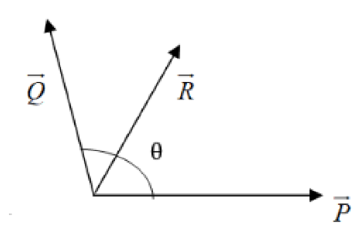
$= \overrightarrow{ad} + \overrightarrow{de}$ (as $\overrightarrow{ac} + \overrightarrow{cd} = \overrightarrow{ad}$)

$= \overrightarrow{ae}$ (as $\overrightarrow{ad} + \overrightarrow{de} = \overrightarrow{ae}$)

We find that $\vec{R}$ is given by $\overrightarrow{ae}$ which is closing side of the polygon taken in the opposite order.

14. The resultant of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If the magnitude of $\vec{Q}$ is doubled, the new resultant becomes perpendicular to $\vec{P}$, then find the magnitude of $\vec{R}$

Sol.



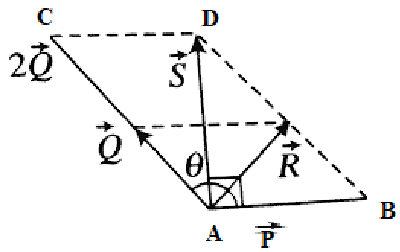
$\vec{P} + \vec{Q} = \vec{R}$

$\tan\alpha = \frac{Q\sin\theta}{P+Q\sin\theta}$ and $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ …(i)

When $\vec{Q}$ is doubled, resultant $\vec{R}_1$ is perpendicular to $\vec{P}$

Therefore $R_1^2 = P^2 + 4Q^2 + PQ\cos\theta$ …(ii)

From the right angles triangle BAD



$4Q^2 = R_1^2 + P^2$,
$R_1^2 = 4Q^2 – P^2$
Substituting in (ii) and solving we get $P^2 + 2PQ \cos\theta = 0$ …(iii)
Substituting (iii) in (i) we get $R^2 = Q^2$ or $R = Q$.

15. Find a unit vector parallel to the resultant of the vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$.

Sol. Given $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{B} = 3\hat{i} – 5\hat{j} + \hat{k}$. Let $\vec{R}$ be the resultant vector of $\vec{A}$ and $\vec{B}$
Therefore $\vec{R} = \vec{A} + \vec{B}$
$\vec{R} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + (3\hat{i} – 5\hat{j} + \hat{k}) = 5\hat{i} - 2\hat{j} + 5\hat{k}$
Unit vector along $\vec{R}$ is given by, $\vec{R} = \frac{\vec{R}}{|\vec{R}|}$
$\vec{R} = \frac{5\hat{i}-2\hat{j}+5\hat{k}}{\sqrt{5^2+2^2+5^2}} = \frac{1}{\sqrt{54}}(5\hat{i} - 2\hat{j} + 5\hat{k})$

16. Two forces 5N and 7N act on a particle with an angle of 60° between them. Find the resultant force.

Sol. Here P = 5N, Q = 7N, θ = 60°
The magnitude of resultant force is R
$= \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{5^2 + 7^2 + 2 \times 5 \times 7 \times \cos 60°} = \sqrt{109} = 10.44N$
If $\vec{R}$ makes angle β with the force $\vec{P}$, then
$\tan\beta = \frac{Q\sin\theta}{P+Q\sin\theta} = \frac{7\sin 60°}{5+7\cos 60°} = 0.7132$ or $\beta = \tan^{-1} 0.7132 = 35°29'$.

17. Two forces equal to P and 2P act on a particle. If the first be doubled and the second be increased by 20 Newton, the direction of the resultant is unaltered. Find the value of P.

Sol. Let the resultant make angle β with the force P.
Therefore in first case, $\tan\beta = \frac{2P\sin\theta}{P+2P\cos\theta}$
In the second case, $\tan\beta = \frac{(2P+2)\sin\theta}{2P+(2P+20)\cos\theta}$
Hence $\frac{(2P+2)\sin\theta}{2P+(2P+20)\cos\theta} = \frac{2P\sin\theta}{P+2P\cos\theta}$

Or $\frac{2P\sin\theta}{P+2P\cos\theta} = \frac{20\sin\theta}{P+20\sin\theta}$ $\left[\because \frac{a}{b} = \frac{c}{d} = \frac{a-c}{b-d}\right]$
From the above equation, 2P = 20 or P = 10N.

18. Find unit vector parallel to the resultant of the vectors $\vec{A} = \hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{B} = 3\hat{i} - 5\hat{j} + \hat{k}$.

Sol. The resultant of $\vec{A}$ and $\vec{B}$ is
$\vec{R} = \vec{A} + \vec{B} = (\hat{i} + 4\hat{j} - 2\hat{k}) + (3\hat{i} - 5\hat{j} + \hat{k}) - (1+3)\,\hat{i} + (4-5)\,\hat{j} + (-2+1)\,\hat{k} = 4\hat{i} - \hat{j} - \hat{k}$
$|\vec{R}| = \sqrt{4^2 + (-1)^2 + (-1)^2} = \sqrt{16+1+1} = 3\sqrt{2}$
The unit vector parallel to $\vec{R}$ is $\hat{R} = \frac{\vec{R}}{|\vec{R}+} = \frac{1}{3\sqrt{2}}(4\hat{i} - \hat{j} - \hat{k})$

19. If $\vec{A} = 3\hat{i} + 5\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$.

Sol. $|\vec{A}| = \sqrt{3^2 + 4^2} = 5$
Unit vector in the direction of $\vec{A}$ is $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{1}{5}(3\hat{i} + 4\hat{j})$
Also $|\vec{B}| = \sqrt{7^2 + 24^2} = 25$
The vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$
$= |\vec{B}|\hat{A} = 25 \times \frac{1}{5}(3\hat{i} + 4\hat{j}) = 15\hat{i} + 20\hat{j}$.

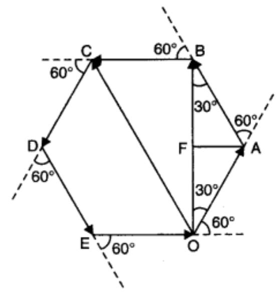
20. A train is moving with a velocity of 30 km/h due east and a car is moving with a velocity of 40km/h due north. What is the velocity of car as appears to a passenger in the train?

Sol. As the given details, we can consider the a train is moving with a velocity of 30 km/hr due east and a car moving with velocity 40 km / hr due north.
The velocity of car is given as $-30\hat{i} + 40\hat{j}$ km/h.
The next step is $|\vec{v}| = \sqrt{(30^2 + 40^2)} = 50$ km/h.
Hence, it will actually appear going North West at a speed of 50km/h.

21. A plane is travelling eastward at a speed of 500 km/h. But a 90km/h wind is blowing southward. What is the direction and speed of the plane relative to the ground?

