CLASS – 12

WORKSHEET- ELECTROMAGNETIC INDUCTION

A. MAGNETIC FLUX

(1 Mark Questions)

1. When is the flux linked with a closed coil held in a magnetic field zero?

Sol. The magnetic flux will be zero when the plane of the coil is along the magnetic field.

2. Name the SI units of (i) magnetic flux and (ii) magnetic induction (or magnetic flux density).

Sol. (i) Weber (Wb) (ii) Tesla (T).

3. Write the dimensional formula of magnetic flux.

Sol. $[M^1L^2T^{-2}I^{-1}]$.

4. A square of side L meters lies in the x-y plane in a region, where the magnetic field is given by $B = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})T$ where B_0 is constant. The magnitude of flux passing through the square is

(a) $2\ B_0\ L^2$ Wb. (b) $3\ B_0\ L^2$ Wb. (c) $4\ B_0\ L^2$ Wb. (d) $\sqrt{29}\ B_0\ L^2$ Wb

Sol. (c)

As we know that the magnetic flux linked with uniform surface area A in uniform magnetic field is ϕ = B.A. The direction of A is perpendicular to the plane of square and square line in x-y plane in a region.

$A = l^2k$

As given that, $B = B_0\ (2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$

So, $\phi = B.A. = B_0(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}).L^2\hat{k} = 4B_0L^2Wb$

5. A loop, made of straight edges has six corners at A(0,0,0), B(L,O,0) C(L,L,0), D(0,L,0) E(0,L,L) and F(0,0,L). A magnetic field $B = B_0(\hat{i} + \hat{k})T$ is present in the region. The flux passing through the loop ABCDEFA (in that order) is

(a) $B_0\ L^2$ Wb. (b) $2\ B_0\ L^2$ Wb (c) $\sqrt{2}\ B_0\ L^2$ Wb. (d) $4\ B_0\ L^2$ Wb.

Sol. (b)

(2 Marks Questions)

6. A rectangular loop of area 20cm×30cm is placed in a magnetic field of 0.3T with its plane (i) normal to the field (ii) inclined 30° to the field and (iii) parallel to the field. Find the flux linked with the coil in each case.

Sol. Her $A = 20cm \times 30cm = 6 \times 10^{-2}\ m^2$, B = 0.3T.

Let θ be the angle made by the field B with the normal to the plane of the coil.

(i) Here q = 90° - 90° = 0°

Therefore, $\phi = BA \cos\theta = 0.3 \times 6 \times 10^{-2} \times \cos 0° = 1.8 \times 10^{-2}$ Wb

(ii) Here q = 90° - 30° = 60°

Therefore, $\phi = 0.3 \times 6 \times 10^{-2} \times \cos 60° = 0.9 \times 10^{-2}$ Wb

(iii) Here q = 90°

Therefore $\phi = 0.3 \times 6 \times 10^{-2} \times \cos 90°$ = zero.

7. Find the magnetic flux linked with a rectangular coil of size 6cm×8cm placed at right angle to a magnetic field of 0.5 Wbm^{-2}.

Sol. $\phi = BA \cos 0° = 0.5 \times 0.06 \times 0.08 \times 1 = 2.4 \times 10^{-3}$ Wb

(3 Marks Questions)

8. A square coil of 600 turns each of side 20cm, is placed with its plane inclined at 30° to a uniform magnetic field of 4.5×10^{-4} Wbm^{-2}. Find the flux through the coil.

Sol. $\phi = NBA \cos\theta = 600 \times 4.5 \times 10^{-4} \times 0.20 \times 0.20 \times \cos(90° - 30°) = 5.4 \times 10^{-3}$ Wb

B. FARADAY'S LAW AND LENZ'S RULE

(1 Mark Questions)

1. On what factors does the magnitude of the emf induced in the circuit due to magnetic flux depend?

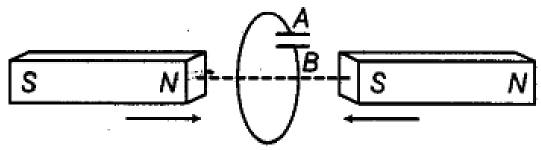
Sol. The magnitude of the emf induced in the circuit due to magnetic flux depends on the rate of change of magnetic flux with time through the circuit.

$|\varepsilon| = \frac{\Delta\phi}{\Delta t}$

2 A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop? Justify.

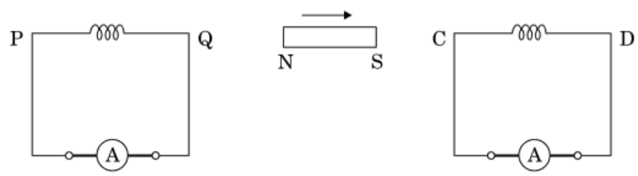
Sol. The magnetic flux of force due to current are parallel to the plane of the loop. So angle between magnetic field and area vector is 90°. Hence, the flux linked with the loop is zero. Hence there will be no induced emf in the loop.

3. Predict the polarity of the capacitor in the situation described below.



Sol. Polarity of plate A will be positive with respect to plate B in the capacitor, as induced current is in clockwise direction.

4. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the direction of the induced current in each coil.



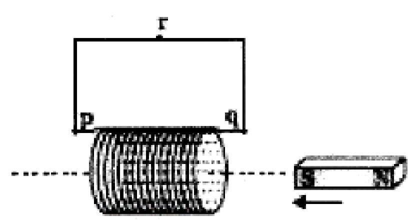
Sol. According to Lenz's law, the direction of current in loop PQ is from P to Q and in loop CD is from C to D.

5. State Lenz's law.

Sol. This law gives us the direction of induced emf. According to this law, the direction of induced emf in a circuit is such that it opposes the change in magnetic flux responsible for its production. Lenz's law is in accordance with the principle of conservative of energy.

6. Predict the direction of induced current in the situations described by the following figures: (1 mark each)

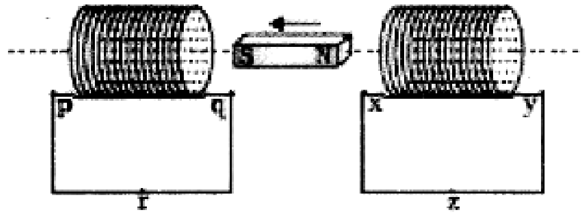
(i)



(a)

Sol As the S-pole of the magnet approaches the coil be Lenz's law, the induced current in the coil develops S-pole at the end q. For this the induced current should flow clockwise when seen from the magnetic side. Hence the induced current flows along qrpq.

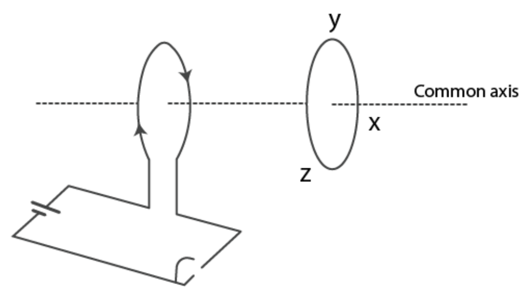
(ii)



(b)

Sol. As the magnet is moved, its S-pole moves towards pq coil and its N pole moves away from xy coil. By Lenz's law the induced current should develop S pole at the end q and also a S pole at the end x. For this the current in the two coils should flow clockwise when seen from the magnet side. Hence the current flows along prq in the one coil and along yzx in the other coil.

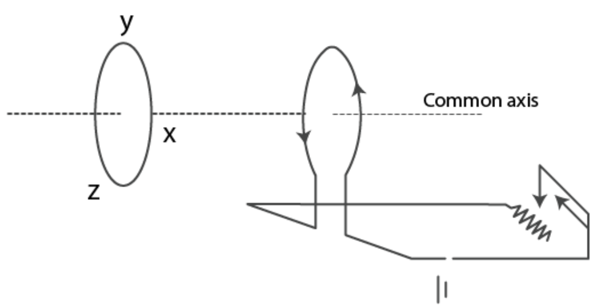
(iii)



Tapping key just closed

Sol. As the circuit of the left loop is completed, current flows in the direction of arrows shown. The current develops south polarity on the side of right loop. The induced current should flow clockwise (when seen from left) in the right loop so as to oppose the growth of current in left loop. Hence the induced current flows along yzx.

