CLASS – 11

WORKSHEET- WORK, ENERGY, POWER

A. WORK

(1 Mark Questions)

1. An artificial satellite is at a height of 36,500km above earth's surface. What is the work done by earth's gravitational force in keeping it in its orbit?

Sol. The gravitational force always act in perpendicular to the direction of motion of the satellite throughout the orbital motion of the satellite so the work done by Earth gravitational force in keeping the satellite in its orbit is equal to zero.

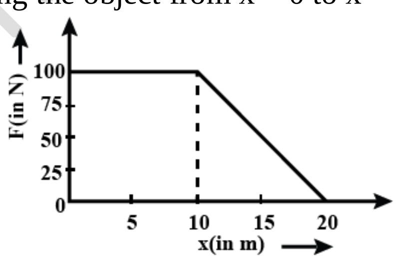
2. A body is being raised to a height h from the surface of earth. What is the sign of work done by (i) applied force (ii) gravitational force?
(a) Positive, positive (b) Positive, Negative (c) Negative, Positive (d) Negative, negative

Sol. (b)

3. A weightlifter lifts a weight off the ground and holds it up
(a) work is done in lifting as well as holding the weight.
(b) no work is done in both lifting and holding the weight.
(c) work is done in lifting the weight but no work is required to be done in holding it up.
(d) no work is done in lifting the weight by work is required to be done in holding it up.

Sol. (c)

4. A force F acting on an object varies with distance x as shown in the figure. The work done by the force is moving the object from x = 0 to x = 20m is



(a) 500J (b) 1000J (c) 1500J (d) 2000J

Sol. (c)
Work done = 10 × 100 + ½ × 10 × 100 = 1000 + 500 = 1500J

5. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative: (1 mark each)
(a) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket,
(b) Work done by gravitational force in the above case,

(c) Work done by friction on a body sliding down an inclined plane,
(d) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
(e) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

Sol. (a) positive (b) negative (c)negative (d) positive (e) negative.

6. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is
(a) + 2000J (b) – 200J (c) zero (d) – 20,000J

Sol. (c)
According to the question, work done by the frictional force on the cycle is
= 200×10 = -2000J
As the road is not moving, hence work done by the cycle on the road is zero.

7. A body is being raised to a height h from the surface of earth. What is the sign of work done by (a) applied force (b) gravitational force?

Sol. Sign of work done by (a) applied force is positive (b) gravitational force is negative.

(2 Marks Questions)

8. A force $\vec{F} = \hat{i} + 5\hat{j} + 7\hat{k}$ acts on a particle and displaces it through $\vec{s} = 6\hat{i} + 9\hat{k}$. Calculate the work done if the force is in newton and displacement in metre.

Sol. $W = \vec{F}.\vec{S} = (\hat{i} + 5\hat{j} + 7\hat{k}).(6\hat{i} + 0\hat{j} + 9\hat{k}) = 1 \times 6 + 5 \times 0 + 7 \times 9 = 69J.$

9. Calculate the work done in lifting a 300N weight to a height of 10m with an acceleration $0.5\ ms^{-2}$. Take $g = 10\ ms^{-2}$.

Sol. Here $m = \frac{W}{g} = \frac{300N}{10ms^{-2}} = 30kg$
W = ,(g +a)h = 30 × (10 + 0.5) × 10 = 3150J.

10. If a force $\vec{F} = (-2\hat{i} + 3\hat{j} + \hat{k})$, causes a displacement $\vec{S} = (\hat{i} + 2\hat{j} - 4\hat{k})$, of an object, what will be the work done on the object?

Sol. $\vec{F} = (-2\hat{i} + 3\hat{j} + \hat{k})$ and $\vec{S} = (\hat{i} + 2\hat{j} - 4\hat{k})$
Work done, $W = \vec{F}.\vec{S} = (-2\hat{i} + 3\hat{j} + \hat{k}).((\hat{i} + 2\hat{j} - 4\hat{k}) = -2 + 6 - 4 = 0.$

11. Find the work done if a particle moves from position $\vec{r_1} = (4\hat{i} + 3\hat{j} + 6\hat{k})m$ to a position $\vec{r_2} = (14\hat{i} + 13\hat{j} + 16\hat{k})m$ under the effect of force $\vec{F} = (4\hat{i} + 4\hat{j} - 4\hat{k})N$?