Sol. The speed of the plane: $v = \sqrt{((500^2) + (90^2))}$ km/h= 508km/h
The direction of the plane :$\tan\theta = 90/500 \Rightarrow \theta = \tan^{-1}(90/500) = 10.2°$(South of East).
Hence, the speed of the plane relative to ground will be 508 km/h and the direction of the plane will be 10.2°(South of East).

22. On an open ground, a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of the displacement with the total path length covered by the motorist in each case.



Sol. Clearly he will follow the horizontal path ABCDEFA. The orders of the turns taken by him are indicated at the vertices of the hexagon.

(i) At the third turn, the motorist will be at D. The magnitude of displacement **AD** will be = AP + PQ + QD = AB sin 30°+ BC + CD sin 30° = 500 × ½ + 500 + 500 × ½ = 1000m = 1km

The direction of **AD** is 60° left of the initial direction **AB**.

Total path length = AB + BC + CD = 500×3 = 1500m = 1.5km

(ii) At the sixth turn, the motorist comes back to the starting point A, so magnitude of displacement is zero.

Total path length = AB + BC + CD + DE + EF + FA = 500×6 = 3000m = 3km

(iii) At the end of eighth turn, the motorist will be at C. The magnitude of his displacement **AC** is

|**AC**| = AR + RC = AB sin60° + BC sin 60° = $500\times\frac{\sqrt{3}}{2}+500\times\frac{\sqrt{3}}{2} = 500\sqrt{3}$ = 866m.

The direction of **AC** is 30° left fo the initial direction **AB**. Total path length = 500×8 = 4000m = 4km.

C. MULTIPLICATION OF VECTOR

(1 Mark Questions)

1. What is the maximum of components into which a vector can be resolved?

Sol. A vector can be split into infinite components (but only 3 orthogonal ones).

2. What is the dot product of two similar unit vectors?

Sol. Unity For example, $\hat{i}.\hat{i}$ = (1)(1)cos0° = 1.

3. What is the value of $\hat{i}.(\hat{j}\times 2\hat{k})$?

Sol. The value of $\hat{i}.(\hat{j}\times 2\hat{k})$ is zero.

4. State with reasons, whether the following algebraic operations with scalar and vector physical quantities are meaningful:

(a) adding any two scalars

Sol. No. Only two such scalars can be added which represent the same physical quantity.

(b) adding a scalar to a vector of the same dimensions,

Sol. No. A scalar cannot be added to a vector even of same dimensions because a vector has a direction while a scalar has no direction e.g., speed cannot be added to velocity

(c) multiplying any vector by any scalar,

Sol. Yes. We can multiply any vector by a scalar. For example, when mass (scalar) is multiplied with acceleration (vector), we get force (vector).

(d) multiplying any two scalars

Sol. Yes. We can multiply any two scalars. When we multiply power (scalar) with time (scalar), we get work done (scalar) i.e., W = Pt.

(e) adding any two vectors

Sol. No. Only two vectors of same nature can be added by using the law of vector addition.

(f) adding a component of a vector to the same vector

Sol. No. A component of a vector can be added to the same vector only by using the law of vector addition. So the addition of a component of a vector to the same vector is not a meaningful algebraic operation.

5. The angle between $\vec{A} = \hat{i} + \hat{j}$ and $\vec{B} = \hat{i} - \hat{j}$, is

(a) 45° (b) 90° (c) –45° (d) 180°

Ans. (b)

Given, **A** = $\hat{i} + \hat{j}$; **B** = $\hat{i} - \hat{j}$

As we know that $\vec{A}.\vec{B} = |\mathbf{A}||\mathbf{B}| \cos\theta$

$(\hat{i} + \hat{j}).(\hat{i} - \hat{j}) = (\sqrt{1^2 + 1^2})(\sqrt{1^2 + 1^2})\cos\theta$

$(\hat{i} + \hat{j})(\hat{i} - \hat{j}) = \cos\theta$

$\sqrt{2} \times \sqrt{2} = \cos\theta$

Where θ is the angle between **A** and **B**

$\cos\theta = \frac{1-0+0-1}{\sqrt{2}\sqrt{2}} = 0$

Therefore θ = 90°

6. It is found that $|\vec{A}+\vec{B}|=|\vec{A}|$. This necessarily implies,

(a) $\vec{B} = 0$ (b) $\vec{A},\vec{B}$ are antiparallel

(c) $\vec{A},\vec{B}$ are perpendicular (d) $\vec{A}.\vec{B} \leq 0$

Ans. (a)

It means that $\vec{B}$ will be zero magnitude and there will be no direction.

(2 Marks Questions)

7. Find the angle between the vectors $\vec{A} = 2\hat{i} - 4\hat{j} + 6\hat{k}$ and $\vec{B} = 3\hat{i} + \hat{j} + 2\hat{k}$.

Sol. We have two vectors

$\vec{a} = 2i - 4j + 6k$

$\vec{b} = 3i + j + 2k$

We know that

$\vec{a}.\vec{b} = |a||b|\ cos\theta$

$|a| = \sqrt{(2)^2 + (-4)^2 + (6)^2} = \sqrt{56}$

$|b| = \sqrt{(3)^2 + (1)^2 + (2)^2} = \sqrt{14}$

$\cos\theta = \dfrac{(2i - 4j + 6k)(3i + j + 2k)}{\sqrt{56}\sqrt{14}}$

$cos\theta = \dfrac{1}{2}$

$\theta = 60^0$

8. For what value of a are the vectors $\vec{A} = a\hat{i} - 2\hat{j} + \hat{k}$, $\vec{B} = 2a\hat{i} + a\hat{j} - 4\hat{k}$ and perpendicular to each other.

Sol. To prove perpendicular dot product of vector A and vector B should be zero.

So, A. B=0

⇒(ai^-2j^+k^) .(2ai^+aj^-4k^)=0

⇒$2a^2$-2a-4=0

⇒a^2-a-2=0

⇒a^2-2a+a-2=0

⇒a(a-2)+1(a-2)=0

⇒(a-2)(a+1)=0

⇒a=2 or -1

Hence, value of a will be 2 or -1 so that two vectors will be perpendicular to each other.

9. Find the angles between the following pairs of vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = -2\hat{i} - 2\hat{j} - 2\hat{k}$.

Sol. Let vector $\vec{A}$ = î+ĵ+$\hat{k}$, then | $\vec{A}$ | = √{$1^2+1^2+1^2$} = √3. units.

Vector $\vec{B}$ = -2î-2ĵ-2$\hat{k}$, then | $\vec{B}$ | = √{$(-2)^2+(-2)^2+(-2)^2$} = 2.√3.units.

Angle between vector $\vec{A}$ and $\vec{B}$ = $\cos^{-1}$[vector $\vec{A}$.vector $\vec{B}$ / | $\vec{A}$ |.| $\vec{B}$ |].

= $\cos^{-1}$[(î+ĵ+$\hat{k}$).(-2î-2ĵ-2$\hat{k}$)/√3.2.√3.]

= $\cos^{-1}$ [(-2–2–2)/6.]

= $\cos^{-1}$ [-1].