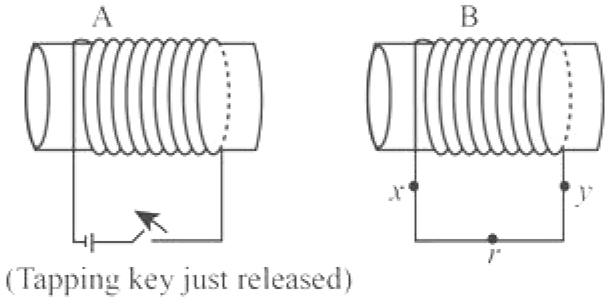
(iv)



Rheostat setting being charged

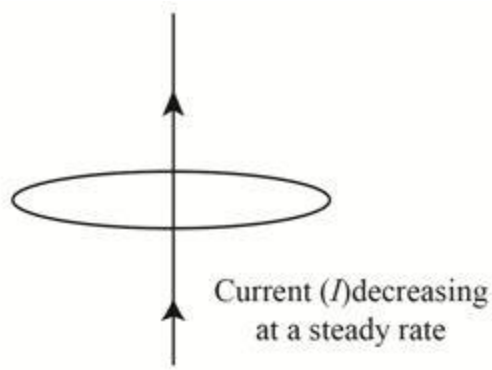
Sol. A the rheostat is adjusted the resistance decreases and current increases. This increase the flux should be opposed. The induced current should produce flux in the opposite direction of original flux. Hence the induced circuit flows along zyx.

(v)



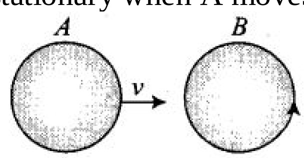
Sol. As the circuit breaks, the flux decreases. The induced current should flow along xry to supplement the flux.

(vi)



Sol. The circular field lines set up around the current carrying wire lie in the plane of the loop. The flux threading the coil in the perpendicular direction is zero. Any change in current will not change this flux. Hence no induced current is set up in the coil.

7. There are two coils A and B as shown in Figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counterclockwise. B is kept stationary when A moves. We can infer that

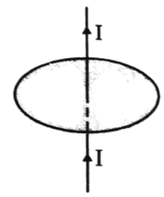


(a) there is a constant current in the clockwise direction in A.
(b) there is a varying current in A. (c) there is no current in A.
(d) there is a constant current in the counterclockwise direction in A.

Sol. (d)

(2 Marks Questions)

8. The current I in a wire passing normally though the centre of conducting loop is increasing at a constant rate. Will any current be induced in the loop?



Sol. The magnetic lines of force due to the current I are parallel to the plane of the loop. The flux linked with the loop is zero. Hence no current is induced in the loop.

9. Two identical loops, one of copper and another of aluminium are rotated with the same speed in the same magnetic field. In which case (i) the induced emf (ii) the induced current, will be more and why?

Sol. (i) Induced emf wil be same in both the loops because ti does not depend on the nature of the material of the loops.

(ii) Induced current will be more in copper loop because its resistance is less than that of aluminium loop.

10. State Faraday's laws of electromagnetic induction. Express them mathematically.

Sol. These can be stated as follows:

First law: Whenever the magnetic flux linked with a closed circuit changes, an emf (an hence a current) is induced in it which lasts only so long as the change in flux is taking place. This phenomenon is called electromagnetic induction.

Second law: The magnitude of the induced emf is equal to the rate of change in magnetic flux linked with the closed circuit. Mathematically, $|\varepsilon| = \frac{d\phi}{dt}$

Mathematical form of the laws of electromagnetic induction: Expression for induced emf

According to the Faraday's flux rule,

Magnitude of induced emf = Rate of change of magnetic flux

Or $|\varepsilon| = \frac{d\phi}{dt}$

Taking into account Lenz's rule for the dirction of induced emf. Faraday's law takes the form, $\varepsilon = -\frac{d\phi}{dt}$

The negative sign indicates that the direction of induced emf is such that it opposes the change in magnetic flux.

If the coil consists of N tightly wound turns then the emfs developed in all these turns will be equal and in the same direction and hence get added up. Total induced emf will be

$\varepsilon = -N\frac{d\phi}{dt}$

If the flux changes from ϕ_1 to ϕ_2 in time t, then the average induced emf will be

$\varepsilon = -N\frac{\phi_2 - \phi_1}{t}$

If ϕ is webers and t is seconds, ε will be in volts.

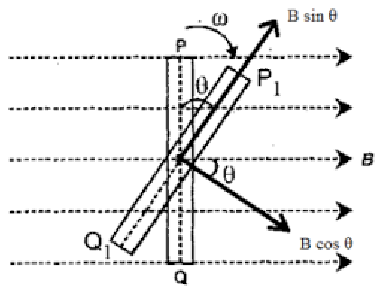
11. State Lenz's law. On which law of conservation it is based?

Sol. Lenz's law states that the direction of induced current in a circuit is such tht it opposes the cause or the change which produces it.

Lenz's law is based on the law of conservation of energy. From the definition of Lenz's law, we know that the induced current is always opposed by the cause that produces it. Therefore, there is extra work done against the opposing force.

12. A rectangular coil of N turns, area A is held in a uniform magnetic field B. If the coil is rotated at a steady angular speed ω, deduce an expression for the induced emf in the coil at any instant of time.

Sol. Consider the coil PQRS free to rotate in a uniform magnetic field $\vec{B}$. The axis of the rotation of the coil is perpendicular to field $\vec{B}$. The flux through the coil, when its normal makes an angle θ with the field, is given by $\phi = BA\cos\theta$ where A is the face area of the coil.



If the coil rotates with an angular velocity ω and turns through an angle θ in the time t, then $\theta = \omega t$, so $\phi = BA\cos\omega t$

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by $\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA\cos\omega t) = BA\,\omega\sin\omega t$

If the coil has N turns, then the total induced emf will be

$\varepsilon = NBA\,\omega\sin\omega t$

Thus the induced emf varies sinusoidally with time t. The value of induced emf is maximum when $\sin\omega t = 1$ or $\omega t = 90°$ i.e., when the plane of the coil is parallel to the field $\vec{B}$. Devoting this maximum value by ε_0 we have $\varepsilon_0 = NBA\omega$

Therefore $\varepsilon = \varepsilon_0 \sin\omega t = \varepsilon_0$ win 2π ft where f is the frequency of rotation of the coil.

(3 Marks Questions)

13. A small piece of metal wire is dragged across the gap between the pole of a magnet is 0.5s. The magnetic flux between the pole pieces is known to be 8×10^{-4}Wb. Estimate the emf induced in the wire.

Sol. Here dt = 0.5s, $d\phi = 8 \times 10^{-4} - 0 - 8$

The emf induced in the wire, $|\varepsilon| = \frac{d\phi}{dt} = \frac{8\times10^{-4}}{0.5} = 1.6\times10^{-3}$V

14. The magnetic flux through a coil perpendicular to the plane is varying according to the relation: $\phi = (5t^3 + 4t^2 + 2t - 5)$Wb

Calculate the induced current through the coil at t = 2s, if the resistance of the coil is 5Ω.

