CLASS – 12

WORKSHEET- MAGNETISM AND MATTER

A. BAR MAGNET & MAGNETIC FIELD LINES

(1 Mark Questions)

1. What is source of magnetic field?

Sol. The three sources of magnetic force are electromagnet, permanent magnet, current carrying conductor.

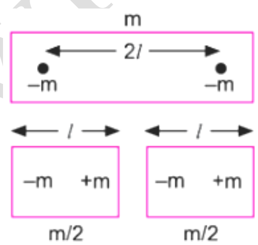
2. Write S.I. unit of (i) Pole strength and (ii) Magnetic dipole moment.

Ans. S.I. unit of (i) Pole strength is ampere-metre (A-m). (ii) SI unit of magnetic dipole moment is Ampere metre^2 (Am^2).

(2 Marks Questions)

3. How does the (i) pole strength and (ii) magnetic moment of each part of bar magnet change if it is cut into two equal pieces transverse to its length?

Sol. (i) Pole strength of each part remains same as that of the original magnet.
(ii) Magnetic moment of each part is half of that of the original magnet because length of each part is halved.



4. A magnetized needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why?

Sol. In the first case, the magnetic field is uniform, and forces acting on the two ends of the needle are equal and opposite. So the net force is zero. However, a torque acts on the needle.
In the case of iron nail, there is an induced magnetism. The induced (say) south pole in the nail, being closer to the north pole of the bar magnet , experiences a larger attractive force than the induced north pole. So the nail experiences both a net attractive force and a torque.

5. A short bar magnet placed with its axis at 30º with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnitude of magnetic moment of the magnet?

Sol. Here $\theta = 30°$, B = 0.25T, $\tau = 4.5\times10^{-2}$J, m = ?

As $\tau = mB \sin\theta$, $m = \frac{\tau}{B \sin\theta} = \frac{4.5\times10^{-2}}{0.25\times\sin30°} = 0.36\ JT^{-1}$

6. A short bar magnet of magnetic moment m = 0.32 J T^{-1} is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (a) stable, and (b) unstable equilibrium? What is the potential energy of the magnet in each case?

Sol. Here m = 0.32 JT^{-1}, B = 0.15T

(a) The bar will be in stable equilibrium when its magnetic moment $\vec{m}$ is parallel to $\vec{B}$ (θ = 0°). Its potential energy is then minimum and is given by

$U_{min} = -mB \cos 0° = -0.32 \times 0.15 \times 1 = -4.8 \times 10^{-2}$J.

(b) The bar magnet will be in unstable equilibrium when the magnetic moment $\vec{m}$ is antiparallel to $\vec{B}$ (θ = 180°). Its potential energy is then maximum and is given by

$U_{max} = -mB \cos 180° = -0.32 \times 0.15 \times (-1) = +4.8 \times 10^{-2}$J

7. A closely wound solenoid of 800 turns and area of cross section $2.5 \times 10^{-4}\ m^2$ carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Sol. Here N = 900, A = $2.5 \times 10^{-4} m^2$, I = 3.0A,

m = NIA = $800 \times 3 \times 2.5 \times 10^{-4} = 0.60\ JT^{-1}$.

The magnetic field of a solenoid has the same pattern as that of a bar magnet. It acts along the axis of the solenoid. Its direction is determined by the sense of flow of current.

8. If the solenoid in previous Question is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?

Sol. Here m = 0.60 JT^{-1}, B = 0.25T, θ = 30°

$\tau = mB \sin\theta = 0.60 \times 0.25 \times \sin 30° = 7.5 \times 10^{-2}$J

9. How does the (i) pole strength and (ii) magnetic moment of each part of a bar magnet change if it is cut into two equal pieces transverse to its length?

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10. A mangetised needle in a uniform magnetic field experiences a torque but no net force. An iron nail near a bar magnet, however, experiences a force of attraction in addition to a torque. Why?

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11. A short bar magnet placed with its axis at 30° with a uniform external magnetic field of 0.25T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnetic moment of the magnet? [Ans. 0.36 JT^{-1}]

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12. A short bar magnet of magnetic moment m = 0.32 JT^{-1} is placed in a uniform external magnetic field of 0.15T. If the bar is free to rotate in the plane of the field, which orientations would correspond to its (i) stable and (ii) unstable equilibrium? What is the potential energy of the magnet in each case? [Ans. – 4.8×10^{-2}J, +4.8×10^{-2}J]

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13. A closely wound solenoid of 800 turns and area of cross section 2.5×10^{-4} m^2 carries a current of 3.0A. Explain in what sense does the solenoid act like a bar magnet. What is its associated magnetic moment? [Ans. 0.60 JT^{-1}]

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14. If the solenoid in previous question is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25T is applied, what is the magnitude of the

torque on the solenoid when its axis makes an angle of 30° with the direction of the magnetic field? [Ans. 7.5×10^{-2} J]

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(3 Marks Questions)

15. State Gauss's law for magnetism. Explain its significance.

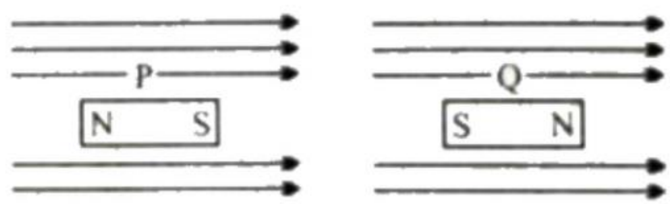
Sol. Gauss' law of magnetism: Gauss's law for magnetism states that the net magnetic flux through any closed surface is zero.

$\phi = \Sigma \vec{B}.\Delta\vec{S} = 0$

Physical significance: This law establishes that isolated magnetic pole do not exist.

16. Two identical bar magnets P and Q are placed in two identical uniform magnetic fields. Justify that both the magnets are in equilibrium. Which one of these is in stable equilibrium? Give reasons for your answer.

Sol.



Let q_m be the pole strength of each magnet.

In case of magnet P, Force on N pole, q_mB(towards right), Force on S pole = q_mB (towards left)

Therefore Net force on P = 0

The net force on P = 0

The dipole moment $\vec{m}$ of P points in the opposite direction of $\vec{B}$.