= 180°.

10. (a) If $\hat{i}$ and $\hat{j}$ are unit vectors along X- and Y-axis respectively, then what is the magnitude and direction of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$?

(b) Find the components of along the directions of vectors and $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$.

[Ans. (a)45°,45°, (b) 5/2($\hat{i}+\hat{j}$), - ½ ($\hat{i}-\hat{j}$).]

____
____
____
____

(3 Marks Questions)

11. Two vectors both equal in magnitude, have their resultant equal in magnitude of the either. Find the angle between the two vectors.

Sol. Here P = Q = R

As $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\therefore P = \sqrt{P^2 + P^2 + 2P.P\cos\theta}$

Or $P^2 = 2P^2(1 + \cos\theta)$

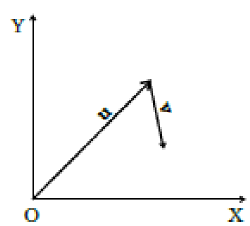
Or $1 + \cos\theta = ½$

Or $\cos\theta = -½ = \cos 120°$ or $\theta = 120°$.

D. INTRODUCTION OF MOTION IN PLANE

(1 Mark Questions)

1. Figure shows the orientation of two vectors u and v in the XY plane. If $u = a\hat{i} + b\hat{j}$ and $v = p\hat{i} + q\hat{j}$



which of the following is correct?

(a) a and p are positive while b and q are negative.

(b) a, p and b are positive while q is negative.

(c) a, q and b are positive while p is negative. (d) a, b, p and q are all positive.

Ans. (b)

From the diagram $u = a\hat{i} + b\hat{j}$

As u is the first quadrant, so both components a and b will be positive

For $v = p\hat{i} + q\hat{j}$ as it is in positive x direction and located component q will be negative.
Hence a, b and p are positive but q is negative

2. For any arbitrary motion in space, which of the following relations are true:

(a) $\vec{v}_{average} = (1/2)[\vec{v}(t_1) + \vec{v}(t_2)]$

Sol. Invalid for non uniform acceleration.

(b) $\vec{v}_{average} = [\vec{r}(t_2) - \vec{r}(t_1)]/(t_2 - t_1)$

Sol. This is mathematical definition of average velocity

(c) $\vec{v}(t) = \vec{v}(0) + \vec{a}t$

Sol. This is valid for constant acceleration

(d) $\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + (1/2)\vec{a}t^2$

Sol. Valid only for constant acceleration

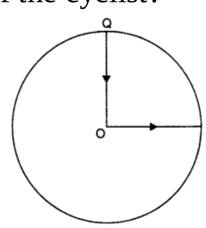
(e) $\vec{a}_{average} = [\vec{v}(t_2) - \vec{v}(t_1)]/(t_2 - t_1)$

(The average stands for average of the quantity over the time interval t_1 to t_2)

Sol. Mathematical definition of average acceleration.
Only (b) and (e) are valid.

(3 Marks Questions)

3. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist?



Sol. (a) Net Displacement is zero as both initial and final positions are same

(b) Average velocity = displacement/time taken. As displacement is zero, average velocity of the cyclist is also zero.

(c)Total distance covered = OP + arc PQ + OQ = $2\pi r/4 + 4$

$= 1 + \frac{2\times 22\times 1}{7\times 4} + 1 = \frac{25}{4}$km

Time taken = 10 min = 1/6 hr

Average speed = $\frac{\textbf{Total distance covered}}{\textbf{time taken}} = \frac{\textbf{25/7km}}{\textbf{1/6h}}$ = 21.43 kmh^{-1}.

4. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cab man takes him along a

circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal?

Sol. Magnitude of displacement = 10km, total path length = 23km

Time taken = 28min = 28/60h = 7/15 h

(i) Average speed = $\frac{\text{Total path length}}{\text{Time taken}} = \frac{23\text{km}}{\frac{7}{15}\text{h}} = 49.3\text{kmh}^{-1}$

(ii) Magnitude of average velocity = $\frac{\textbf{Displacement}}{\textbf{Time taken}} = \frac{\textbf{10km}}{\frac{7}{15}\text{h}} = \mathbf{21.43\ kmh^{-1}}$

Clearly the average speed and magnitude of the average are not equal. They will be equal only for straight path.

5. The position of a particle is given by

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k}\ \text{m}$$

Where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the value of $\vec{v}$ and $\vec{a}$ of the particle,

(b) What is the magnitude and direction of velocity of the particle at t = 2.0s?

Sol. Same as 6.

The direction of velocity is given by θ = $\tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{8}{3}\right)$

= $\tan^{-1}$ (- 2.6667) = -70° with x axis.

(5 Marks Questions)

6. The position vector of a particle is given by: $\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\hat{k}$

Where t is in seconds and the coefficients have the proper units for $\vec{r}$ to be in metres.

(a) Find the $\vec{v}$ and $\vec{a}$ of the particle.

(b) What is the magnitude and direction of velocity of the particle at t = 2s?

Sol. $\vec{r} = 3.0t\hat{i} - 2.0t^2\,\hat{j} + 4\hat{k}$

(a) Velocity $\vec{v} = \frac{d\vec{r}}{dt} = 3.01\hat{i} - 4.01\hat{j}$ m/s =

Acceleration $\vec{a} = \frac{d\vec{v}}{dt} = -4.0\hat{j}$ m/s^2 =

(b) magnitude of the velocity at t = 2s

$\vec{v} = 3.0\,\hat{i} - 8.0\,\hat{j}$

$|\vec{v}| = \sqrt{(3)^2 + (-8)^2} = \sqrt{9 + 64} = \sqrt{73}$ m/s.

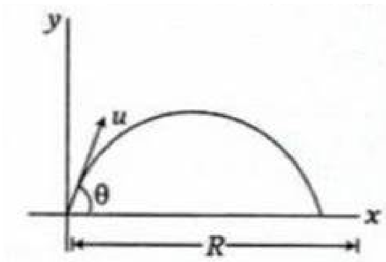
E. PROJECTILE MOTION

(1 Mark Questions)

1. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be

(a) 60 m (b) 71 m (c) 100 m (d) 141 m

Ans. (c)



Consider projectile is fired at an angle □. According to the question, □ = 15° and R = 50m.