Sol. The magnitude of induced emf set up at any instant t will be

$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}(5t^3 + 4t^2 + 2t - 5) = 15t^2 + 8t + 2$

At t = 2s, $|\varepsilon| = 15(2)^2 + 8(2) + 2 = 60 + 16 + 2 = 78V$

Resistance of the coil, R = 5Ω

Induced current, $I = \frac{|\varepsilon|}{R} = \frac{78}{5} = 15.6A$

15. A 50 turn coil of area $500cm^2$ is rotating at a rate of 50 rounds per second perpendicular to a magnetic field of 0.5 Wbm^{-2}. Calculate the maximum value of induced emf.

Sol. $\varepsilon_0 = NBA \times 2\pi f = 50 \times 0.5 \times 0.5 \times 2 \times 3.14 \times 50 = 392.5$ V

16. A long solenoid with 15 turns per cm has a small loop of area 20 cm^2 placed inside, normal to the axis of the solenoid. If the current carried by the solenoid changes steadily from 2A to 4A in 0.1s, what is the induced voltage in the loop while the current is changing.

Sol. Here n = 15 turns/cm = 1500 turns/m, A = $20cm^2 = 2 \times 10^{-4}m^2$

$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A.\frac{dB}{dt} = A.\frac{d}{dt}(\mu_0 NI) = \mu_0 NA\frac{dI}{dt}$

$= 4\pi \times 10^{-7} \times 1500 \times 2 \times 10^{-4} \times \left(\frac{4-2}{0.2}\right) = 7.5 \times 10^{-6}V.$

17. We have an air-cored solenoid having a length of 30 cm, whose area is $\mathbf{25}\boldsymbol{cm^2}$ and a number of turns are 500. And the solenoid has carried a current of 2.5 A. Suddenly the current is turned off and the time is taken for it is $\mathbf{10^{-3}}$s. What would be the average value of the induced back-emf by the ends of the open switch in the circuit? (Neglect the variation in the magnetic fields near the ends of the solenoid.) [Ans. 6.54V]

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(5 Marks Questions)

18. We have a powerful loudspeaker magnet and have to measure the magnitude of the field between the poles of the speaker. And a small search coil is placed normal to the field direction and then quickly removed out of the field region, the coil is of $\mathbf{2}\boldsymbol{cm^2}$ area and has 25 closely wound turns. Similarly, we can give the coil a quick **90∘** turn to bring its plane parallel to the field direction. We have measured the total charge flown in the coil by using a ballistic galvanometer and it comes to 7.5 mC. Total resistance after combining the coil and the galvanometer is **0.50Ω**. Estimate the field strength of the magnet. [Ans. 0.75 Wb m^{-2}]

C. MOTIONAL EMF

(1 Mark Questions)

1. A horizontal conducting rod 10m long extending from east to west is falling with a speed 5.0 ms^{-1} at right angles to the horizontal component of the Earth's magnetic field,0.3×10^{-4} Wb m^{-2}. Find the instantaneous value of the emf induced in the rod.

Sol. $\varepsilon = Blv \sin \theta$

Given $B = 0.3\times10^{-4}$ Wb m^{-2}, $v = 5$ ms^{-1}, $l = 10m$, $\theta = 90°$,

ε = Blvsinq = $0.3\times10^{-4} \times 10 \times 5 \times \sin90° = 1.5 \times 10^{-4}V = 1.5 \times 10^{-3}V = 1.5$ mV

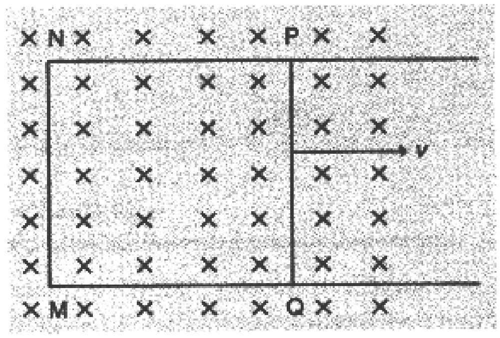
2 State the factors on which the induced emf in a coil rotating in a uniform magnetic field depends.

Sol. Induced emf, $\varepsilon = NBA\omega \sin \omega t$

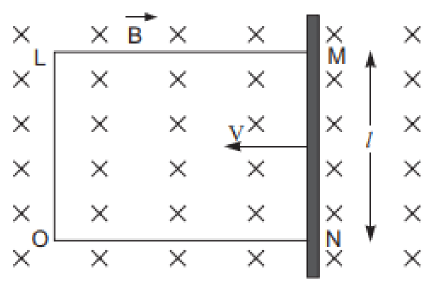
Clearly induced emf set up in the coil depends on (i) number of turns of the coil (ii) area of the coil (iii) angular speed of rotation of the coil and (iv) strength of the magnetic field.

(2 Marks Questions)

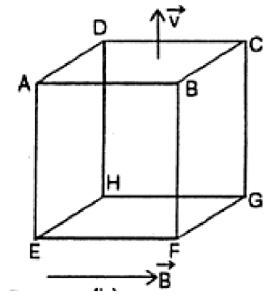
3. A rectangular loop PQMN with movable arm PQ of length 10cm and resistance 2Ω is placed in a uniform magnetic field of 0.1T acting perpendicular to the plane of the loop as shown in the figure. The resistances of the arms MN, NP and MQ are negligible. Calculate the (i) emf induced in the arm PQ and (ii) current induced in the loop when arm PQ is moved with velocity 20m/s.



4. A rectangular conductor LMNO is placed in a uniform field of 0.5T. The field is directed perpendicular to the plane of the conductor. When the arm MN of length of 20cm is moved towards left with a velocity of 10 ms^{-2}, calculate the emf induced in the arm. Given the resistance of the arm to be 5Ω (assuming that other arms are of negligible resistance) find the value of the current in the arm.



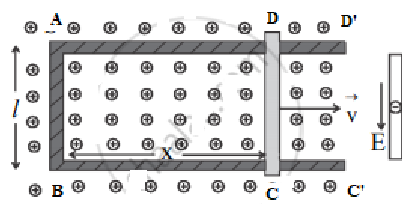
5. Twelve wires of equal lengths are connected in the form of a skeleton-cube which is moving with a velocity $\vec{v}$ in the direction of a magnetic field $\vec{B}$. Find the emf in each arm of the cube.



Sol. As the direction of velocity $\vec{v}$ of any arm of the cube is parallel to the direction of field $\vec{B}$, so no magnetic Lorentz force is exerted on the free electrons of any arm. Hence no emf is induced in any arm.

6. Prove that the magnitude of the emf induced in a conductor of length *l* when it moves at v m/s perpendicular to a uniform magnetic field B is B*l*v.

Sol. Consider a conductor CD of length l moving with a velocity v towards right on U shaped conducting rails situated in a magnetic field B as shown in the figure. The field is uniform and points into the plane of the paper. As the conductor slides, the area of the circuit changes from ABCD to ABC'D' in time dt.



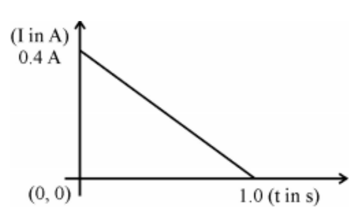
The increase in flux, dϕ = B × change in area = B × area CDD'C' = B.I.vdt

This set ups induced emf in the loop of magnitude, $|\varepsilon| = \frac{d\phi}{dt} = Blv$

According to Fleming's right hand rule, the induced current flows in the anticlockwise direction.

(3 Marks Questions)

7. When a conducting loop of resistance 10Ω and area $10cm^2$ is removed from an external magnetic field acting normally, the variation of induced current in the loop with time is shown in the figure.



Find the (i) total charge passed through the loop (ii) change in magnetic flux through the loop (iii) magnitude of the magnetic field applied.

Sol. (i) We know that I = $\frac{dq}{dt} \Rightarrow dq = Idt$

$\therefore$ q = $\int$ Idt = Area under the I-t curve = ½ × 0.4 × 1 = 0.2C.