Therefore torque, τ = mB sin 180° = 0

PE of P = - mB cos 180° = + mB

In case of magnet Q, Force on N pole = q_mB (towards right), force on S pole = q_mB (towards left)

Therefore net force on Q = 0

The dipole moment $\vec{m}$ of Q point in the direction $\vec{B}$.

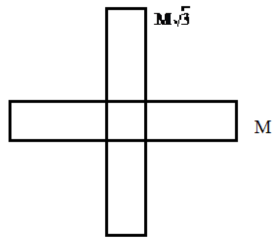
Therefore torque τ = mB sin 0° = 0

PE of Q = - mB cos 0° = 0

PE of Q = - mB cos 0° = - mB

As the net force and net torque on both of the magnets are zero, so both P and Q are in equilibrium. But Q is in stable equilibrium because its PE is minimum. If the magnet P (in unstable equilibrium) is disturbed, a torque acts on it to align it in the opposite direction, whereas the torque on magnet Q brings it back to the original alignment.

17. Two magnets of magnetic moments M and $M\sqrt{3}$ are joined to form a cross. The combination is suspended in a uniform magnetic field B. The magnetic moment M now makes and angle θ with the field direction. Find the value of angle θ.



Sol. When the magnet of moment M makes angle θ with the field B, the other magneti of moment M√3 will make angle (90° – θ) with the field B. In the uniform position,

Torque experienced by first magnet = Torque experienced by second magnet

Or mB sin θ = √3 mB sin (90° - θ)

Or sin θ = $\sqrt{3}$ cos θ

Or sinθ/cosθ = √3 or tan θ = √3

Therefore θ = 60°.

18. A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment: (i) normal to the field direction, (ii) opposite to the field direction?

(b) What is the torque on the magnet in cases (i) and (ii)?

Sol. Here m = 1.5 JT^{-1}, B = 0.22T

(a) Given θ_1 = 0°, θ = 90°

∴ W = - mB ($\cos\theta_2 - \cos\theta_1$) = - 1.5 × 0.22 (cos 90° - cos0°)

= - 0.33 × (0 – 1) = + 0.33J

Torque τ = mB sin 90° = 1.5 × 0.22 × 1 = 0.33Nm

(b) Given $_1$ = 0°, θ_2 = 180°

W = - 1.5 × 0.22 × cos (180° - cos 0°) = - 0.33 × (- 1 – 1) = 0.66J.

Torque τ = mB sin 180° = 1.5 × 0.22 × 0 = 0.

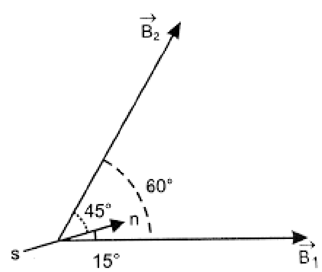
19. A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the

centre of the magnet on (a) the axis, (b) the equatorial lines (normal bisector) of the magnet. [Ans. 0.48G]

20. A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is 60º, and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15º with this field, what is the magnitude of the other field? [Ans. 4.4×10^{-3}T]

21. A bar magnet of magnetic moment 1.5 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.22T. (a) What is the amount of work required to turn the magnet so as to align its magnetic moment (i) normal to the field direction (ii) opposite to the field direction? (b) What is the torque on the magnet in case (i) and (ii)?

[Ans. (i) +0.33J, 0.33Nm (ii) 0.66J, 0]



22. A Rowland ring of mean radius 15cm has 3500 turns of wire wound on a ferromagnetic wire of relative permeability 800. What is the magnetic field (B) in the core for a magnetizing current of 1.2A?

Sol. A Rowland ring is a circular ring of a magnetic material over which is wound a toroidal solenoid. The magnitude of the magnetic field in the core is given by B = μnI where n = 3500/2πr, is the number of turns per unit length.

Now relative permeability, $\mu_r = \frac{\mu}{\mu_0}$ so that $\mu = \mu_0\mu$,

Hence, B = m_0m_r, $\frac{3500}{2\pi r}$. I = 4p × 10^{-7} × 800 × $\frac{3500}{2\pi\times15\times10^{-2}}$×1.2T = 4.48T.

(5 Marks Questions)

23. A closely wound solenoid of 2000 turns and area of cross section 1.6×10^{-4} m^2, carrying a current of 4.0A, is suspended through its centre allowing it to turn in a horizontal plane.
(a) What is the magnetic moment associated with the solenoid?
(b) What are the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2}T is set up at an angle of 30° with the axis of the solenoid?

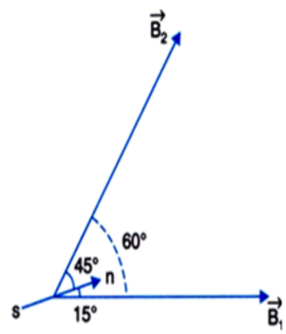
Sol. Here N = 2000, A = 1.6 × $10^{-4}m^2$. I = 4.0A
(a) Magnetic moment of solenoid of turns N, area of cross section A and carrying current I is, m = NIA = 2000 × 4.0 × 1.6 × $10^{-4}Am^2$ = 1.28 Am^2.
This magnetic moment acts along the axis of the solenoid in a direction related to the sense of current via the right hand screw rule.
(b) Net force experienced by the magnetic dipole in the uniform magnetic field = 0
The magnitude of the torque τ exerted by the magnetic field $\vec{B}$ on the solenoid is given by τ = mB sin θ = 1.28 × 7.5 × 10^{-2} × sin 30° = 0.048Nm.
The torque tends to align the axis of the solenoid (i.e., its magnetic moment vector $\vec{m}$) along the field $\vec{B}$.

24. A circular coil of 16 turns and radius 10cm carrying a current of 0.75A rests with its plane normal to an external field of magnitude 5.0×10^{-2}T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2.0 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?

Sol. Here N = 16, r = 10cm = 0.10m, I = 0.75A, B = 5.0 × 10^{-2}T, v = 2.0 s^{-1}
Magnetic moment of the coil is m = NIA = NI.πr^2
Frequency of oscillation, $v = \frac{1}{2\pi}\sqrt{\frac{mB}{I}}$
∴ Moment of inertia is $I = \frac{mB}{4\pi^2v^2} = \frac{NI\pi r^2.B}{\pi^2v^2} = \frac{16\times0.75\times(0.1)^2\times5\times10^{-2}}{4\times3.14\times4}$ = 1.2×10^{-4} kg m^2.