Range = 50m = $\frac{u^2 \sin(2\times15°)}{g}$+

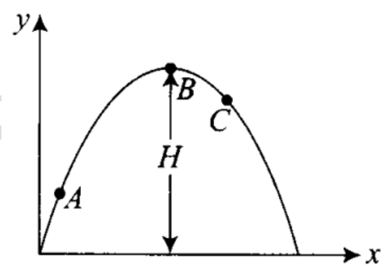
$50\times g = u^2$ in30° = $u^2 \times ½ \Rightarrow 50\times g\times2 = u^2$

$u^2 = 50\times9.8\times2 = 100\times9.8 = 980$

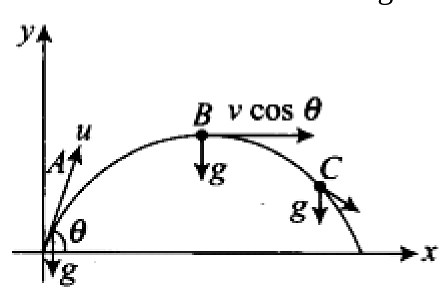
$u = \sqrt{980} = 31.304$m/s $= 14\sqrt{5}$ (since $g = 9.8 m/s^2$)

Now θ = 45°, R = $\frac{u^2 \sin 2\times45°}{g} = u^2/g = 100$m

2. A particle is projected in air at some angle to the horizontal, moves along parabola as shown in Figure, where x and y indicate horizontal and vertical directions, respectively. Show in the diagram, direction of velocity and acceleration at points A, B and C.



Sol. Direction of the velocities at point A, B and C are along the tangent of the path. Acceleration at each point is the acceleration due to gravity vertically downwards.



3. A football is kicked into the air vertically upwards. What is its (a) acceleration, and (b) velocity at the highest point?

Sol. We have velocity of the football at the highest point as zero, i.e. v = 0. Hence the acceleration of the football at highest point is acceleration due to gravity (g) and velocity of the football at highest point is zero.

4. In case of a projectile motion, what is the angle between the velocity and acceleration at the highest point?

(a) 0° (b) 45° (c) 90° (d) 180°

Sol. (c)

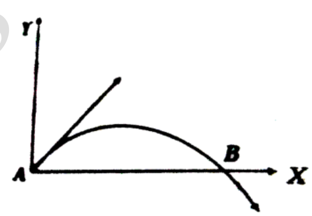
In projectile motion acceleration is directed towards earth and at the peal point the angel between velocity and acceleration is 90° because at peak point vertical component of velocity is 0.

5. From a certain height above the ground a stone A is dropped gently. Simultaneously another stone B is fired horizontally. Which of the two stone will arrive on the ground earlier??

Sol. Both the stones will hit the ground simultaneously.

(2 Marks Questions)

6. The velocity of a projectile at the initial point A is $(2\,\hat{i} + 3\,\hat{j})$ m/s. What will be its velocity (in m/s) at point B?



Sol. At point B, X Component of velocity remains unchanged while Y component reverses its direction.

Therefore the velocity of the projectile at point B is $2\hat{i} - 3\hat{j}$ m/s.

7. Prove that the maximum horizontal range is four times the maximum height attained by a projectile which is fired along the required oblique direction.

Sol. Horizontal range, $R = \frac{u^2 \sin\theta}{g}$

For range to be maximum, sin 2θ = I

$\Rightarrow R_{max} = \frac{u^2}{g}$ …(i)

Maximum height attained by particle projected at θ = 45°,

$h_{max} = \frac{u^2 \sin\theta}{2g} = \frac{u^2 \sin^2 45°}{2g} = \frac{u^2}{4g}$ …(ii)

From equations (i) and (ii)

$R_{max} = 4h_{max}$. Hence proved.

(3 Marks Questions)

8. Show that motion of one projectile as seen from another projectile will be a straight line.

Sol. Let u_1 and θ_1 are the initial velocity and angle of projection of projectile 1. Let u_2 and θ_2 be the initial velocity and angle of projection of projectile 2.

For projectile 1,

$x = u_1 \cos \theta_1 \times t$, $y = u_1 \sin \theta_1 t = ½ gt^2$

So, the position vector can be written as $\vec{R}_1 = (u_1\cos\theta_1 t)\hat{i} + (u_1\sin\theta_1 t – ½ gt^2)\hat{j}$

Similarly for projectile 2, $\vec{R}_2 = (u_2 \cos\theta_2 t)\, \hat{i} + (u_2\sin\theta_2 t – ½ gt^2)\, \hat{j}$

Relative position vector of projectile 1 w.r.t. 2

$\vec{R}_1 – \vec{R}_2 = t(u_1\cos\theta_1 – u_2\cos\theta_2)\, \hat{i} + t(u_1\sin\theta_1 – u_2\sin\theta_2)\, \hat{j}$

$\vec{R}_1 – \vec{R}_2 = k_1 t\hat{i} + k_2 t\hat{j}$ where $k_2 = u_1\sin\theta_1 – u_2\sin\theta_2$ and $k_1 = u_1\cos\theta_1 – u_2\cos\theta_2$

Since the relative position vector is independent of acceleration so, the path of one projectile will appears to be a straight line when observed from another projectile.

9. The equations of motion of a projectile are given by x = 36t m and $2y = 96t – 9.8t^2$ m. Find the angle of projection.

Sol. Given x = 36t, $2y = 96t – 9.8t^2$ or $y = 48t – 4.9t^2$

Let the initial velocity of projectile be u and angle of projection is θ. Then

Initial horizontal component of velocity,

$u_x = u \cos \theta = \left(\frac{dx}{dt}\right)_{t=0} = 36$ or u cos θ = 36 …(i)

Initial vertical component of velocity,

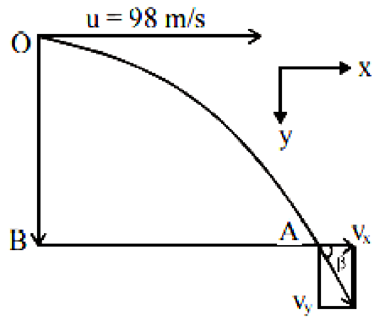
$u_y = u \sin \theta = \left(\frac{dy}{dt}\right)_{t=0} = 48$ or u sin θ = 48 …(ii)

Dividing (i) by (ii) we get tan θ = 48/36 = 4/3

Therefore sin θ = 4/5 or $\theta = \sin^{-1}(4/5)$.

10. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground.

Sol.



(i) As shown in figure the projectile is fired from the top O of a hill with a velocity u = 9.8 ms^{-1} along the horizontal OX. It reaches the target P in time t.

Initial velocity in the downward direction = 0

Vertical distance, OA = y = 490m

As y = ½ gt^2

Therefore 490 = ½ × $9.8t^2$

Or t = $\sqrt{100}$ = 10s

(ii) Distance of the target from the hill, AP = x = Horizontal velocity × time

= 98 × 10 = 980m.

(iii) The horizontal and vertical components of velocity v of the projectile at point P are

$v_x = u = 98\ ms^{-1}$

$v_y = u_x + gt = 0 + 9.8 \times 10 = 98 ms^{-1}$

Therefore v = $\sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2}$ = 138.59ms^{-1}.

11. A projectile has a range of 50m and reaches a maximum height of 10m. Calculate the angle at which the projectile is fired.

Sol. Here R = 50m, H = 10m, θ = ?

Horizontal range, R = $\frac{u^2\sin2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g}$ …(1)

Maximum height, H = $\frac{u^2\sin^2\theta}{2g}$ …(2)

Dividing (1) by (2) we get

$\frac{H}{R} = \frac{u^2\sin^2\theta}{2g} \times \frac{g}{2u^2\sin\theta\cos\theta} = \frac{1}{4}\tan\theta$

Or tan θ = $\frac{4H}{R} = \frac{4\times10}{50} = 0.8$

Or θ = $\tan^{-1}(0.8)$ = 38.66°.