(ii) As we know $|\varepsilon| = \frac{d\phi}{dt} \Rightarrow \frac{dq}{dt} \times R = \frac{d\phi}{dt}$

$\Rightarrow \phi = qR = 0.2 \times 10 = 2Wb$.

8. A 0.4m long straight conductor is moved in a magnetic field of induction 0.9 Wbm^{-2} with velocity of 7m/s. Calculate the maximum emf induced in the conductor.

Sol. Maximum induced emf, $\varepsilon = Blv = 0.9 \times 0.4 \times 7 = 2.52V$.

9. A rectangular wire loop of sides 8cm and 2cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is $1cms^{-1}$ in a direction normal to the (i) longer side (ii) shorter side of the loop? For how long does the induced voltage last in each case?

Sol. Here B = 0.3T, v = $1cms^{-1}$ = $0.01ms^{-1}$, l = 8cm = 0.08m, b = 2cm = 0.02m

(i) When the loop moves normal to the longer side:

Induced emf, $\varepsilon = Nlv = 0.3 \times 0.08 \times 0.01 = 2.4 \times 10^{-4}V$

Therefore time for which emf lasts = Time in which shorter side moves out of the field

$= \frac{b}{v} = \frac{0.02}{0.01} = 2s$

(ii) When the loop moves normal to the shorter side,

Induced emf, $\varepsilon = Rbv = 0.3 \times 0.02 \times 0.01 = 0.6 \times 10^{-4}V$

Therefore, time for which emf last = Time in which the longer side moves out of the field

$= \frac{l}{v} = \frac{0.08}{0.01} = 8s$

10. A jet plane is travelling towards the west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°.

Sol. $v = 1800\ kmh^{-1} = \frac{900 \times 1000}{3600}\ ms^{-1} = 500ms^{-1}$

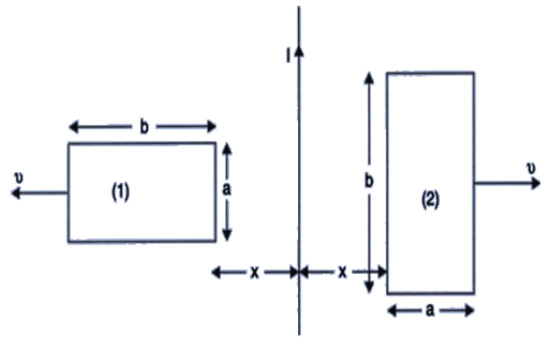
$l = 25m$, $B_H = 5.0 \times 10^{-4}T$, $\delta = 30°$,

$B_V = B \sin\delta = B_H \tan\delta = 5.0 \times 10^{-4} \times \sin 30° = 2.5 \times 10^{-4}$ T

Only the flux lines of vertical component of field B are cutting across the horizontally moving the plane. Therefore induced emf, $\varepsilon = B_V l_v = 2.5 \times 10^{-4} \times 25 \times 500 = 3.1V$.

(5 Marks Questions)

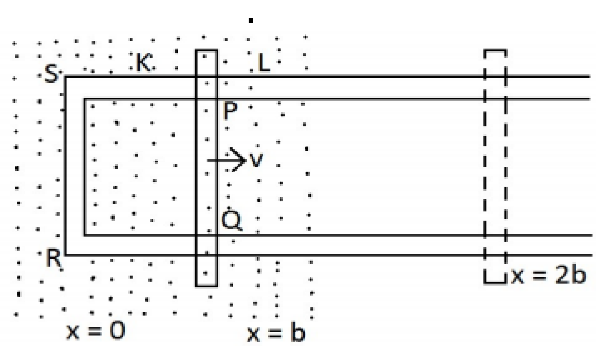
11. Figure shows two identical rectangular loops (1) and (2) placed on a table along with a straight lone current carrying conductor between them. (i) What will be the directions of the induced currents in the loops when they are pulled away from the conductor with same velocity v? (ii) Will the emfs induced in the two loops be equal. Justify your answer.



Sol. (i) By Lenz's law, the direction of induced current wil be such that it tends to maintain the original flux. So, induced current flows anticlockwise in loop (1) and clockwise in loop (2).

(ii) No the emfs induced in the two coils will not be equal because the rats of change of flux linked with the two loops are different.

12. Refer to the figure. The arm PQ of the rectangular conductor is moved from x = 0 to the right side. The uniform magnetic field perpendicular to the plane and extends from x = 0 to x = b and is zero for x>b. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from x = 0 to x = 2b and is then moved back to x = 0 with constant speed v. Obtain expressions for the flux, the induced emf, the force necessary to pull the arm and the power dissipated as joule heat. Sketch the variation of these quantities with time.



13. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50rads^{-1} in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 **Ω**, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Sol. Here r = 8cm = 0.08m, N = 20, $\omega = 50s^{-1}$, $B = 3.0\times10^{-2}T$

At any instant, emf induced in the coil is given by $\varepsilon = NBA\omega \sin\omega t$

Therefore maximum induced emf in the coil is

$\varepsilon_0 = NBA\omega = NB\pi r^2\omega = 20 \times 3.0 \times 10^{-2} \times 3.14 \times (0.08)^2 \times 50V = 0.603V$

Since the average value of sin ωt over a cycle is zero, therefore $\varepsilon_{av} = 0$

Maximum induced current, $I_0 = \varepsilon_0/R$

Therefore power dissipated as heat $= \frac{1}{2}\varepsilon_0 I_0 = \frac{1}{2}\frac{\varepsilon_0^2}{R} = \frac{1}{2} \times \frac{(0.603)^2}{10}W = 0.018W.$

The source of this power is the external agent which keeps he coil rotating.

14. A horizontal straight wire 10 m long extending from east to west is falling with a speed of 5.0 m s^{-1}, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb m^{-2}.

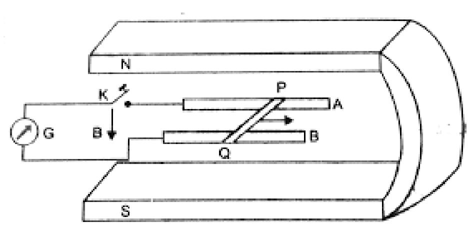
(a) What is the instantaneous value of the emf induced in the wire?
(b) What is the direction of the emf?
(c) Which end of the wire is at the higher electrical potential?

Sol. (a) Here l = 10m, v = 5.0ms^{-1}, $B_H = 0.30 \times 10^{-4}$ Wbm^{-2}

$\varepsilon = B_H lv = 0.30 \times 10^{-4} \times 10 \times 5.0 = 1.5 \times 10^{-3}V.$

(b) According to Fleming's right hand rule, the direction of emf is from west to east.
(c) Western end of the wire is at the higher electric potential.

15. In the given figure we have a metal rod PQ which is put on the smooth rails AB and these are kept in between the two poles of permanent magnets. All these three (rod, rails and the magnetic field) are in mutually perpendicular direction. There is a galvanometer 'G' connected through the rails by using a switch 'K'. Given, Rod's length = 15 cm, Magnetic field strength, B = 0.50 T, Resistance produced by the closed-loop 9.0$m\Omega$. Let's consider the field is uniform.



(i) Determine the polarity and the magnitude of the induced emf if we will keep the K open and the rod will be moved with the speed of 12 cm/s in the direction shown in the figure.

(ii) When the K was open is there any excess charge built up? Assume that K is closed then what will happen after it?

(iii) When the rod was moving uniformly and the K was open, then on the electron in the rod PQ there was no net force even though they did not experience any magnetic field because of the motion of the rod. Explain.

(iv) After closing the K, calculate the retarding force.

(v) When the K will be closed calculate the total external power which will be required to keep moving the rod with the same speed (12 cm/s)? and also calculate the power required when K will be closed.

(vi)What would be the power loss (in form of heat) when the circuit is closed? What would be the source of this power?

(vii) Calculate the emf induced in the moving rod if the direction of the magnetic field is changed from perpendicular to parallel to the rails?