Sol. Work done by the force is $\vec{F}.\vec{S} = (4\hat{i} + 4\hat{j} - 4\hat{k}).(\vec{r}_2 - \vec{r}_1)$
$= (4\hat{i} + 4\hat{j} - 4\hat{k}).\{(14\hat{i} + 13\hat{j} + 16\hat{k}) - (4\hat{i} + 3\hat{j} + 6\hat{k})\}$
= 40 + 40 – 40 = 40J

(3 Marks Questions)

12. A am weighing 50 kg f supports a body of 25 kg f on his head. What is the work done when he moves a distance of 20m up in an incline of 1 in 10? Take $g = 9.8\ ms^{-2}$.

Sol. Here m = 50+25 = 75kg, $\sin\theta = 1/10$, s = 10m, $g = 9.8\ ms^{-2}$.

Force needed to be applied against gravity = $mg \sin \theta$

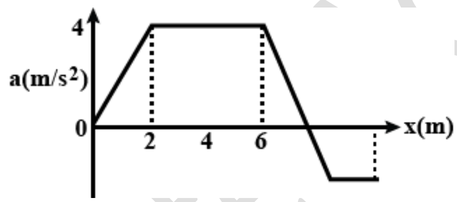
Therefore $W = Fs = mg \sin \theta \times s = 75 \times 9.8 \times 1/10 \times 20 = 1470J$.

13. While catching a cricket ball of mass 200g moving with a velocity of 20m/s, the player draws his hands backwards through 20cm. Find the work done in catching the ball and the average force exerted by the ball on the hand.

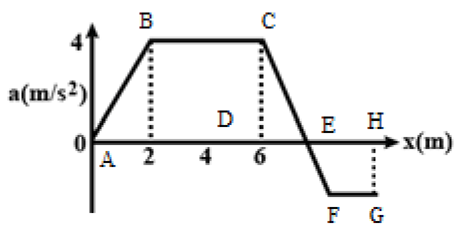
Sol. W = Change in KE = $\frac{1}{2} mv^2 = \frac{1}{2} \times 0.2 \times 20 = 40J$

Therefore F = W/S = 40/0.20 = 200N.

14. Figure gives the acceleration of a 2.0kg body as it moves from rest along x-axis while a valuable force acts on it from x = 0m to x = 9m. Find the work done by the force on the body when it reaches (i) x = 4m and (ii) x = 7m.



Sol. Total work done =- Mass × Areas under acceleration vs displacement graph.



Work done by the force on the body when it reaches at x = 4m is

W_{x-4} = Mass of the body × Area under ABCD = $2 \times [(1/2 \times 1 \times 6) + (3 \times 6)] = 42J$

Work done by the force on the body when it reaches at x = 7m is

$W_{x-7} = W_{x-4}$ + Mass of the body (Area under CDE – Area under EFGH)

$= 42J + 2[(1/2 \times 1 \times 6) – (1/2 \times 1 \times 6) – (1 \times 6)]J = 30J$

15. A body constrained to move along the z-axis of a coordinate system is subject to a constant force F given by

$$\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}\ N$$

where $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along the x- y- and z-axis of the system respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis?

Sol. Here $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$ N

As the body moves a distance of 4m along Z axis, so $\vec{S} = 4\hat{k}$ m.

$\therefore$ W = $\vec{F}.\vec{S} = (-\hat{i} + 2\hat{j} + 3\hat{k}).(4\hat{k})$

$= (-\hat{i} + 2\hat{j} + 3\hat{k}).(0\hat{i} + 0\hat{j} + 4\hat{k}) = -1 \times 0 + 2 \times 0 + 3 \times 4 = 12$J.

(5 Marks Questions)

16. A body of mass 2 kg initially at rest moves under the action of an applied horizontal force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the
(a) Work done by the applied force in 10 s
(b) Work done by friction in 10 s
(c) Work done by the net force on the body in 10 s
(d) Change in kinetic energy of the body in 10 s and interpret your results.