12. Prove that the maximum horizontal range is four times the maximum height attained by the projectile, when fired at an inclination so as to have maximum horizontal range.

Sol. For θ = 45°, the horizontal range is maximum and is given by $R_{max} = \frac{u^2}{g}$

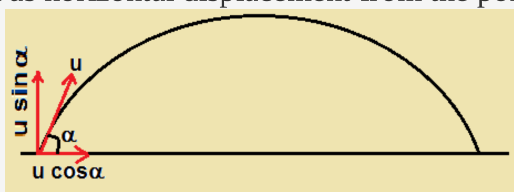
Maximum height attained, $H_{max} = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g} = \frac{R_{max}}{4}$

Or $R_{max} = 4H_{max}$.

13. A projectile is fired with a velocity 'u' making an angle θ with the horizontal. Show that its trajectory is a parabola.

Sol. Projectile is an object projected with some velocity and its initial direction of motion makes an angle α (α >0 and α <90°) with horizontal direction.

Projetile moves in a two dimensional plane so that at given time it has vertical displacement as well as horizontal displacement from the point of projection.



Let the projetion velocity be u and let it make an angle α with horizontal x-direction

velocity component :- $u_x = u \cos \alpha$ and $u_y = u \sin \alpha$

displacement component :- $x = u \cos(\alpha) \times t$ and $y = u \sin(\alpha) \times t - (1/2) \times g \times t^2$;

by eliminating t (substitue $t = x / [u \cos(\alpha)]$ in y-displacement),

we get for vertical displacement,

$$y = (\tan \alpha)\, x - \left(\frac{g}{2u^2\cos^2\alpha}\right)x^2$$

above equation is similar to $y = bx - cx^2$, which is equation of parabola

14. Find the angle of projection at which the horizontal range and maximum height of a projectile are equal.

Sol. Given horizontal range = maximum range

Or $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin\theta}{2g}$

Or $2 \sin \theta \cos\theta = \frac{\sin^2\theta}{2}$

Or $\frac{\sin\theta}{\cos\theta} = 4$ or $\tan\theta = 4$

So, θ = 75°58'.

15. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m s^{-1} can go without hitting the ceiling of the hall?

Sol. Here H = 25m, u = $40ms^{-1}$.

If the ball is thrown at an angle θ with the horizontal, then maximum height of the flight

$H = \frac{u^2 \sin^2\theta}{2g}$

So, $25 = \frac{(40)^2 \sin^2\theta}{2\times9.8}$

Or $\sin^2\theta = \frac{25\times2\times9.8}{(40)^2} = 0.306$

Or $\sin\theta = \sqrt{0.306} = 0.554$

And $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-0.306} = \sqrt{0.694}$=0.833

The maximum horizontal distance is given by

$R = \frac{u^2\sin^2 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{2\times(40)^2\times0.554\times0.833}{9.8}$ = 150.7m.

16. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?

Sol. $R_{max} = u^2/g$ = 100m or u^2 = 100g or $u = \sqrt{\mathbf{100g}}$

For upward throw the ball we have, $u = \sqrt{\mathbf{100g}}$, v = 0, a = - g, s = ?

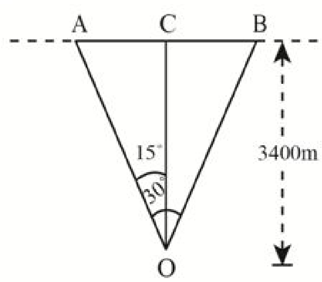
As $v^2 - u^2 = 2as$

Therefore, 0 – 100g = 2(- g)s

Or s = - 100g/- 2g = 50m.

17. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10 s apart is 30°, what is the speed of the aircraft? Time taken by aircraft from A to B is 10 s.

Sol. Let A and B represent the two aircraft positions separated 10s apart.



Then tan15° = x/3400

Or x = 3400 tan15° = 3400×0.2679 = 910.86m

Speed of aircraft = 910.86m/5s = 182.2 ms^{-1}.

18. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, can one hope to hit a target 5 km away? Assume the muzzle speed to the fixed, and neglect air resistance.

Sol. In the first case, R = 3km = 3000m, θ = 30°

Horizontal range, $R = \frac{\mathbf{u^2 \sin 2\theta}}{\mathbf{g}}$

Therefore, $3000 = \frac{\mathbf{u^2 \sin 60}}{\mathbf{g}}$

Or $\frac{u^2}{g} = \frac{3000}{\sin 60°} = \frac{3000\times 2}{\sqrt{3}} = \mathbf{2000\sqrt{3}}$

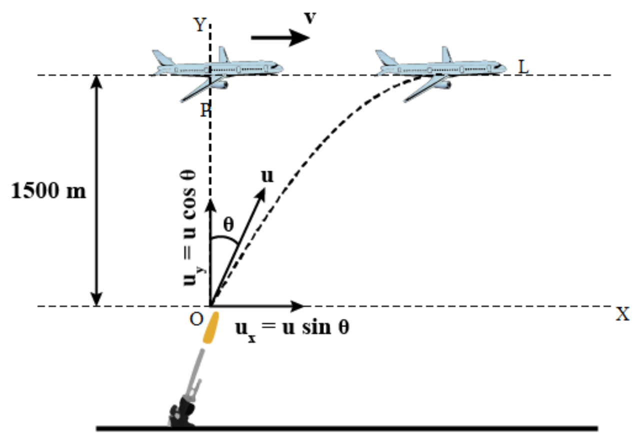
Maximum horizontal range, $R_{max} = \frac{u^2}{g} = \mathbf{2000\sqrt{3}}$ = 3464m = 3.46km

But the distance of the target (5km) is greater than the maximum horizontal range of 3.46km, so, the target cannot hit by adjusting the angle of projection.

19. A fighter plane flying horizontally at an altitude of 1.5 km with speed 720 km h^{-1} passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gun be fired for the shell with muzzle speed 600 m s^{-1} to hit the plane? At what minimum altitude should the pilot fly the plane to avoid being hit? (Take g = 10 m s^{-2})?

Sol. Speed of plane = 720kmh^{-1} = 200ms^{-1}

The shell moves along the curve OL. The plame moves along PL. Let them hit after time t.



For hitting the horizontal distance travelled by the plane = horizontal distance travelled by the shell

Or horizontal velocity of plane ×t = horizontal velocity of shell × t

200 ×t = 600 cos θ × t

cos θ = 200/600 = 1/3 or θ = 70°30'

The shell should fired at an angle of 70°30' with the horizontal of 19°30' with the vertical.