[Ans. (i) 9.0 mV, (iv) 75×10^{-3}V, (v) 9.0×10^{-3}W, (vi) 9.0×10^{-3}W, (vii) 0]

D. SELF INDUCTANCE

(1 Mark Questions)

1. Define the term 'self-inductance' of a coil. Write its SI unit.

Sol. Self inductance: When the current in a coil is changed, a back emf is induced in the same coil. This phenomenon is called self inductance. If L is self inductance of the coil, then

$N\phi \propto I$

$N\phi = LI$

$L = N\phi/I$

The unit of self inductance in Henry (H).

2 Define self-inductance. Give its SI units.

Sol. Same as 1.

3. The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as
(a) *l* and A increase. (b) *l* decreases and A increases.
(c) *l* increases and A decreases. (d) both *l* and A decrease.

Sol. (b)

(2 Marks Questions)

4. A 12V battery connected to a 6Ω, 10H coil through a switch drives a constant current through the circuit. The switch is suddenly opened, if it takes 1ms to open the switch, find the average emf induced across the coil.

Sol. Steady state current = $\frac{12V}{6\Omega}$ = 2A

Final current = 0

$\therefore \frac{dl}{dt} = \frac{(0-2)A}{1ms}$ = - 2 × 10^{-3} As^{-1}

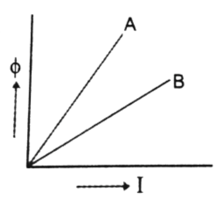
Induced emf, ε = $-L\frac{dt}{dt}$ = - 10 × (- 2 × 10^3) = 20000 V

Such a high emf usually causes sparks the open switch.

5. A coil of inductance 0.5H is connected to a 18V battery. Calculate the rate of growth of current.

Sol. $\frac{dl}{dt} = \frac{\varepsilon}{L} = \frac{18}{0.5}$ = 36As^{-1}.

6. A plot of magnetic flux (ϕ) versus current (I) is shown in figure for inductors A and B. Which of the tow has greater value of self-inductance?



Sol. Self inductance L = $\frac{\phi}{I}$ = Slope of ϕ-I graph

Slope of ϕ-I graph for A > slope of ϕ-I graph for B. Therefore inductor A has larger value of self inductance.

7. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

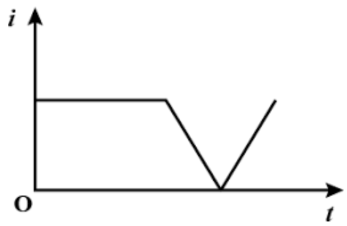
Sol. Here I_1 = 50A, I_2 = 0.0A, t = 0.1s, ε = 100V

As ε = $L\frac{dI}{dt} = L\frac{I_2-I_1}{t}$

Therefore 200 = $-L\frac{0.0-5.0}{0.1}$ = +50L

Or L = 4H.

8. The current i in an induction coil varies with time t according to the graph Draw the graph of induced e.m.f. with time.



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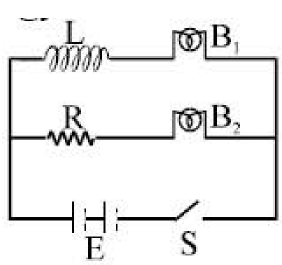
(3 Marks Questions)

9. The self inductance of an inductor coil having 100 turns is 20mH. Calculate the magnetic flux through the cross-section of the coil corresponding to a current of 4mA. Also, find the total flux. [Ans. 8×10^{-7} Wb, 8×10^{-5} Wb]

10. Figure shows an inductor L and a resistor R connected in parallel to a battery through a switch. The resistance of R is same as that of the coil that makes L. Two identical bulbs are put in each arm of the circuit.



(i) Which of the bulbs lights up bright when S is closed?
(ii) Will the two bulbs be equally bright after some time? Give reason for your answer.

Sol. (i) The bulb B_2 in R arm lights up earlier than the bulb B_1 in L arm. This is because when S is closed, the induced current set up in L opposes the growth of current in L arm.

(ii) After steady state is attained, the self inductance plays no role and the two bulbs will be equally bright.

11. Derive the expression for the self inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length.

Sol. Consider a long solenoid of length l and radius r with r << l and having n turns per unit length. If a current I flows through the coil, then the magnetic field inside the coil is almost constant and is given by, $B = \mu_0 NI$

Magnetic flux linked with each turn = BA = $\mu_0 nIA$ wher $A = \pi r^2$ = the cross sectional area of the solenoid.

Therefore magnetic flux linked with the entire solenoid is $L = \mu_0 n^2 IA$

If N is the total number of turns in the solenoid, then n = N/l and so $L = \frac{\mu_0 N^2 A}{l}$

If the coil is wound over a material of high relative magnetic permeability μ_r (eg soft iron) then

$L = \mu_r \mu_0 n^2 IA = \frac{\mu_r \mu_0 N^2 A}{l}$

12. Derive the expression of energy across an Inductor.

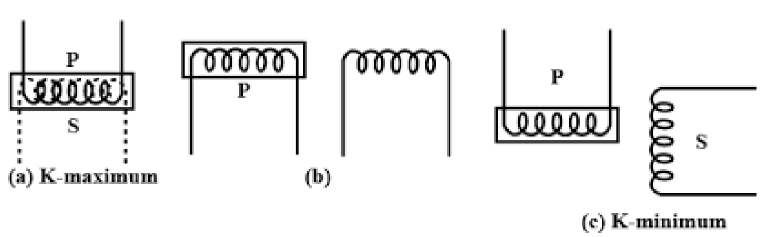
E. MUTUAL INDUCTANCE

(1 Mark Questions)

1. Define mutual inductance.

Sol. The phenomena of inducing current in a circuit by changing the current or a flux in a neighbouring circuit is called mutual induction. Si unit of mutual inductance is Henry denoted by H.

2. In which of the following cases will the mutual inductance be (i) minimum (ii) maximum?



Sol. Minimum in (b) and maximum in (c).

(2 Marks Questions)

3. A large circular coil, of radius R and a small circular coil, of radius r, are put in vicinity of each other. If the coefficient of mutual induction, for this pair, equals 1mH, what would be the flux linked with the larger coil when a current of 0.5A flows through the smaller coil?

Sol. Her M = 1mH, I = 0.5A, flux ϕ = MI = 10^{-3}H × 0.5A = 5 × 10^{-4} Wb.

4. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Sol. Here M = 1.5JH, I_1 = 0A, I_2 = 20A, t = 0.5s

As $\varepsilon = -\frac{d\phi}{dt} = -L\frac{dl}{dt}$

Therefore, dϕ = Ldl = 1.5 × (20 – 0) = 30Wb.

5. There are two coils A and B seperated by some distance. If a current of 2 A flows through A, a magnetic flux of 10-2 Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A?

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(3 Marks Questions)

6. A pair of adjacent coils has a mutual inductance of 1.5H. If the current in one coil changes from 0 to 20A in 0.5s, what is the change of flux linkage with the other coil?

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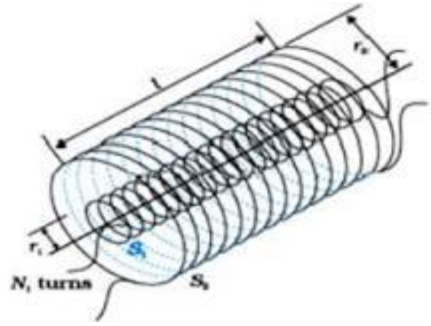
7. Define the term 'mutual inductance' between the two coils. Obtain the expression for mutual inductance of a pair of long coaxial solenoids each of length l and radii r_1 and r_2 ($r_2 >> r_1$). Total number of turns in two solenoids are N_1 and N_2 respectively.

Sol. The phenomena of inducing current in a circuit by changing the current or a flux in a neighbouring circuit is called mutual induction. Si unit of mutual inductance is Henry denoted by H.