Sol. Here m = 2kg, u = 0, F – 7N, $\mu_k = 0.1$, t = 10s

Force of friction, $f_k = \mu_k R = \mu_k mg = 0.1 \times 2 \times 9.8 = 1.96$N

Net force with which the body moves, $F' = F - f_k = 7 - 1.96 = 5.04$N

Acceleration, $a = F'/m = 5.04/2 = 2.52$ ms^{-1}

Distance, $s = ut + ½ at^2 = 0 + ½ \times 2.52 \times (10)^2 = 126$m

(a) Work done by the applied force, $W_1 = Fs = 7 \times 126 = 885$J

(b) Work done by the friction = $W_2 = - f_2 \times s = - 1.96 \times 126 = - 246.9$J

(c) Work done by the net force, $W_3 = F's = 5.04 \times 126 = 635$J

(d) Final velocity acquired by the body after 10s,

$v = u + at = 0 + 2.52 \times 10 = 25.2$ ms^{-1}

Change in KE of the body = $½ mv^2 - ½ mu^2 = ½ \times 2 \times (25.2)^2 - 0 = 635$ J

Thus the change in KE of the body is equal to the work done by the net force on the body.

B. POWER

(1 Mark Questions)

1. Define power and its SI unit.

Sol. Power as the rate of doing work, it is the work done in unit time. The SI unit of power is Watt (W) which is joules per second (J/s).

2. How many watts are there in one horse power?

Sol. There are 746 watts in one horse power.

3. The power of a water pump is 2kW. If g = 10 ms^{-2}, the amount of water it can raise in one minute to a height of 10m is
(a) 2000 litre (b) 1000 litre (c) 100 litre (d) 1200 litre

Sol. (d)

Volume = $1.2\text{m}^3 = 1.2 \times 10^3$ litre = 1200 litre.

4. Water is flowing in a river at 2m/s. The river is 50m wide and has an average depth of 5m. The power available from the current in the river is (density of water = 1000 kg m^{-2})

(a) 0.5 MW (b) 1 MW (c) 1.5 MW (d) 2 MW

Sol. (b)

Given data, speed of water = v = 2m/s, area of flow = 5m×50m=250m^2, Density of water, ρ=1000 kg/m^3

Form of the continuity equation the volume of water flowing per second v=Av

Hence, mass of water m = vρ = Avρ

K.E= ½ mv^2 = ½ $Av\rho v^2$ = ½ $A\rho v^3$

P = ½ $A\rho v^3$ = ½ ×250 × $(2)^3$×1000

=10^6 W=1MW

5. A body is initially at rest. It undergoes a one-dimensional motion with constant acceleration. The power delivered to it at time t is proportional to

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

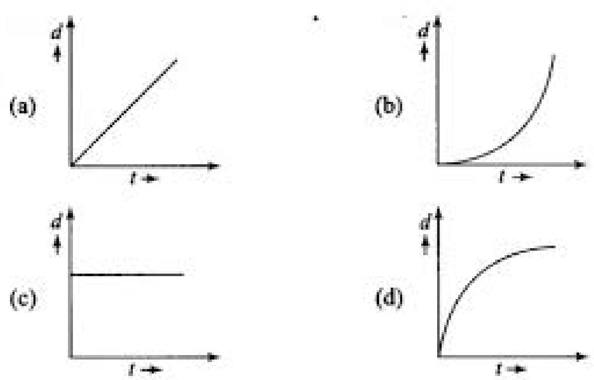
Sol. (ii)

6. A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to

(i) $t^{1/2}$ (ii) t (iii) $t^{3/2}$ (iv) t^2

Sol. (iii)

7. A body is moving unidirectionally under the influence of a source of constant power supplying energy. Which of the diagrams shown in Fig. 6.5 correctly shows the displacement-time curve for its motion?



Sol. (b)

As given that power = constant

As we know that power (P)

P = $\frac{dW}{dt} = \frac{\vec{F}.\vec{dx}}{dt} = \frac{F\,dx}{dt}$

As the body is moving unidirectionally. Hence f.dx = fdxcos0° = Fdx

P = Fdx/dt = constant (since P = constant by question)
$L^2 \propto T^3 \Rightarrow L \propto T^{3/2} \Rightarrow$ Displacement (d) $\propto t^{3/2}$

8. Calculate the power of a crane in watts, which lifts a mass of 100 kg to a height of 10 m in 20s.

Sol. m = 100kg, g = 10m/s, h = 10m, W = mgh = 100 × 10 × 10 = 10000J
P = W/t = 1000/20 = 50W.