The maximum height of the flight of the shell is

$$h = \frac{u^2\sin^2\theta}{2g} = \frac{u^2(1-\cos^2\theta)}{2g} = \frac{(600)^2\times(1-\frac{1}{9})}{2\times 10} = \mathbf{16000m = 16km}.$$

(5 Marks Questions)

20. A body is projected with velocity v at an angle θ with the horizontal. Find the (a) Time of flight (b) Maximum height attained (c) Maximum range for the body.

Sol. Let the projectile is fired at an angle θ with initial velocity u.

Let u_x and u_y be the component of u along x and y direction respectively.
Using equation of motion along horizontal and vertical direction

$v_x = u_x = u \cos \theta$ …(i)

$v_y = u \sin \theta - gt$ …(ii)

At maximum $v_y = 0$, so, $u \sin \theta = gt_m$

Where t_m is the time of maximum height.

$t_m = u\sin \theta / g$ …(iii)

Since $y = u_y t - \frac{1}{2} gt^2$

At maximum height, $H = u \sin\theta\, t_m - \frac{1}{2} g_m^2$

$= (u\sin\theta)\left(\frac{u\sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{u^2\sin^2\theta}{g^2}\right)$ or $H = \frac{u^2\sin^2\theta}{2g}$

Since total time ascend is equal to total time of descend, then total time of flight

$T_f = 2t_m =$, $T_f = \frac{2u\sin\theta}{g}$

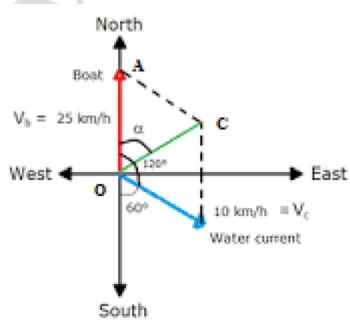
Horizontal range $= u_x T_f = u\cos\theta\left(\frac{2u\sin\theta}{g}\right) = \frac{u^2\sin^2\theta}{g}$

F. RELATIVE VELOCITY IN TWO DIMENSIONS

(3 Marks Questions)

1. A motorboat is racing towards north at 25 km/h and the water current in that region is 10km/h in the direction of 60° east of south. Find the resultant velocity of the boat.

Sol.



Let the motorboat start moving form O as shown in figure

$\vec{v}_b$ = velocity of motorboat, = 25kmh^{-1} due north.

$\vec{v}_c$ = velocity of water current = 10 kmh^{-1}, 60° east of south.

By parallelogram law, the resultant velocity $\vec{v}$ is equal to the diagonal $\overrightarrow{OC}$.

Its magnitude is $v = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cos 120°}$

$= \sqrt{25^2 + 10^2 + 2 \times 25 \times 10(-\frac{1}{2})}$ = 21.8 kmh^{-1}

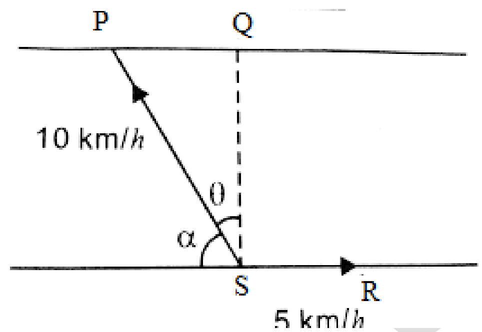
Suppose the resultant velocity $\vec{v}$ meals angle β with the north direction. Then

$\tan\beta = \frac{v_c \sin 120°}{v_b + v_c \cos 120°} = \frac{10\times(\frac{\sqrt{3}}{2})}{25+10\times(-\frac{1}{2})} = \frac{\sqrt{3}}{4} = 0.433$

Therefore $\beta = \tan^{-1}(0.433) = 23.4°$.

2. A boatman can row with a speed of 10km/h in still water. If the river flows steadily at 5km/h, in which direction should the boatman row in order to reach a point on the other bank directly opposite to the point from where he started? The width of the river is 2km.

Sol. As shown in figure, the boatman starts from S. He should reach Q. Since the river flows along PQ with a velocity of $5 kmh^{-1}$, he should travel along SP.



Speed of boatman is shown by vector $\overrightarrow{SP}$, $\overrightarrow{SP} = 10\ kmh^{-1}$

$\overrightarrow{SQ}$ is the resultant of $\overrightarrow{SP}$ and $\overrightarrow{PQ}$

$\angle QSP = \alpha$, $\angle PQS = 90°$

Since Q and S are directly opposite

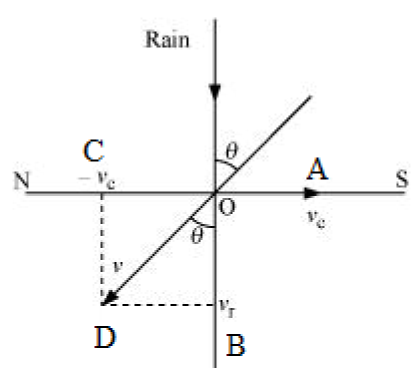
$\sin\alpha = \frac{PQ}{SP} = \frac{5}{10} = \frac{1}{2}$ i.e $\alpha = 30°$

Therefore $\alpha = 90° + \alpha = 120°$

Thus the boatman must row the boat in a direction at an angle of 120↑8 with the direction of river flow. The direction does not depend on width of the river.

3. Rain is falling vertically with a speed of 30 m s^{-1}. A woman rides a bicycle with a speed of 10 m s^{-1} in the north to south direction. What is the direction in which she should hold her umbrella?

Sol.



The situation is shown in figure.

Here $\overrightarrow{\mathbf{OA}} = \vec{\mathbf{v}}_w$ = velocity of woman cyclist = 10ms^{-1} due south

$\overrightarrow{\mathbf{OB}} = \vec{\mathbf{v}}_R$ = velocity of rain = 30ms^{-1}, vertically downward

$\overrightarrow{\mathbf{OC}} = \vec{\mathbf{v}}_W$ = opposite velocity of the woman cyclist

$\overrightarrow{\mathbf{OD}} = \vec{\mathbf{v}}_R + (-\vec{\mathbf{v}}_W) = \vec{\mathbf{v}}_R - \vec{\mathbf{v}}_W = \vec{\mathbf{v}}_{RW}$

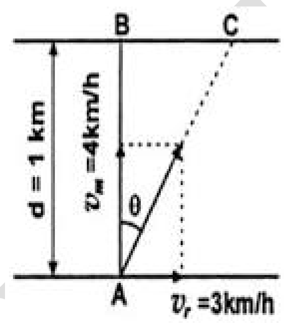
= velocity of rain relative to woman cyclist

$\vec{\mathbf{v}}_{RW} = \overrightarrow{\mathbf{OD}} = \sqrt{\mathbf{OC^2 + OB^2}} = \sqrt{\mathbf{10^2 + 30^2}} = \mathbf{10\sqrt{10} = 31.6ms^{-1}}$

If OD makes angle β with the vertical, then tan β = $\frac{\mathbf{BD}}{\mathbf{OB}} = \frac{\mathbf{OC}}{\mathbf{OB}} = \frac{\mathbf{10}}{\mathbf{30}} = \mathbf{0.333}$ or β = 18°26'.