Consider two long coaxial solenoids each of length l, Let n_1 be the number of turns per unit length of inner solenoid S_1 of radius r_1, n_2 be the number of turns per unit length of the outer solenoid S_2 of radius r_2. Let N_1 and N_2 be the total number of turns of solenoids S_1 and S_2 respectively.



When a current $I_2$is passed through S_1 the magnetic flux linked with solenoid $S_1$is
$N_1\phi_1 = M_{12}I_2$ ..(i)
where M_{12} is called mutual inductance of solenoid $S_1$with respect to solenoid S_2.
It is also referred as the coefficient of mutual induction.
The magnetic field due to current I_2 in S_2 is $B_2 = \mu_0 n_2 I_2$ …(ii)
So, the magnetic flux linked with S_2 is $N_1\phi_1 = B_2(\pi r_1^2)n_2 I = \mu_0 n_1 n_2 \pi r_1^2 l l_2$ …(iii)
Where $n_1 l$ is the total number of turns in solenoid S_1
From (i) and (iii) we get
$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 L$ …(iv) which is required expression.
Similarly $M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$ …(v)
From (iv) & (v) we get
$M_{12} = M_{21} = M$
Hence coefficient of mutual induction between two coaxial solenoids is
$M = \mu_0 n_1 n_2 p r_1^2 l$ or $M = \mu_0 N_1 N_2 \pi r_1^2/l$.

F. EDDY CURRENT

(1 Mark Questions)

1. Give one example of use of eddy current.

Sol. An eddy current is used for magnetic braking in trains. Strong electromagnets are situated in the train, just above the rails. When the electromagnets are activated, the eddy currents induced in the rails opposes the motion of the train.

2. A metallic piece gets hot when surrounded by a coil carrying high frequency alternating current. Why?

Sol. It becomes hot due to eddy current produced in it.

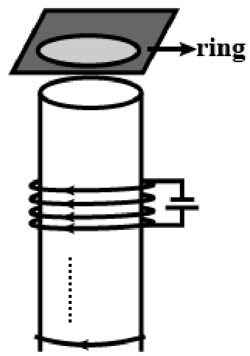
3. How are eddy currents reduced in a metallic core?

Sol. Eddy currents are reduced by using multiple plates instead of using a large plate.

4. Why does a metallic piece become very hot when it is surrounded by a coil carrying high frequency alternating current?

Sol. High frequency alternating current produces a rapidly changing magnetic field which induces large eddy currents in the metallic piece which is turn gets heated up.

5. Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I (see Figure). The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring?



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6. Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

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(2 Marks Questions)

7. What are eddy currents? Write any two applications of eddy currents.

Sol. When magnetic flux linked with a metallic conductor in the form of solid mass changes, then induced currents are set up in the conductor in the form of closed loops known as 'eddy currents'. Two applications of eddy currents are: (i) Electromagnetic braking in trains (ii) Induction furnace.

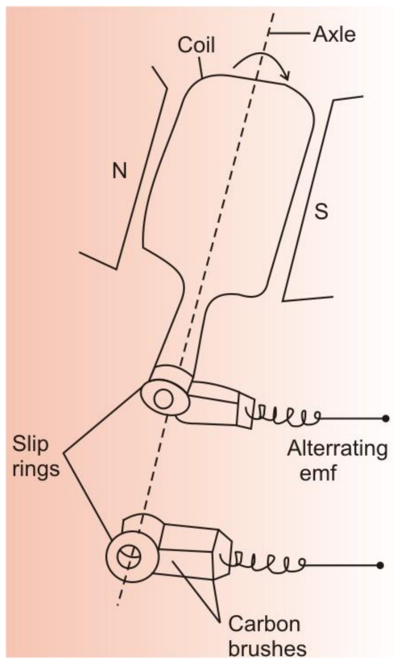
G. AC GENERATOR

(5 Marks Questions)

1. (a) Draw a schematic diagram for an ac generator. Explain its working and obtain the expression for the instantaneous value of the emf in terms of the magnetic field B, number of turns N of the coil of the area A rotating with angular frequency ω. Show how an alternating emf is generated by a loop of wire rotating in a magnetic field.

(b) A circular coil of radius 10cm and 20 turns is rotated about its vertical diameter with angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of 3.0×10^{-2}T.
(i) Calculate the maximum and average emf induced in the coil.
(ii) If the coil forms a closed loop of resistance 10Ω, calculate the maximum current in the coil and the average power loss due to joule heating.

Sol. (a)



Principle: AC generator is based on the principle of electromagnetic induction. It converts mechanical energy into electrical energy.
IT consists of (i) Armature coil of large number of turns of copper wire wound over soft iron core. Soft iron core is used to increase magnetic flux. (ii) Field magnets used to apply magnetic field, in which armature coil is rotated with its axis perpendicular to field lines. (iii) Slip rings used to provide movable contact of armature coil with external circuit containing load. (iv) Brushes which are metallic pieces used pass on electric current from the armature coil to the external circuit containing load.
When armature is rotated in the magnetic field due to change in orientation of the coil magnetic flux through it changes. Due to change in flux an emf is induced.
$\varepsilon = - N\frac{d\phi}{dt}$
$\varepsilon = NBA\omega \sin \omega t$ [since $\phi = BA \cos \omega t$]
$i = \frac{\varepsilon}{R} = \frac{NBA\omega}{R}\sin\omega t$
(b) Radius of coil, r = 10cm = 0.1m, Area, $A = \pi r^2 = 3.14 \times (0.1)^2 = 0.0314$ m^2, Number of turns, N = 20, Angular speed, $\omega = 50$ $rads^{-1}$, Magnetic field = 3.0×10^{-2}T.
(i) Maximum induced emf $\varepsilon_{max} = NAB\omega = 20\times0.0314\times3.0\times10^{-2}\times50 = 0.942V$
Average induced emf = $\frac{\varepsilon_{max}+\varepsilon_{min}}{2} = \frac{0.942+0}{2} = 0.471V$
(ii) Given R – 10Ω
Maximum current in the coil, $I_0 = \frac{\varepsilon_0}{R} = \frac{0.942}{10} = 0.0942A$

Average current, $I = \frac{\varepsilon}{R} = \frac{0.471}{10} = 0.0471A$
Now average power loss, $P = I^2R = (0.0471)^2 \times 10 = 0.022W$.

H. CASE STUDY

1. **Bottle Dynamo**: A d\bottle dynamo is a small generator to generate electricity to power the bicycle light.

It is not a dynamo. Dynamo generates dc but a bottle dynamo generates ac. Newer models are now available with a rectifier,. The available dc can power the light and small electronic gadgets. This is also known as sidewall generator since it operates using a roller placed on the sidewall of bicycle tyre. When the bicycle is in motion, the dynamo roller is engaged and electricity is generated as the tyre spins the roller. When engaged, a dynamo requires the bicycle rider to exert more effort to maintain a given speed than would otherwise be necessary when the dynamo is not present or disengaged.

Bottle dynamos can be completely disengaged during day time when cycle light is not in use. In wet conditions, the roller on a bottle dynamo can slip against the surface of the tyre, which interrupts the electricity generated. This cause the lights to go out intermittently.

(i) Why is bottle dynamo is not a dynamo?

(a) It generates ac only
(b) TI generates dc only
(c) It looks like a bottle
(d) It requires no fule to operate

Sol. (a)

Dynamo generates dc but bottle dynamo generates ac. SO, it is not a dynamo in that sense. Bu it generates electricity for bicycle light.

(ii) Can you recharge the battery of your mobile phone with the help of bottle dynamo?

(a) Yes
(b) No
(c) Yes, when a rectifier is sued
(d) Yes, when a transformer is used

Sol. (c)

Newer models of bottle generators are available with a rectifier dc available from such bottle generator can be used directly for charging mobile phone. Otherwise with the old models, a rectifier to be attached to convert ac to dc.