25. A magnetic dipole is under the influence of two magnetic fields. The angle between the field direction is 60° and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15° with this field, what is the magnitude of the other field?

Sol.



Here $B_1 = 1.2\times10^{-2}$T, $\theta_1 = 15°$, $\theta_2 = 60° - 15° = 45°$

In equilibrium, $\tau_1 = \tau_2$ or $mB_1\sin\theta_1 = mB_2\sin\theta_2$ or $B_2 = \frac{B_1\sin\theta_1}{\sin\theta_2}$

$= \frac{1.2\times10^{-2}\sin15°}{\sin45°} = \frac{1.2\times10^{-2}\times0.2588}{0.7071} = 4.4\times10^{-3}$T

B. EARTH'S MAGNETISM

(1 Mark Questions)

1. Where on the surface of Earth is the vertical component of Earth's magnetic field zero?

Sol. Vertical component of earth's magnetic field is zero at magnetic equator.

2. The material which is not suitable for making a permanent magnet is

(a) Steel (b) Ticonal (c) Lead (d) Alnico

Ans. (c)

3. If the horizontal and vertical components of the Earth's magnetic field are equal at a certain place, what would be the angle of dip at that place?

Sol. Given $B_V = B_H = 1$

$\text{Tan}\ \delta = \frac{B_V}{B_H} = \tan\delta = 1$

$\delta = 45°$.

4. Answer the following questions regarding earth's magnetism: (1 mark each)

(a) A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

Sol. The three independent quantities used to specify the earth's magnetic field are (i) magnetic declination, (ii) angle of dip, and (iii) horizontal component o earth's magnetic field.

(b) The angle of dip at a location in southern India is about 18º.Would you expect a greater or smaller dip angle in Britain?

Sol. Britain is closer to the magnetic north pole. So the angle of dip is greater in Britain than that in India. It is about 70° in Britain.

(c) If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

Sol. Magnetic lines of force of earth's magnetism will seem to come out of the ground at Melbourne in Australia because this region lies in the southern hemisphere of the earth where the earth's magnetic north pole lies.

(d) In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?

Sol. Earth's magnetic field is exactly vertical at the poles and so the horizontal component of earth's field is zero which makes the compass needle in any direction at the geometric north of south pole.

(e) The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8×10^{22} J T^{-1} located at its centre. Check the order of magnitude of this number in some way.

Sol. Magnetic field $\vec{B}$ at and equatorial point of the earth's magnetic dipole is given by

$B = \frac{\mu_0}{4\pi}.\frac{m}{r^3}$

Now $m = 8\times10^{22}$ JT^{-1}, $r = 6.4\times10^6$m

Therefore $B = 10^{-7} \times \frac{8\times10^{22}}{(6.4\times10^6)^3} T = 0.3\times10^{-4} T = 0.3G$ which is of the same order of magnitude as hat of the observed field on the earth.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

Sol. The earth's field is only approximately a dipole field. Local N-S poles may arise due to the different deposits of magnetized minerals.

5. How does the angle of dip vary from equator to poles?

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6. Answer the following: (1 mark each)

(a) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

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(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of earth's magnetism. Why?

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(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the battery (i.e., the source of energy) to sustain these currents?

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(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

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(e) The earth's field departs from the dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

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(f) Interstellar space has an extremely work magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence? Explain.

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7. Name the elements of parameters of earth's magnetic field.

Sol. The elements of earth's magnetic field are (i) Decliantion (ii) Dip (iii) Horizontal component of earth's magnetic field.

8. Define angle of dip (or magnetic inclination) at a place.

Sol. The angle made by the earth's total magnetic field with ht horizontal direction is called angle of dip (δ) or magnetic inclination at that place.

9. Define declination at a place.

Sol. The angle between the geographical meridian and the magnetic meridian at a place is called then magnetic declination (α) at that place.

10. Horizontal components of Earth's magnetic field at a place is √3 times the vertical component. What is the value of angle of dip at this place?

Sol. Her $\tan\delta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3}B_V} = \frac{1}{\sqrt{3}}$

∴ Angle of dip, δ = 30°

11. Where on the surface of the earth is the angle of dip (i) 0° and (ii) 90°?

Sol. (i) Angle of dip is 0° at magnetic equator. (ii) Angle of dip is 90° at magnetic poles of the earth.

(2 Marks Questions)

12. At a certain location in Africa, a compass points 12º west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60º above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Sol. Here B_H = 0.16G, δ = 60°

So, $B = \frac{B_H}{\cos\delta} = \frac{0.16}{\cos 60°} = \frac{0.16}{0.5}$ = 0.32G.

Thus the earth's magnetic field has a magnitude of 0.32G and lies in a vertical plane 12° west of the geographic meridian making an angle of 60° (upwards) with the horizontal (magnetic south to magnetic north) direction.

(3 Marks Questions)

13. A bar magnet of magnetic moment 6 JT^{-1} is aligned at 60° with a uniform external magnetic field of 0.44T. Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final, orientation in case (ii)

Sol. Here m = 6J T^{-1}, θ_1 = 60°, B = 0.44T

(a) Work done in turning the magnet, W = - mB(cos θ_2 =- cos θ_1)

(i) $\theta_1 = 60°$, $\theta_2 = 90°$. So, W = - 6×0.44(cos 90° - cos 60°) = -6 × 0.44 (0 – ½) = 1.32J
(ii)$\theta_1 = 60°$, $\theta_2 = 180°$. So W = - 6 × 0.44 (cos 180° - cos 60°) = -6 × 0.44(-1 – ½) = 3.96J
(b) $\tau = mB \sin\theta = mB \sin 180° = 0$

14. A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22º with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.

Sol. Here $\delta = 22°$, $B_H = 0.35G$, B = ?