(2 Marks Questions)

9. A man weighs 60 kg climbs up a staircase carrying a load of 20kg on his head. The stair case has 20 steps each of height 0.2m. If he takes 10s to climb, find his power.

Sol. Here m = 60+20 = 80kg, h = 20×0.2 = 4m, g = 9.8 ms^{-2}, t = 10s
$P = \frac{W}{t} = \frac{mgh}{t} = \frac{80\times9.8\times4}{10} = \frac{3136}{10}$ = 313.6W.

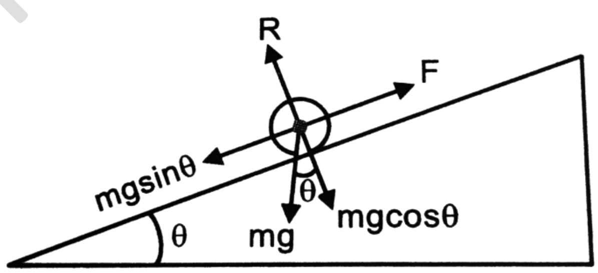
(3 Marks Questions)

10. The human heart discharges 75ml of blood at each beat against a pressure of 0.1m of Hg. Calculate power of heart assuming that pulse frequency is 80 beats per minute. Density of Hg = 13.6×10^3 kgm^{-3}.

Sol. Volume of blood discharged per beat, V = 75ml = $75 \times 10^{-6} m^{-3}$
Pressure, P = 0.1 mm of Hg = $0.1 \times 13.6 \times 10^3 \times 9.8$ Nm^2 [since P = hρg]
Work done per beat = PV, Work done I 80 beats = 80 × PV, Time , t = 1min = 60s
Power = $\frac{Work}{Time} = \frac{80\times PV}{t} = \frac{80\times0.1\times13.6\times10^3\times9.8\times75\times10^{-6}}{60}$ = 1.33W

11. A man cycles up a hill, whose slope is 1m in 20 with it velocity of 6.4 kmh^{-1}. The weight of man and the cycle is 98kg. What work per minute is he doing? What is his horse power?

Sol. Refer to figure. If the inclination of the hill with the horizontal is θ, then



Sin θ = 1/20, v = 6.4 kmh^{-1} = $\frac{6.4\times5}{18} ms^{-1} = \frac{16}{9} ms^{-1}$, m = 98kg, t = 1min = 60s
As the velocity of the cyclist is uniform, so the only force he has to exert is against gravity. It is given by F = mg sin θ
Power of the man, P = Fv = mg sin θ × v

$= 98 \times 9.8 \times \frac{1}{20} \times \frac{16}{9}W = \frac{98\times9.6\times16}{746\times20\times9}$ hp = 0.144 hp

Work done per minute, $W = Pt = \frac{98\times9.8\times16}{20\times9}\times60 = 5122.1J$.

12. The turbine pits at the Niagra falls are 50m deep. The average horse power developed is 5000. If the efficiency of the generator is 85%, how much water passes through the turbines per minute? Take $g = 10\ ms^{-2}$.

Sol. Useful power developed = 5000 hp, Efficiency = 85%

Therefore total power generated $= \frac{100}{85} \times 5000hp = \frac{100\times5000\times746}{85}W$

Total work done by the falling water in 1 min or 60s W = Pt =

$\frac{100\times5000\times746}{85} \times 60 = 26.33 \times 10^7 J$

Now mgh = W

Therefore $m = \frac{W}{gh} = \frac{26.33\times10^7}{10\times50} = 5.26\times10^5 kg$

13. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump?

Sol. Mass of water = Volume × density = 30 × 1000 = 3×10^4 kg

$\therefore$ Output power $= \frac{\text{Work done}}{\text{Time}} = \frac{mgh}{t} = \frac{3\times10^4\times9.8\times40}{15\times60} = \frac{39200}{3}W$

As efficiency $= \frac{\text{Output power}}{\text{Input power}} \times 100$

$\therefore$ Input power $= \frac{\text{Output power}}{\text{Efficiency}} \times 100 = \frac{39200}{3\times30} \times 100 = 43.6 \times 10^3 W = 43.6$ kW.