4. A man can swim with a speed of 4.0 km h^{-1} in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km h^{-1} and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank?

Sol.



In the figure, $\vec{\mathbf{v}}_m$ and $\vec{\mathbf{v}}_R$ are the velocities of man and river. Clearly $\vec{\mathbf{v}}$ is the resultant of these velocities. If the man begins to swim along AB, he will be deflected to the path AC by flowing river.

Time taken to cover AC with velocity $\vec{\mathbf{v}}$ will be same as the time taken to cover distance AB with velocity $\vec{\mathbf{v}}_M$.

So, time taken by the man to cross the river is t = $\frac{\mathbf{AB}}{\mathbf{v_m}} = \frac{\mathbf{1km}}{\mathbf{4kmh^{-1}}} = \frac{\mathbf{1}}{\mathbf{4}}\mathbf{h = 15min}$

Distance through which the man goes down the river is BC = $v_R \times t$ = 3kmh^{-1} × ¼ h = 0.75km.

G. UNIFORM CIRCULAR MOTION

(1 Mark Questions)

1. A stone tied to the end of a string 100cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 22s, then the acceleration of the stone is

(a) 16 ms^{-2} (b) 4 ms^{-2} (c) 12 ms^{-2} (d) 8 ms^{-2}

Ans. (a)

Here r = 100cm = 1m, frequency, v = 14/22 rps,

Therefore $\omega = 2\pi v = 2 \times \frac{22}{7} \times \frac{14}{22} = 4 rads^{-1}$

The acceleration of the stone = $a_c = \omega^2 r = (4)^2(1) = 16\ ms^{-2}$.

2. Velocity vector and acceleration vector in a uniform circular motion are related as

(a) both in same direction (b) perpendicular to each other

(c) both in opposite direction (d) not related to each other

Ans. (a)

3. Can an object be accelerated without speeding up or slowing down? Explain.

Sol. Yes, an object can be accelerated without speeding up or slowing down. We know that acceleration is referred as the rate of change of velocity and the velocity is the speed in a particular direction and is a vector quantity. So, if the velocity can either changes its speed or direction, there is some acceleration.

4. What the angular frequency of the object if it complete 100 revolutions in 50 seconds?

Sol. Angular velocity = $(100 x 2\pi) / 50 = 4\pi$.

5. Read each statement below carefully and state, with reasons, if it is true or false:

(a) The net acceleration of a particle in circular motion is always along the radius of the circle towards the centre.

Sol. False—The net acceleration of a particle in circular motion is towards the centre only if its speed is constant.

(b) The velocity vector of a particle at a point is always along the tangent to the path of the particle at that poin.

Sol. True—A particle released at any point of its path will always move along the tangent to the path at the point.

(c) The acceleration vector of a particle in uniform circular motion averaged over one cycle is a null vector

Sol. True—For any two diametrically opposite points on the circumference, the acceleration vectors are equal and opposite. Hence, the acceleration vector average over one completely cycle is null vector.

(2 Marks Questions)

6. A car is moving along a circular road at speed of 20m/s. The radius of the circular road is 10m. If the speed is increased at the rate of $30m/s^2$, what is the resultant acceleration?

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(3 Marks Questions)

7. Assuming that the moon completes one revolution in a circular orbit around the earth in 27.3 days, calculate the acceleration of the moon towards the earth. The radius of the circular orbit can be taken as 3.85×10^5 km.

Sol. $a = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = 3.85\times10^8\times\left[\frac{2\times3.14}{27.3\times24\times60\times60}\right]^2 = 2.73\times10^{-3}\ ms^{-2}$.

8. **A** stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s, what is the magnitude and direction of acceleration of the stone?

Sol. Here r = 80cm, v = 14/25 rps

So, $\omega = 2\pi\nu = \mathbf{2\times\frac{22}{7}\times\frac{14}{25} = \frac{8}{25}\,rads^{-1}}$

The acceleration of the stone is $a = r\omega^2 = \mathbf{80\times\left(\frac{88}{25}\right)^2 = 9991.2\ cms^{-2}}$

This acceleration is directed along the radius of the circular path towards the centre fo the circle.

9. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 km/h. Compare its centripetal acceleration with the acceleration due to gravity.

Sol. Here r = 1km = 1000m, v = $900kmh^{-1} = \frac{900\times5}{18} = 250ms^{-1}$

Centripetal acceleration, $a = \frac{v^2}{r} = \frac{(250)^2}{1000} = 62.5\ ms^{-2}$

Therefore $\frac{\text{Centripetal acceleration}}{\text{Acceleration due to gravity}} = \frac{62.5}{9.8} = 6.38$

(5 Marks Questions)

10. (i) Derive an expression for the centripetal acceleration of a body moving with uniform speed v along a circular path of radius r.

(ii) A stone tied to the end of the string 80cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 seconds, what is the magnitude and direction of acceleration of the stone?

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H. CASE STUDY

1. Relative velocity: If two objects A and B are moving in a straight line with velocities v_A and v_b respectively, the relative velocity of object A with respect to object B is given by $v_{AB} = v_A - v_B$. It follows that the relative velocity of object B with respect to object A will be $v_{BA} = v_B - v_A$. FI objects A and B are moving alogn different directions in a plane, the relative velocity of one objet with respect to the other is found by using the parallelogram law fo vector addidion. To find v_{AB}, we find the resultant of vectors v_A and - v_B. To find v_{BA} we find the resultant of vectors v_B and – v_A.

(i) Two persons P and Q are standing 54m apart on a long horizontal belt moving with a speed of 4m/s in the direction from P to Q. Person P rolls a round stone towards person Q with a speed of 9m/s with respect to the belt. The velocity of stone with respect to an observer on a stationary platform is

(a) 13m/s in the direction to motion of the belt

(b) 13 m/s opposite to the direction of motion of the belt

(c) 5m/s in the direction of motion of the belt

(d) 5m/s opposite to the direction of motion of the belt.

Sol. Let us choose the positive direction to be the direction from P to Q, i.e. the direction of motion of the belt. Then the velocity of the belt is $v_B = +\ 4$ m/s, speed of the stone with respect to the belt, $v_{SB} = +\ 9$m/s. If v_s is the speed fo the stone with respect to a stationary observer, we have

$V_{SB} = v_S - v_B$ or $v_S = v_{SB} + v_B = +9+4 = +13$m/s

The positive sign shows that the direction of velocity of stone is from P to Q i.e. in the direction fo motion of the belt. Hence the correct choice is (a).

(ii) What will be the answer to (i) above if person Q rolls the stone towards person P with a speed of 9m/s with respect to the belt?

(a) 13m/s in the direction to motion of the belt

(b) 13 m/s opposite to the direction of motion of the belt

(c) 5m/s in the direction of motion of the belt

(d) 5m/s opposite to the direction of motion of the belt.