$B = \frac{B_H}{\cos\delta} = \frac{0.35G}{\cos 22°} = \frac{0.35G}{0.9272} = 0.38G$

15. A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null−point (i.e., 14 cm) from the centre of the magnet? (At *null points*, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

Sol. As the null point lies on the axis of the magnet, therefore,

$B_{axial} = \frac{\mu_0}{4\pi}.\frac{2m}{r^3} = B_H$

Magnetic field of the magnet on its normal bisector at the same distance will be

$B_{equa} = \frac{\mu_0}{4\pi}.\frac{m}{r^3} = \frac{B_H}{2} = \frac{0.36}{2} = 0.18G$

Therefore total magnetic field at the required point on the normal bisector is $B_{equa} + B_H$ = 0.18+0.36 -= 0.54G.

16. If the bar magnet in previous question is turned around by 180º, where will the new null points be located?

Sol. When the magnet is turned around by 180°, its south pole will lie in the geographical south direction. The null points will now lie on the equatorial line of the magnet, ay at distance x from the centre of the magnet. Then

$B_{aqu} = \frac{\mu_0}{4\pi}.\frac{m}{x^3} = B_H$

But $B_H = \frac{\mu_0}{4\pi}.\frac{2m}{r^3}$

Therefore, $\frac{\mu_0}{4\pi}.\frac{m}{x^3} = \frac{\mu_0}{4\pi}.\frac{2m}{r^3}$ or $\frac{2}{r^3} = \frac{1}{x^3}$

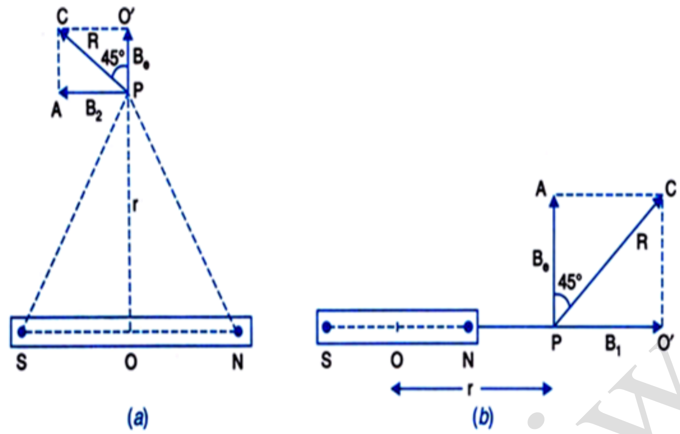
Or $x^3 = r^3/2$

Therefore $x = r/2^{1/3}$ = 14cm/1.26 = 11.1cm

17. A short bar magnet of magnetic moment 5.25×10^{-2} J T^{-1} is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45º with earth's field on (a) its normal bisector

and (b) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.

Sol.



Sol. Here m = $5.25\times10^{-2}JT^{-1}$, $B_0 = 0.42G = 0.42 \times 10^{-4}T$

(a) Figure (a) shows a point P on the normal bisector of a magnet where the resultant field is located at 45° with the earth's field B_0. As point P is an equatorial point, therefore, the resultant field must be such that $\frac{B_{equa}}{B_0} = \tan 45°$

Or $B_{aqua} = B \tan 45° = B_0$

But for short magnet, $B_{aqua} = \frac{\mu_0}{4\pi}.\frac{m}{r^3}$ $\therefore \frac{\mu_0}{4\pi}.\frac{m}{r^3} = B_0$

Or $r^3 = \frac{\mu_0}{4\pi}.\frac{m}{B_0} = 10^{-7}.\frac{5.25\times10^{-2}}{0.42\times10^{-4}} = 125\times10^{-6}$

Hence $r = 5 \times 10^{-2}m = 5cm$.

(b) Figure (b) shows a point Q on the axis of the magnet where the resultant field is inclined at 45° with the earth's field B_0. In this condition, $\frac{B_{axial}}{B_0} = \tan 45°$ or $B_{axial} = B_0$

But for a short magnet, $B_{axial} = \frac{\mu_0}{4\pi}.\frac{2m}{r^3}$

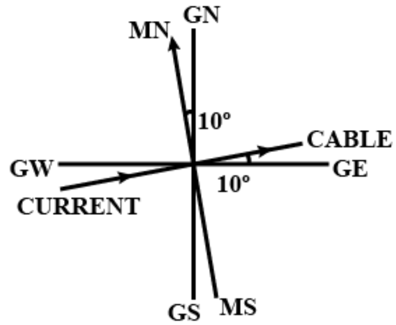
$\therefore \frac{\mu_0}{4\pi}.\frac{2m}{r^3} = B_0$

Or $r^3 = \frac{\mu_0}{4\pi}.\frac{2m}{B_0} = 125\times10^{-6} \times 2 = 250 \times 10^{-6}$

Here $r = (250)^{1/3} \times 10^{-2}$ m = 6.3×10^{-2}m = 6.3cm.

18. A long straight horizontal cable carries a current of 2.5 A in the direction 10º south of west to 10° north of east. The magnetic meridian of the place happens to be 10º west of the geographic meridian. The earth's magnetic field at the location is 0.33 G, and the angle of dip is zero. Locate the line of neutral points (ignore the thickness of the cable). (At *neutral points*, magnetic field due to a current-carrying cable is equal and opposite to the horizontal component of earth's magnetic field.) [Ans. 1.5 cm]

Sol.



Suppose the neutral point lies at a distance r from the cable. Then at the neutral point,
$\frac{\mu_0 I}{2\pi r} = B_H$ or $r = \frac{\mu_0 I}{2\pi B_H} = \frac{4\pi\times10^{-7}\times2.5}{2\pi\times0.33\times10^{-4}} = 1.5\times10^{-2}$m = 1.5cm
As the direction of the magnetic field of the cable is opposite to that of B_H at point above the cable, so the line of neutral points lies parallel to an above the cable at a distance of 1.5cm from it.

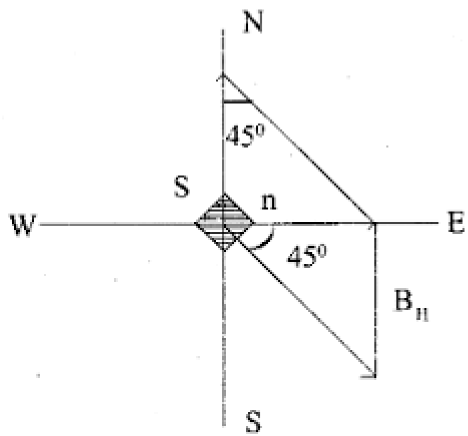
19. In the magnetic meridian of a certain place, the horizontal component of the earth's magnetic field is 0.26G and the dip angle is 60°. What is the magnetic field of earth in this location?