14. A family uses 8 kW of power, (a) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square meter. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (b) Compare this area to that of the roof of a typical house.

Sol. (a) Let the area needed to supply 8kW = Am^2, Energy incident per unit area = 200W, Energy incident on area A = 200 × AW

Energy converted into useful electrical energy = 20% of 200 × A = 40A W

But 40 AW = 8kW = 8000W or A = 8000/40 = $200m^2$.

(b) Area of the roof of the given house, A' = 70% of 20m × 15m

$= \frac{700\times20\times15}{100} = 210\ m^2$

Required ratio = A/A' = 200/210 = 20:21.

C. KINETIC ENERGY AND WORK ENERGY THEOREM

(1 Mark Questions)

1. Can a body have energy without momentum?

Sol. If either of the mass or velocity is zero, then kinetic energy ($K = \frac{1}{2} mv^2$) will be zero. Therefore it is not possible a body has kinetic energy without having momentum.

2. Can a body have momentum without energy?

Sol. A body cannot have momentum without kinetic energy. This is because a body that does not has kinetic energy has zero velocity, and therefore, zero momentum.

3. A light body and a heavy body have the same momentum. Which one will have greater kinetic energy?

Sol. The kinetic energy of lighter body is higher than the kinetic energy of heavier body.

4. The work energy theorem states that the change in

(a) kinetic energy of a particle is equal to the work done on it by the net force.

(b) kinetic energy of a particle is equal to the work done by one of the forces acting on it.

(c) potential energy of a particle is equal to the work done on it by the net force.

(d) potential energy of a particle is equal to the work done by one of the forces acting on it.

Sol. (a)

5. How will the momentum of a body change if its kinetic energy is doubled?

Sol. The moment of a body is given by, $p = \sqrt{2mK}$ …(i)

When kinetic energy is doubled, the momentum, $p' = \sqrt{2m(2K)}$ …(ii)

On dividing (i) by (ii), $\frac{p}{p'} = \frac{\sqrt{2mK}}{\sqrt{4mK}}$ or $p' = \sqrt{2}\, p$.

6. A mass of 5 kg is moving along a circular path of radius 1 m. If the mass moves with 300 revolutions per minute, its kinetic energy would be

(a) $250\pi^2$ (b) $100\pi^2$ (c) $5\pi^2$ (d) 0

Sol. (a)

As given that, mass (m) = 5kg, n = 300 revolution. Radius (R) = 1m, t = 60sec

$\omega = (2\pi n/t) = (300\times2\times\pi)\text{rad}/60\text{s} = 10\pi$ rad/s

Linear speed (v) = $\omega R = (10\pi \times 1) \Rightarrow v = 10\pi$ m/s

$KE = \frac{1}{2} mv^2 = \frac{1}{2} \times 5 \times (10\pi)^2 = 250\pi^2$ J

7. Give example of a situation in which an applied force does not result in a change in kinetic energy.

Sol. Work done by tension when a body tied with string moving in a vertical circle, horizontal circle, work done by gravity when a ball is tied to a string and is moving in a horizontal circle etc.

(2 Marks Questions)

8. Two bodies of masses m_1 and m_2 have the same linear momentum. What is the ratio of their kinetic energies?

Sol. If v_1 and v_2 are the velocities of two bodies having masses m_1 and m_2 respectively, then $m_1v_1 = m_2v_2$ or $m_1^2\, v_1^2 = m_2^2\, v_2^2$

or $\frac{m_1 v_1^2}{m_2 v_2^2} = \frac{m_2}{m_1}$ or $\frac{\left(\frac{m_1 v_1^2}{2}\right)}{\left(\frac{m_2 v_2^2}{2}\right)} = \frac{m_2}{m_1}$

or $\frac{K_1}{K_2} = \frac{m_2}{m_1}$

9. A particle is projected making an angle of 45° with the horizontal having kinetic energy K. What is the kinetic energy of highest point?