Sol. In the case v_B = +4m/s by v_{SB} – 9 m/s. Hence $v_S = v_{SB} + v_B$ = - 9 + 4 = - 5m/s.
The negative sign indicates that the direction of velocity of stone is opposite to the direction of motion of the belt. Hence the correct choice is (d).

(iii) In (i) above, what is the time taken by the stone to travel from P to Q?

(a) 54/13s (b) 54/5s (c) 6s (d) 27/2s

Sol. Since persons P and Q and the stone are located on the belt, the speed of the stone relative to P or Q will be 9m/s.
So, the time taken = 54m/9m/s = 6s which is choice (c).

I. ASSERTION REASON QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true

1. Assertion: Vector addition is commutative.
Reason: $(\vec{A} + \vec{B}) \neq (\vec{B} + \vec{A})$

Ans. (c) Assertion is true but reason is false.
Since vector addition is commutative, therefore $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

2. Assertion: If $\vec{A}$ is parallel to $\vec{B}$ then $\vec{A} \times \vec{B}$ is a null vector.
Reason: The magnitude cross product of two vectors is given by $|\vec{A} \times \vec{B}|$ = AB sin q.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
As $\vec{A} \| \vec{B}$, ∴ θ = 0 ⇒ $\vec{A} \times \vec{B} = AB \sin 0 = \vec{0}$
i.e. $\vec{A} \times \vec{B}$ is a null vector. Where null vector is a vector whose magnitude is zero but has a direction.

3. Assertion: Two equal vectors have their resultant equal to three fourth of either of them. The angle between them is 136°.
Reason: The minimum number of numerically equal vectors whose sum can be zero is three.

Ans. (c) Assertion is true but reason is false
Let $|\vec{A}| = |\vec{B}|$ = x and $|\vec{R}|$ = ¾ x then
$R^2 = \vec{A}^2 + B^2 + 2AB\cos\theta$
Therefore $9/16x^2 = x^2 + x^2 + 2\times x\times x\times\cos\theta \Rightarrow 9/16x^2 = 2x^2(1+\cos\theta) \Rightarrow \cos\theta = 9/32 - 1 = 23/32 = -0.7188$, therefore θ = 136°
Two vectors numerically equal but opposite in direction can give sum as zero.

4. Assertion: When a body is dropped or thrown horizontally from the same height, it would reach the ground at the same time.

Reason: Horizontal velocity has no effect on the vertical direction.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion. Horizontally velocity provides the horizontal range. It does not effect time taken in the vertical direction.

5. Assertion: The maximum height of a projectile is 25 percent of maximum range.
Reason: The maximum height is independent of initial velocity of projectile.

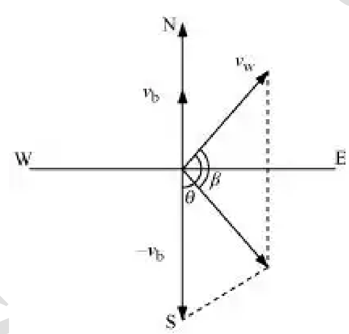
Ans. (c) Assertion is true but reason is false.

For maximum range, θ = 45°. In that case maximum height = $\frac{u^2 sin^2 45°}{2g} = \frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{u^2}{4g} = \frac{1}{4}\left(\frac{u^2}{g}\right)$ = 25% maximum range. Since angle for maximum range is 45°, therefore the percentage also cannot vary.

J. CHALLENGING PROBLEMS

1. In a harbour, wind is blowing at the speed of 72 km/h and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

Sol.



When the boat is stationary, the flag flutters along NE direction. This shows that velocity of the wind is along NE direction. When the boat moves the flag flutters along the direction of relative velocity of wind w.r.t. boat. Thus **v_w = OA** = wind velocity = 72km/h due NE direction.

. Boat velocity = **v_B = OB**= 51km/h due north

Realtive velocity of wind w.r.t. boat is given by **$v_{WB} = v_W - v_B = v_W + (-v_B)$ = OA + OC = OD**

Clearly the flag will flutter in the direction of **OD** on the mast of the moving boat.

Angle between **v_W** and **–v_B**, θ = 45°+90°=135°

If **v_{WB}** makes angle β with **v_w**, then

$\tan\beta = \frac{v_B \sin\theta}{v_W + v_B\cos\theta} = \frac{51\sin 135°}{72+51\cos 135°} = \frac{51\sin 45°}{72+51(-\cos 45°)} = \frac{51\times 1/\sqrt{2}}{72-51\times 1/\sqrt{2}} = \frac{51}{72\sqrt{2}-51} = 1.0037$

or β = 45.01°

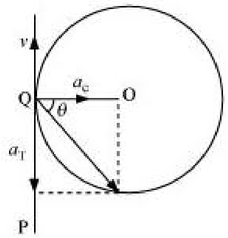
Angle w.r.t. east direction = 45.01° - 45° = 0.01°.

Hence the flag will flutter almost in east direction.

2. A cyclist is riding with a speed of 27 km/h. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m/s every second. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Sol. Here r = 80m, v = 27 kmh^{-1} = $\frac{\mathbf{27\times5}}{\mathbf{18}}ms^{-1}$ = 7.5ms^{-1}

Centripetal acceleration, a_c = $v^2/r = (7.5)^2/80 = 0.7\ ms^{-2}$



Suppose the cyclist applies the brakes at point A of the circular turn, then tangential acceleration a_T (negative) will act opposite to velocity.

Give $a_T = 0.5\ ms^{-2}$

As the acceleration a_C and a_T are perpendicular to each other so the net acceleration of the cyclist is a = $\sqrt{\mathbf{a_C^2+a_T^2}} = \sqrt{(\mathbf{0.7})^2+(\mathbf{0.5})^2} = \sqrt{\mathbf{0.49+0.25}} = \sqrt{\mathbf{0.74}}$ = 0.86 ms^{-2}

If θ is the angle between the total acceleration and the velocity of the cyclist, then

$\tan\theta = a_C/a_T = 0.7/0.5 = 1.4$ or θ = 54°28'.

3. (a) Show that for a projectile the angle between the velocity and the x-axis as a function of time is given by

$$\theta(t) = \tan^{-1}\left(\frac{v_{0y} - gt}{v_{0x}}\right)$$

(b) Shows that the projection angle θ_0 for a projectile launched from the origin is given by: $\theta_0 = \tan^{-1}\left(\frac{4h_m}{R}\right)$

Where the symbols have their usual meaning.

4. A vector has both magnitude and direction. Does that mean anything that has magnitude and direction is necessarily a vector? The rotation of a body can be specified by the direction of the axis of rotation and the angle of rotation about the axis. Does that make any rotation a vector?

Sol. No, anything that has both magnitude and direction is not necessarily a vector. It must obey the laws of vector addition.

Rotation is not generally considered a vector even though it has magnitude and direction because the addition of two finite rotations does not obey commutative law. However, infinitesimally small rotations obey commutative law and hence an infinitesimally small rotation is considered a vector.

SPACE FOR ROUGH WORK

SPACE FOR NOTES