Sol. Here B_H = 0.26G, δ = 60°
As $B_H = B\cos\delta$
Therefore $B = \frac{B_H}{\cos\delta} = \frac{0.26G}{\cos 60°} = \frac{0.26G}{0.5}$ = 0.52G.

(5 Marks Questions)

20. A compass needle free to turn in a horizontal plane is placed at the centre of circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle of 45º with the magnetic meridian. When the current in the coil is 0.35 A, the needle points west to east.
(a) Determine the horizontal component of the earth's magnetic field at the location.
(b) The current in the coil is reversed, and the coil is rotated about its vertical axis by an angle of 90º in the anticlockwise sense looking from above. Predict the direction of the needle. Take the magnetic declination at the places to be zero.

Sol. (a) Magnetic field set up at the centre of the coil is $B = \frac{\mu_0 NI}{2r}$
The field acts along the axis perpendicular to the plane of the coil is in a vertical plane making an angle of 45° with the magnetic meridian and the needle point in the west east direction, it is obvious from the figure, that the needle is oriented at an angle of 45° with the field B. Using law of sines for a triangle , we get



$\frac{B_H}{\sin 45°} = \frac{B}{\sin 90°}$

Or B_H = B sin 45° = $\frac{\mu_0 NI}{2r}.\sin 45° = \frac{4\pi\times10^{-7}\times30\times0.35}{12\times10^{-2}}\times 0.7071T$ = 3.8876×10^{-5}T = 0.39G.

(b) The needle will reverse its original direction i.e. it will point east to west.

21. A telephone cable at a place has four long straight horizontal wires carrying a current of 1.0A in the same direction east to west. The earth's magnetic field at the place is 0.39G and the angle of dip is 35°. The magnetic declination is nearly zero. What are the resultant magnetic fields at points 4.0cm below, and above the cable?

Sol. Earth's field B = 0.39G, δ = 35°, B_H = B cos δ = 0.39 cos 35° = 0.319G, B_V = B sin δ = 0.39 sin 35° = 0.224G

For the cable, we have I = 1.0A, N – 4, r = 4.0cm = 4×10^{-2}m

Therefore magnetic field produced by the cable wires is

B' = $\frac{\mu_0 NI}{2\pi r} = \frac{4\pi\times10^{-7}\times4\times1.0}{2\pi\times4\times10^{-2}}$ = 0.2×10^{-4}T = 0.2G.

Resultant field below the cable: At points below the cable the field B' is in the opposite direction of B_H. So the horizontal component of the resultant field is

$R_H = B_H - B'$ = 0.319 – 0.2 = 0.119G

The vertical component of the earth's field remains unaffected, $R_V = B_V$ = 0.224G

Therefore magnitude of the resultant field is R = $\sqrt{R_H^2 + R_V^2} = \sqrt{(0.119)^2 + (0.224)^2}$ = 0.254G

The angle that R makes with the horizontal is δ = $\tan^{-1}\frac{R_V}{R_H} = \tan^{-1}\frac{0.224}{0.119} = \tan^{-1}18.8$ = 62°

Resultant field above the cable: In this case the field of the cable acts as in the direction of B_H.

Therefore $R_H = B_H + B'$ = 0.319 + 0.2 = 0.519G

R_V= 0.224G

R = $\sqrt{(0.519)^2 + (0.224)^2}$ = 0.566G

Angle o dip, δ = $\tan^{-1}\frac{0.224}{0.519}$ = $\tan^{-1}$ 0.4316 = 23°.

C. MAGNETIC MATERIAL

(1 Mark Questions)

1. The magnetic field lines are _______ by a diamagnetic substance.

Sol. Magnetic field lines are repelled by diamagnetic substances.

2. Answer the following questions: (1 mark each)

(a) Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?

Sol. The tendency to disrupt the alignment of dipoles (with the magnetizing field) arising from random thermal motion is reduced at lower temperatures.

(b) Why is diamagnetism, in contrast, almost independent of temperature?

Sol The induced dipole moment in a diamagnetic sample is always opposite t the magnetizing field, no matter what the internal motion of the atoms is.

(c) If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

Sol. As bismuth is diamagnetic so the field in the toroid with bismuth core will be slightly less than when the core is empty.

(d) Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?

Sol No, permeability of a ferromagnetic material is not independent of the magnetic field. This is evident from the B h curve which has greater slope (hence greater μ) at lower fields.

(e) Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point.) Why?

Sol. The proof of the important fact is basd on boundary conditions of magnetic fields ($\vec{B}$ and $\vec{H}$) at the interface of two media. When on eof the media has $\mu >> 1$, the field lines must this medium nearly normally.

(f) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetization of a ferromagnet?

Sol. Yes, apart from minor differences in the strength of the individual atomic dipoles of two different materials, a paramagnetic sample with saturated magnetization will have the same order of magnetization. But saturation requires impractically high magnetizing fields.

3. Answer the following questions: (1 mark each)

(a) Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.

Sol. In a ferromagnetic substance, the atomic dipoles are grouped together in domains. All the dipoles of a domain are aligned in the same direction and have net magnetic moment. In an unmagnetized substance these domains are randomly distributed so that the resultant magnetization is zero. When the substance is placed in an external magnetic field, these domains align themselves in the direction of the field. Some energy is spent in the process of alignment. When the external field is removed, these domains do not come back into their original random positions completely. The substance retains some magnetization. The energy spent in the process of magnetization is not fully recovered. The balance of energy is lost as heat. This the basic cause for irreversibility of the magnetization curve of a ferromagnetic substance.

(b) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?

Sol. Carbon steel piece, because heat lost per cycle is proportional to the area of the hysteresis loop.

(c) 'A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory?' Explain the meaning of this statement.