Sol. Initial kinetic energy, K = ½ mu^2

Velocity at the highest point = horizontal component of u – u cos 45° = $\frac{u}{\sqrt{2}}$

Hence KE at the highest point, K' = ½ mu'^2

K' = $\frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2}\left(\frac{1}{2}mu^2\right) = \frac{K}{2}$

KE at highest point will be half of the initial kinetic energy.

10. A particle of mass 0.5 kg travels in a straight line with velocity $u = a\, x^{3/2}$, where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from x = 0 to x = 2 m?

Sol. Velocity, $v = ax^{3/2}$

Acceleration, $\frac{dv}{dt} = \frac{3}{2}ax^{1/2}\frac{dx}{dt} = \frac{3}{2}ax^{1/2}v = \frac{3}{2}ax^{1/2}.\,ax^{3/2} = \frac{3}{2}a^2x^2$

Force, F = m × acceleration = $\frac{3}{2}a^2x^2$

Work done, W = $\int_0^2 F.\,dx = \frac{3}{2}\int_0^2 ma^2x^2dx = \frac{3}{2}ma^2\left[\frac{x^3}{3}\right]_0^2$

= $\frac{3\times 0.5\times(5)^2}{2\times 3}[2^3 - 0^3]$ = 50J.

(3 Marks Questions)

11. Prove work-energy theorem.

Sol. Work energy theorem states that the change in kinetic energy of a particle is equal to the work done on it by the net external force.

Proof: From third equation of motion, $v^2 - u^2 = 2ad$, where u and v are initial and final speeds respectively and d is the distance traversed.

Multiplying both sides by m/2, we have ½ mv^2 = ½ mu^2 = mad = F.d or $K_f - K_i = W$
Where K_i and K_f are initial and final kinetic energies of the object respectively. W is the work done by a force on the body over a certain displacement.

12. A shot travelling at the rate of 100 ms^{-1} is just able to pierce a plank 4cm thick. What velocity is required to just pierce a plank 9cm thick?

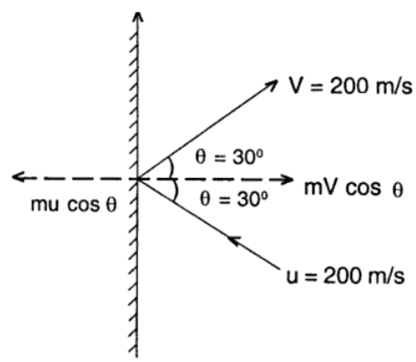
Sol. Here $v_1 = 100ms^{-1}$, s_1 = 4cm, v_2 = ?, s_2 = 9cm.
KE lost by the shot = Work done against Planck's resistance
Therefore ½ $mv_1^2 = F \times s_1$ and ½ $mv_2^2 = F \times s_2$
On dividing, $\frac{v_2^2}{v_1^2} = \frac{s_2}{s_1}$ or $\frac{v_2}{v_1} = \sqrt{\frac{s_2}{s_1}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$
So, $v_2 = \frac{3}{2} \times v_1 = \frac{3}{2} \times 100 = 150\ ms^{-1}$.

13. If the kinetic energy of a body increases by 300%, by what % will the linear momentum of the body increase?

Sol. Initial kinetic energy, K = ½ $mv^2 = \frac{1}{2}\frac{m^2v^2}{m} = \frac{p^2}{2m}$
Therefore initial momentum, p = $\sqrt{2mK}$
Increase in kinetic energy = 300% of K = 3K
Final kinetic energy, K' = K + 3K = 4k
Final momentum, p' = $\sqrt{2mK'} = \sqrt{2m \times 4K} = 2\sqrt{2mK}$ = 2p
% increase in momentum = $\frac{p'-p}{p} \times 100 = \frac{2p-p}{p} \times 100$ = 100%

14. A molecule in a gas container hits a horizontal wall with speed 200 msA and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic?

Sol. Momentum is always conserved, whether the collision is elastic or inelastic.



As the wall is heavy, the molecule rebounding with its own speed does not produce any velocity in the wall. Let m be the mass of the molecule and M that of the wall.
KE before collision, K_i = ½ $m(200)^2$ + ½ $M(0)^2 = 2 \times 10^4$ m
KE after collision, K_f = ½ $m(200)^2$ + ½ M(0) = 2×10^4m
Therefore $K_f = K_i$

As the KE is conserved, the collision is elastic.