(iii) Bottle generator generates electricity:

(a) when fuel is poured in the bottle.
(b) when cycle is in motion

(c) when it is mounted properly (d) when wind blows.

Sol. (b)

Bottle generator is also knon as sidewall generator since it operates using a roller placed on the sidewall of bicycle tyre. When the bicycle is in motion, the dynamo roller is engaged and electricity is generated as the tyre spins the roller.

(iv) Bulb of bicycle light glows

(a) with ac supply wire (b) with d c supply only

(c) with both ac and dc supply (d) only when ac supply is rectified

Sol. (c)

Normal lamps work with both ac and dc. So, generator of older model or newer model can be directly used for bicycle lamp.

(v) Which one of the following is not an advantages of newer model of bottle dynamo?

(a) Works intermittently when it roller slips on tyre

(b) Small electronic gadgets can be charged

(c) Can be easily disengaged during day time

(d) Required no fuel

Sol. (a)

In wet conditions, the roller on a bottle dynamo (old model or new model) can slip against the surface of the tyre, which interrupts the electricity generated. This cause the lights to go out intermittently. This is not an advantage.

2. **The Alternating Current Generator:** The ac generator which is one of the most important applications of th phenomenon of electromagnetic induction converts mechanical energy into electrical energy. A rectangular coil consisting of a large number of turns of copper wire would over a soft iron core is rotated between the pole pieces of a permanent strong magnet. The magnetic flux through the coil changes continuously with time, thus producing induced emf given by $E = E_0 \sin \omega t$.

When a load resistor R is connected across the terminals, a current I flows through the circuit.

$I = \frac{E}{R} = \frac{E_0}{E} \sin\omega t = I_0 in\ \omega t$

Where $I_0 = E_0/R$. Such a current is called ac or alternating current.

(i) In an ac generator, a coil of area A hand having N turns rotates in a magnetic field B. The magnetic flux through the coil is

(a) maximum equal to NAB when the plane of the coil is perpendicular to the magnetic field.

(b) zero when the plane of the coil is parallel to the field.

(b) ½ NAB when the plane of the coil makes an angle of 60° with the field

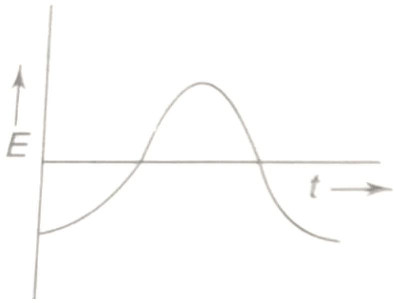
(d) ¼ NAB when the plane of the coil makes and angle of 30° with the field

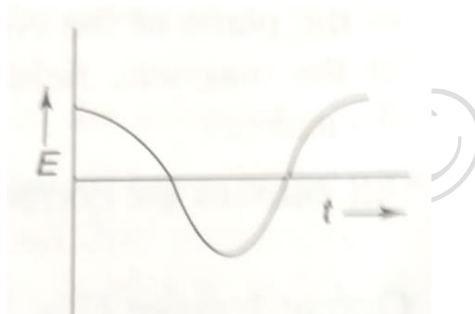
Sol. (a, b, c)

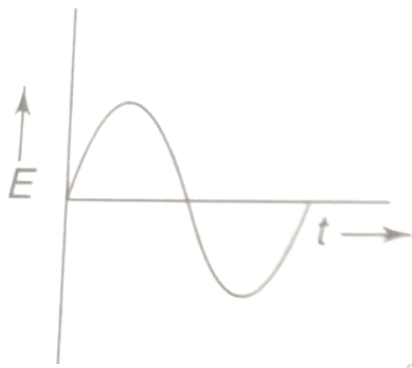
The correct choices are (a, b, c). The magnetic flux is given by $\phi = NAB \cos \theta$

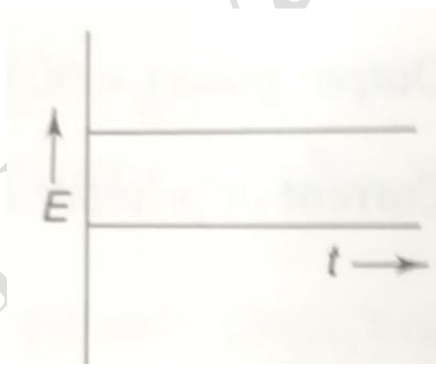
Where θ is the angle which is normal to the plane of the coil subtends with the magnetic field.

(ii) In an ac generator, initially (i.e. at t = 0) the plane of the coil is normal to the magnetic field. Which graph shown in figure represents the variation of induced emf E with time t?

(a)


(b)


(c)


(d)


Sol. (c)

When θ = 0 i.e. when the plane of the coil is perpendicular to the magnetic field, the flux is maximum but induced emf E = 0. Since θ = ωt, E = 0 at t = 0. Hence the correct graph is (c).

(iii) In an ac generator, the peak value of the induced emf depends upon the

(a) frequency of rotation of the coil (b) area of the coil

(c) number of turns in the coil (d) strength of the magnetic field

Sol. all are correct

All four choices are correct.

(iv) In an ac generator

(a) the coil is wound over a sort iron core in order to increase the flux

(b) the electromagnetic field with an alternating current is used.

(c) the output is always taken across a load resistor.

(d) the mechanical energy of the rotating coil is converted into electrical energy.

Sol. (a, c, d)

The correct choices are (a, c, d). A permanent magnet is used.

(v) The emf of an ac generator is given by $E = 100 \sin(100\pi t + \pi/3)$ wher E is in volts and t in seconds.

(a) The peak value of the emf is $100\sqrt{2}$ volts

(b) The frequency of rotation of the armature is 50Hz.

(c) At start (i.e t = 0) the plane of the armature makes an angle of 60° with the magnetic field.

(d) At start, the plane of the coil is perpendicular to the field.

Sol. (b, c)

Comparing the given expression with $E = E_0\sin(\omega t + \theta)$. We find that peak value of $E = E_0 = 100$ volts, angular frequency, $\omega = 100\pi$ or $2\pi v = 100\pi$ or $v = 50$Hz. At t = 0, $E = E_0\sin(\pi/3)$, i.e., $\theta = \pi/3 = 60°$ i.e normal to the plane of the armature makes so angle of 60° with the magnetic field. Hence the correct choices are (b) and (c).

I. ASSERTION REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false **(d) If both assertion and reason are false**

(e) If assertion is false but reason is true.

1. Assertion: Magnetic flux can produce induced e.m.f.

Reason: Faraday established induced e.m.f. experimentally.

Ans. (e) Assertion is false but reason is true.

E.m.f. induces, when there is change in magnetic flux, the magnitude of induced e.m.f. depends upon the rate at which the magnetic flux changes. When magnetic flux is steady or constant no e.m.f. is induced. Faraday did experiment in which there is relative motion between the coil and magnet, ithe flux linked with the coil changes and e.m.f. induces.

2. Assertion: Inductance coil are made of copper.

Reason: Induced current is more in wire having less resistance.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion

The inductance coils made of copper will have very small ohmic resistance. Due to change in magnetic flux a large induced current will be produced in such an inductance, which will offer appreciable opposition to the flow of current.

3. Assertion: When two coils are wound on each other, the mutual induction between the coils is maximum.

Reason: Mutual induction does not depend on the orientation of the coils.

Ans. (c) Assertion is true but reason is false.

The manner in which the two coils are oriented, determines the coefficient of coupling between them, $K = \sqrt{\frac{M}{L_1L_2}}$ where L_1 and L_2 are inductances of two coils. When the two coils are wound on each other, the coefficient of coupling is maximum and hence mutual inductance between the coil is maximum.