Sol. Magnetization of a ferromagnet is not a single valued function of the magnetizing field. Its value for a particular field depends both on the field and also on the history of magnetization i.e. how many cycles of magnetization it has gone through etc. in other words, the value of magnetization is a record or 'memory' of its cycles of magnetization. If information bits can be made to correspond to these cycles, the system displaying such a hysteresis loop can act as a device for storing information.

(d) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or for building 'memory stores' in a modern computer?

Sol. Ceramics (specially treated barium iron oxides) also called farrites.

(e) A certain region of space is to be shielded from magnetic fields. Suggest a method.

Sol. Surround the region by soft iron rings. Magnetic field lines will be drawn into the rings, and the enclosed space will be free of magnetic field, But this shielding is only approximate, unlike the perfect electric shielding fo a cavity in a conductor placed in an external electric field.

4. In a permanent magnet at room temperature

(a) magnetic moment of each molecule is zero.

(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned.

(c) domains are partially aligned. (d) domains are all perfectly aligned.

Sol. (c)

In a permanent magnet at room temperature domains are partially aligned.

5. A paramagnetic sample shows a net magnetisation of 8 Am^{-1} when placed in an external magnetic field of 0.6T at a temperature of 4K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16K, the magnetisation will be

(a) 32/3 Am^{-1} (b) 2/3 Am^{-1} (c) 6 Am^{-1} (d) 2.4 Am^{-1} .

Sol. (b)

6. A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?

____

____

7. If a toroid uses Bismuth at its core, will the field in the core be lesser or greater than when it is empty?

____

____

8. Define magnetic susceptibility?

Sol. Magnetic susceptibility of a material is defined as the ratio of th intensity of magnetization (M) induced in the substance to the magnetisating field intensity (H).

$\chi_m = \frac{M}{H}$

9. The susceptibility of a magnetic material is 1.9×10^{-5}. Name the type of magnetic materials it represents.

Sol. It represents paramagnetic material.

10. What does the area of hysteresis loop indicate?

Sol. The area of the hysteresis loop gives a measure of the energy wasted in a simple when it is taken through a cycle of magnetization.

(2 Marks Questions)

11. If χ stands for the magnetic susceptibility of a given material, identify the class of material for which (i) $-1 \le \chi < 0$ (ii) $0 < \chi < \varepsilon$ (ε stands for a small positive number)

Sol. (i) For $-1 \le \chi < 0$, material is diamagnetic.
(ii) For $0 < \chi < \varepsilon$, material is paramagnetic.

12. Write two properties of a material suitable for making (a) a permanent magnet, and (b) an electromagnet.

Sol. (a) For permanent magnet: (i) Material should have high retentivity and high coercivity. (ii) material should have high permeability.
(b) For electromagnet: (i) Material should have low retentivity. (ii) Material should have high permeability.

13. State and Explain Curie's law of magnetism for paramagnetic material.

Sol. From experiments it is found that the intensity of magnetization (M) of a paramagnetic material is (i) directly proportional to the magnetizing field intensity H, because the latter tens to align the atomic dipole moments. (ii) inversely proportional to absolute temperature T because the latter tends to oppose the alignment o the atomic dipole moments.

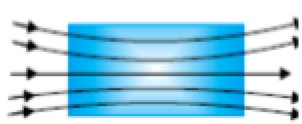
14. What is hysteresis loop? Explain with its help the terms related to it.

Sol. When a ferromagnetic sample is placed in a magnetic field the sample gets magnetized by induction. As the magnetizing field intensity H varies, the magnetic induction B doe not vary linearly with H, i.e. the permeability μ (=B/H) is not constant by varies with H. In fact, it also depends on the past history of the sample.
A study of hysteresis loop provides us information about retentivity, coercivity and hysteresis loss of magnetic material. This helps in proper selection of materials for designing cores of transformers and electromagnets and in making permanent magnets.

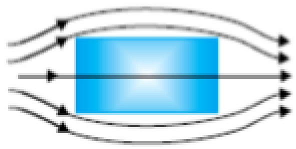
(3 Marks Questions)

15. Show diagrammatically the behavior of magnetic field lines in the presence of (i) paramagnetic, and (ii) diamagnetic substances. How does one explain this distinguishing feature.

Sol. (i) At $\chi = 0.9853$, so material is paramagnetic. The behaviour of magnetic field lines in the presence of paramagnetic substance is shown:

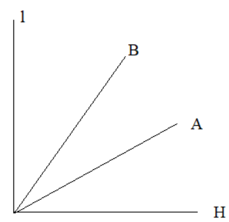


(ii) The behavior of magnetic field line in the presence of diamagnetic substance is shown.



The distinguish feature is because of the difference in their relative permeabilities. The relative permeability of the diamagnetic substance is negative, so the magnetic lines of force do not prefer passing through the substance. The relative permeability of a paramagnetic substance is greater than 1, so, the magnetic lines of force prefer passing through the substance.

16. The following figure show the variation of intensity of magnetization versus the applied magnetic field intensity, H, for two magnetic materials A and B.



(a) Identify the materials A and B

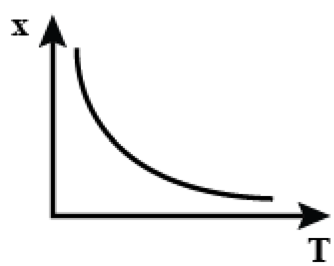
(b) For the materials A, plot the variation of intensity of magnetization versus temperature.

Sol. (a) Slope of I-H graph give susceptibility χ_m (= 1/H) of the material

For material A, the slope is +ve and smaller, ti is likely to be paramagnetic.

For material B, the slope is +ve and larger, it is likely to be ferromagnetic.

(b) The I-T graph for the paramagnetic material A is shown in figure.



17. A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field **B** in the core for a magnetising current of 1.2 A? [Ans. 4.48T]

____

____

____
____
____
____

18. What is relative permeability of a magnetic material? How is it related to the magnetic susceptibility?

Sol. Relative permeability: It is the ratio of the magnetic induction to the magnetizing field intensity $\mu = B/H$. Its SI unit is Tm A^{-1} or Wbm^{-1} A^{-1}.
Magnetic permeability: It is the ratio of the intensity of magnetization (M) induced to the magnetizing field intensity (H).
$\chi_m = \frac{M}{H}$. it can be shown that $\mu = \mu_0(1 + \chi_m)$ and $\mu_r = 1 + \chi_m$

19. What is hysteresis loop? Explain with its help the terms related to it.

Sol. Same as 14

(5 Marks Questions)

20. Define the terms (i) magnetic intensity (ii) magnetization (iii) magnetic induction (iv) magnetic permeability (v) relative permeability and (vi) magnetic susceptibility. Give their SI units, if any. Derive a relation between relative permeability and susceptibility.

Sol. (i) Magnetic field intensity: The ability of magnetizing foiled to magnetise a material medium is expressed by vector $\vec{H}$ called magnetizing field intensity or magnetic intensity. Its magnitude may be defined as the number of ampere-turns (nI) flowing round the unit length of th solenoid required to produce the given magnetizing field . Thus, H = nI
Therefore, $B_0 = m_0nI = m_0H$ or $H = B_0/m_0$. The dimensions of magnetic intensity are $[L^{-1}A]$. Its SI unit is ampere $metre^{-1}(AM^{-1})$ which is equivalent to $Nm^{-2}T^{-1}$ or Jm^{-1} Wb^{-1}.
(ii) Intensity of magnetization: When a magnetic material is placed in a magnetizing field, it gets magnetized. The magnetic moment developed per unit volume of a material when placed in a magnetizing field is called intensity of magnetization or simply magnetization. Thus $\vec{M} = \frac{\vec{m}}{V}$
(iii) Magnetic induction: The total magnetic field inside a magnetic material is the sum of the external magnetizing field and the additional magnetic field produced due to magnetization of material and is called magnetic induction $\vec{B}$. he magnetic induction may also be defined as the total number of magnetic lines of force crossing per unit area normally through a material. Thus the SI unit of magnetic induction is tesla (T) or weber $metre^{-2}$ (Wbm^{-2}) which is equivalent to $Nm^{-1}A^{-1}$ or $JA^{-1}m^{-2}$.

(iv) Magnetic permeability: Permeability is the measure of the extent to which a material can be penetrated or permeated by a magnetic field. The magnetic permeability of a material nay be defined as the ratio of its magnetic induction B to the magnetic intensity H.

$\mu = B/H$

Clearly SI unit of μ = tesla/ampere metre^{-1} = tesla metre ampere^{-1} or TmA^{-1}.

So, dimensions of $\mu = [MLT^{-2}A^{-2}]$.

(v) Relative permeability: Permeability of various magnetic substances can be compared with one another in terms for relative permeability μ_r. It Is defined as the ratio of permeability of the medium to the permeability of free space, Thus $\mu_r = \mu/\mu_0$.

For vacuum, $\mu_r = 1$, for air it is 1.0000004 and for iron, the value of μ_r may exceed 1000.

(vi) Magnetic susceptibility: Magnetic susceptibility measures the ability of a substance to take up magnetization when placed in a magnetic field. It is defined as the ratio of the intensity of magnetization M to the magnetizing field intensity H. It is denoted by χ_m. Thus $\chi_m = M/H$.

As magnetic susceptibility is the ratio of two quantities having the same units (AM^{-1}) so it has no units.

21. Distinguish the magnetic properties of a dia-, para- and ferromagnetic substances in terms of (i) magnetic permeability and (ii) coerctivity. Give one example of each of these materials.

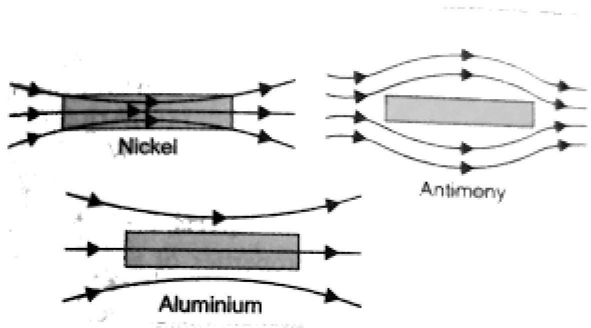
Draw the field lines due to an external magnetic field near a (i) diamagnetic (ii) paramagnetic substance.

Sol.

	Property	Diamagnetic substances	Paramagnetic substances	Ferromagnetic substances
1	Effects of magnets	They are feebly repelled by magnets	They are feebly attracted by magnets	They are strongly attracted by magnets
2	In external magnetic field	Acquire feeble magnetization in the opposite direction of the magnetizing field	Acquire feeble magnetization in the direction of the magnetizing field	Acquire strong magnetization in the direction of the magnetizing field
3	In a non-uniform magnetic field	Tend to move slowly from stronger to weaker parts of the field	Tend to move slowly from weaker to stronger parts of the field	Tend to move quickly from weaker to stronger parts of the field
4	In a uniform magnetic	A freely suspended	A freely	A freely

	field	diamagnetic rod aligns itself perpendicular to the field.	suspended paramagnetic rod aligns itself parallel to the field	suspended ferro-magnetic rod align itself parallel to the field

(b)



D. **ASSERTION REASON TYPE QUESTIONS**

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true.

1. Assertion: Magnetic moment of an atom is due to both, the orbital motion and spin motion of every electron.
Reason: A charged particle produces a magnetic field.

Ans. (c) Assertion is true but reason is false.
In an atom, electrons revolve around the nucleus and as such the circular orbits of electrons may be considered as the small current loops. In addition to orbital motion, an electron has got spin motion also. So the total magnetic moment of electron in the vector sum of its magnetic moments due to orbital and spin motion.

2. Assertion: Soft iron is used as transformer core.
Reason: Soft iron has narrow hysteresis loop.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion
The core of a transformer undergoes cycles of magnetization again and again. During each cycle of magnetization, energy numerically equal to the area of the hysteresis loop is spent per unit volume of the core. Therefore, for high efficiency of transformer, the energy loss will be lesser. If the hysteresis loop is of lesser area, i.e. narrow. That's why the soft iron is used as core, which has narrow hysteresis loop (or area of B-H curve is very small). Also soft iron (ferromagnetic substance) has high permeability, high retentivity, low coercivity and low hysteresis loss.