4. Assertion: An aircraft flies along the meridian, the potential at the ends of its wings will be the same.

Reason: Whenever there is change in the magnetic flux e.m.f. induces.

Ans. (e) Assertion is false but reason is true.

As the aircraft flies, magnetic flux changes through its wings due to vertical component of the earth's magnetic field. Due to this, induced emf is produced across the wings of the aircraft. Therefore, the wings of the aircraft will not be at the same potential.

5. Assertion: Lenz's law violates the principle of conservation of energy.
Reason: Induced e.m.f. opposes always the change in magnetic flux responsible for its production.

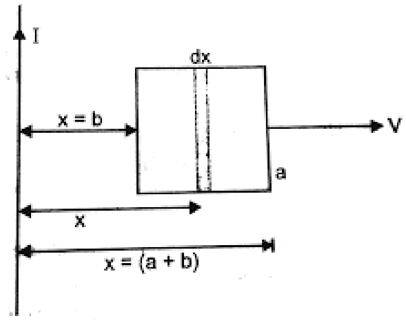
Ans. (e) Assertion is false but reason is true.
Lenz's law states that the direction of induced emf is always such as to oppose the change that cause it and is a direct consequence of the law of conservation of energy. To prove this let us imagine that when the north pole of the magnet approaches the coil, induced current flows in such a direction as to make the face of the coil facing magnet south pole. The magnet would be automatically pulled towards the coil (due to attraction between unlike poles), without expenditure of any mechanical energy. It means electric energy is created out of nothing (or without doing any work) which is against the law of conservation of energy. Therefore it is imperative for the induced current to flow in such a direction that the magnetic effect produced by it tends to oppose the very cause, the relative motion between the coil and the magnet which produce it.

H. CHALLENGING PROBLEMS

1. An athlete peddles a stationary tricycle whose pedals are attached to a coil having 100 turns each of area $0.1 m^2$. The coil, lying in the X-Y plane is rotated, in this plane, at the rate of 50 rpm, about the Y-axis, in a region where a uniform magnetic field $\vec{B} = (0.01)\hat{k}$ tesla, is present. Find the (i) maximum emf (ii) average emf, generated in the coil over one complete revolution.

Sol. Here N = 100, A = $0.1 m^2$, f = 50rpm = 50/60 tps = 5/6 rps
$\vec{B} = (0.01)\hat{k}$ tesla $\Rightarrow$ B = 0.01T
(i) Maximum emf generated in the coil, $\varepsilon_0 = NBA\omega = NBA \times 2\pi f$
$= 100 \times 0.01 \times 0.1 \times 2 \times \pi \times 5/6 = \pi/6$ V = 0.52V
(ii) As the generated emf varies sinusoidally with time, so average emf generated in the coil over one complete cycle = 0.

2. (i) We are given a long straight wire and a square loop of given size (refer to figure). Find out an expression for the mutual inductance between both.
(ii) Now, consider that we passed an electric current through the straight wire of 50 A, and the loop is then moved to the right with constant velocity, v = 10 m/s. Find the emf induced in the loop at an instant where x = 0.2 m. Take a = 0.01 m and assume that the loop has a large resistance.

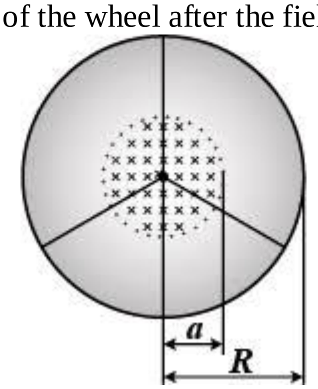


[Ans. (i) $\frac{\mu_0 Ia}{2\pi}\ln\left(1+\frac{a}{x}\right)$, (ii) 1.7×10^{-5}V]

3. A line charge λ per unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (see Fig.). A uniform magnetic field extends over a circular region within the rim. It is given by,

$\vec{B} = B_0\hat{k}\,[r \le a, a < R] = 0$ (otherwise)

What is the angular velocity of the wheel after the field is suddenly switched off?



Sol. According to Faraday's law of electromagnetic induction, the induced emf is $\varepsilon = -\frac{d\phi}{dt}$

This implies the presence of an electric field $\vec{E}$ and tangent to all points of the boundary of the circular region of radius a. If we move a test charge q once round this boundary, a work qε will be done by this force. The electric force on the charge is qE and work done by this force round this boundary is qE × 2πa. Equating the two works done, we get

qε = qE × 2πa or E = ε/2πa

As $\varepsilon = -\frac{d\phi}{dt}$ or E = = $-\frac{1}{2\pi a}.\frac{d}{dt}(\pi r^2 B) = -\frac{a}{2}\frac{dB}{dt}$

In the given problem, total charge on the rim = λ × 2πa. Therefore the force on this charge is

F = λ.2πa.E = - $\lambda.2\pi a.\frac{a}{2}.\frac{dB}{dt}$ [since F = ma = $m\frac{dv}{dt}$]

Or m =.$\frac{d}{dt}(R\omega) = \lambda\pi a^2\frac{dB}{dt}$

Or $mR\frac{d\omega}{dt} = -\lambda\pi a^2\frac{dB}{dt}$

Or $d\omega = -\frac{\lambda\pi a^2}{mR}dB$

Integrating both side $\omega = -\frac{\lambda\pi a^2 B}{mR}$

In vector rotation, $\vec{\omega} = -\frac{B\pi a^2}{mR}\hat{k}$

The negative sign indicates that the vector $\vec{\omega}$ is in the negative z direction.

4. A square loop having side 12 cm with its sides are parallel to x and y-axis moves with a velocity of 8 cm /s in the positive x-direction in a region which have a magnetic field in the direction of positive z-axis. The field is not uniform whether in case of its space or in the case of time. It has a gradient of 10^{-3} T cm^{-1} along the negative x-direction(i.e its value increases by $\mathbf{10^{-3}}\boldsymbol{T}$ as we move from positive to negative direction), and it is reducing in the case of time with the rate of $\mathbf{10^{-3}}\boldsymbol{Ts^{-1}}$. Calculate the magnitude and direction of induced current in the loop (Given: Resistance = **4.5 *m*Ω**).

Sol. Here A = $a^2 = (0.12m)^2 = 144 \times 10^{-4}m^2$, v = $8cms^{-1} = 0.08ms^{-1}$.

Rate of change of magnetic field B with distance is $\frac{dB}{dx} = 10^{-3}Tcm^{-1} = \frac{10^{-3}T}{10^{-2}m} = 10^{-1}Tm^{-1}$

Rate of change of magnetic field B with time t is $\frac{dB}{dt} = 10^{-3}\ Ts^{-1}$

Induced emf due to change in field B with position x is given by

$\varepsilon_1 = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A.\frac{dB}{dt} = A.\frac{dx}{dt}.\frac{dB}{dx} = A.v.\frac{dB}{dx}$

$= 144 \times 10^{-4} \times 0.08 \times 10^{-1}V = 115.2 \times 10^{-6}V$

Induced emf due to change in field B with time t is

$\varepsilon_2 = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A.\frac{dB}{dt}$

$= 144 \times 10^{-4} \times 10^{-3}V = 14.4 \times 10^{-6}V$

Therefore induced emf, $\varepsilon = \varepsilon_1 + \varepsilon_2 = (115.2 + 14.4) \times 10^{-6}V = 129.6 \times 10^{-6}V$

As R = 4.5mΩ = $4.5 \times 10^{-3}\Omega$

Therefore induced current, $I = \frac{\varepsilon}{R} = \frac{129.6\times10^{-6}}{4.5\times10^{-3}}A = 2.9\times10^{-2}A$

The two effects have been added up because both cause a decrease in flux in +ve z direction. The direction of induced current in such as to increase the flux through theloop along +ve z direction. If for the observer the loop moves to the right, the current will be seen to be anticlockwise.

SPACE FOR ROUGH WORK

SPACE FOR NOTES