3. Assertion: A soft iron core is used in a moving coil galvanometer to increase the strength of magnetic field.

Reason: From soft iron more number of the magnetic lines of force passes.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion
A sot iron core increase the strength of magnetic field by forcing large number of magnetic lines through the soft iron core, which in turn increases the sensitivity of the galvanometer.

4. Assertion: A paramagnetic sample display greater magnetism (for the same magnetizing field) when cooled.
Reason: The magnetization doe s not depend on temperature.

Ans. (c) Assertion is true but reason is false.
A paramagnetic sample display greater magnetization when cooled, this is because at lower temperature, the tendency to disrupt the alignment of dipoles (due to magnetizing field) decreases on account of reduced random thermal motion.

5. Assertion: When a magnetic dipole is placed in a non uniform magnetic field, only a torque acts on the dipole.
Reason: Force would also acts on dipole if magnetic field is uniform.

Ans. (d) Both Assertion and Reason are false.
In a non uniform magnetic field, both torque and a net force acts on the dipole. If magnetic field is uniform, net force on dipole would be zero.

E. CHALLENGING PROBLEMS

1. Answer the following questions:
(a) The earth's magnetic field varies from point to point in space.
Does it also change with time? If so, on what time scale does it change appreciably?

Sol. Yes, it does change with time. Time scale for appreciable charge is roughly a few hundred years. But even on a much smaller scale of a few years, its variations are not completely negligible.

(b) The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

Sol. The temperature of earth's core is very high, so iron exists as molten iron which, being at a temperature higher than Curie point is not ferromagnetic.

(c) The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

Sol. Radioactivity may be one of the possible sources for a charged current in the outer conducting regions of the earth's core which are thought to be responsible for earth's magnetism.

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

Sol. Earth's magnetic field gets recorded weakly in certain rocks during their solidification. An analysis of these rocks may reveal the history of earth's magnetism.

(e) The earth's field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

Sol. At large distances, the field gets modified due to the field of ions in motion (in the earth's ionosphere). The field of these ions, in turn, is sensitive to extraterrestrial disturbances such as the solar wind.

(f) Interstellar space has an extremely weak magnetic field of the order of 10^{-12} T. Can such a weak field be of any significant consequence? Explain.

Sol. When a charged particle moves in a magnetic field, it gets deflected along a circular path of radius,

R = mv/eB [since evB = mv^2/R]

A weak field of 10^{-12} T bends the charged particle in a circle of very large radius. Over a small distance, we may not notice the deflection but over very large interstellar distance, the deflection is quite noticeable.

2. A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5×10^{-23} J T^{-1}. The sample is placed under a homogeneous magnetic field of 0.64 T, and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K? (Assume Curie's law) [Ans. 7.9 JT^{-1}]

Sol. Dipole moment of each atomic dipole, m = 1.5×10^{-23} JT^{-1}

Total number of atomic dipoles, N – 2.0×10^{24}

Initial total magnetic moment at temperature T_1 = 4.2 K is

M_1 = 15% of mN = $\frac{15}{100} \times 1.5 \times 10^{-23}$ = $2.0\times10^{24}JT^{-1}$ = $4.5JT^{-1}$

According to Curie's law, M = constant × B/T

$\therefore \frac{M_2}{M_1} = \frac{B_2}{B_1} \times \frac{T_1}{T_2}$

Now B_1 = 0.84T, T_1 = 4.2K, B_2 = 0.98T, T_2 = 2.8K

Hence the final dipole moment at temperature T_2 = 2.8K is

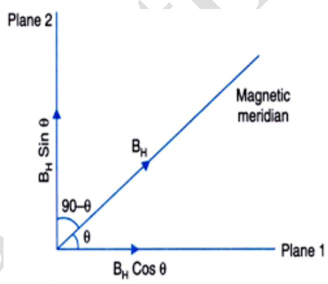
$M_2 = M_1 \times \frac{B_2}{B_1} \times \frac{T_1}{T_2} = 4.5 \times \frac{0.98}{0.84} \times \frac{4.2}{2.8}$ JT^{-1} = 7.9 JT^{-1}

3. (i) A small compass needle of magnetic moment M is free to turn about an axis perpendicular to the direction of uniform magnetic field B. The moment of inertia of the needle about the axis is I. The needle is slightly disturbed from its stable position and then released. Prove that it executes simple harmonic motion. Hence, deduce, the expression for its time period, (ii) A compass needle free to turn in a vertical plane orients itself with its axis vertical at a certain place on the earth. Find out the values of (a) horizontal component of the earth's magnetic field and (b) angle of dip at the place.

4. If δ_1 and δ_2 be the angles of dip observed in two vertical planes at right angles to each other and δ is the true angle of dip, prove that: $\cot^2 \delta_1 + \cot^2 \delta_2 = \cot^2 \delta$.

Sol. Let B_H and B_V be the horizontal and vertical components of earth's magnetic field $\vec{E}$, Sin δ is the true angle of dip, therefore

Tan $\delta = \frac{B_V}{B_H}$ or $\cot \delta = \frac{B_H}{B_V}$ …(i)



As shown in figure, suppose planes 1 and 2 are two mutually perpendicular planes and respectively make angles θ and $90° - \theta$ with the magnetic meridian. The vertical components of earth's magnetic field remain same in the two planes but the effective horizontal components in the two planes will be

$$B_1 = B_H \cos \theta \text{ and } B_2 = B_H \sin \theta$$

The angles of dip δ_1 and δ_2 in the two planes are given by

Tan $\delta_1 = \frac{B_V}{B_1} = \frac{B_V}{B_H \cos\theta}$

Or $\cot \delta_1 = \frac{B_H \cos\theta}{B_V}$ …(2)

Tan $\delta_2 = \frac{B_V}{B_2} = \frac{B_V}{B_H \sin\theta}$

Or $\cot \delta_2 = \frac{B_H \sin\theta}{B_V}$ ..(3)

From equations (2) and (3) we have

$\cot^2\delta_1 + \cot^2\delta_2 = \frac{B_H^2}{B_V^2}(\cos^2\theta + \sin^2\theta) = \frac{B_H^2}{B_V^2}$

or $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$. (using equation (1))

SPACE FOR ROUGH WORK

SPACE FOR NOTES