D. POTENTIAL ENERGY

(1 Mark Questions)

1. Give three examples of forces which are conservative in nature.

Sol. Many forces in nature that we know of like the magnetic force, electrostatic force, gravitational force, etc. are a few examples of conservative forces.

2. The potential energy of a system increases if work is done
(a) upon the system by a non conservative force
(b) by the system against a conservative force
(c) by the system against a non-conservative force
(d) upon the system by a conservative force

Sol. (b)

3. The negative of the work done by the conservative internal forces on a system equals to the change in
(a) total energy (b) kinetic energy (c) potential energy (d) none of these

Sol. (c)

4. Two springs of spring constants 1000N m^{-1} and 2000N m^{-1} are stretched with same force. They will have potential energy in the ratio of
(a) 2:1 (b) $2^2:1^2$ (c) 1:2 (d) $1^2:2^2$

Sol. (a)

$U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{F}{K}\right)^2 = \frac{F^2}{2K}$

$U_1 \times k_1 = U_2 \times k_2$ or $\frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{2000}{1000} = \frac{2}{1}$

(2 Marks Questions)

5. An elastic spring of force constant k is compressed by an amount x. Show that its potential energy is ½ kx^2.

Sol. We know that as we stretch or compress a spring, a restoring force acts on it which increases linearly with the distance from the equilibrium (unstretched position) restoring force F is directly proportional to distance stretched/compressed from unstretched position.
Let this distance be x then F = -kx k is proportionality constant called 'spring constant'. (negative sign shows that restoring force is opposite to displacement)

Now we want to know how much potential energy is stored in the compressed spring. Potential energy of any configuration = work done to get that configuration So we basically have to calculate the work done by us in compressing the spring by a distance x.

W = Force. Distance

The applied force is equal to the restoring force in magnitude. Let us compress the spring by a small distance dx, then small work done in doing so

dW = kx. dx

Total work done will be equal to integrating all the small works dW from x = 0 to x = x

$\int dW = \int kx.\, dx$ [from x = 0 to x = x]

$W = k \int x.dx = \frac{k[x^2]}{2}$

Therefore Potential energy stored in the spring = $W = \frac{kx^2}{2}$

(3 Marks Questions)

6. Two springs have force constants k_1 and k_2 ($k_1 > k_2$). On which spring is more work done, if (i) they are stretched by the same factor and (ii) they are stretched by the same amount?

Sol. (i) Suppose the two springs get stretched by distances x_1 and x_2 by the same force F. then

$F = k_1x_1 = k_2x_2$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} = \frac{k_1x_1.x_1}{k_2x_2.x_2} = \frac{F.x_1}{F.x_2} = \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

As $k_1 > k_2$. Therefore $W_1 < W_2$ or $W_2 > W_1$.

(ii) Suppose the two springs are stretched by the same distance x. Then

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}k_1x^2}{\frac{1}{2}k_2x^2} = \frac{k_1}{k_2}$$

As $k_1 > k_2$. Therefore $W_1 > W_2$.

7. Define the term potential energy, and derive its dimensions. Write an expression for the gravitational potential energy of a body of mass m raised to a height h above the earth's surface.

Sol. Potential energy is the energy stored in a body or a system by virtue of is position in a field of force or by its configuration.

Computation of G.P.E. : Consider a body of mass m lying on the surface of the earth. Let g be the acceleration due to gravity at this place. For heights much smaller than the radius of the earth ($h << r_E$) the value g can be taken constant.

Force needed to lift the body up with zero acceleration, F = weight of the body = mg

Work done on the body in raising it through height h, W = F.h = mg.h

The work done against gravity is stored as the gravitational potential energy (U) of the body.

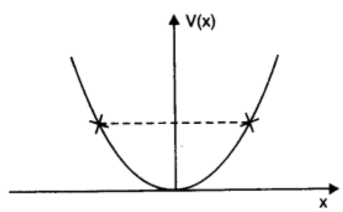
$\therefore U = mgh$

At the surface of the earth, h = 0. ∴ Gravitational PE at the earth's surface = 0.

8. If the elongation in a spring of force constant k is tripled, calculate
(a) ratio of final to initial force in the spring.
(b) ratio of elastic energies stored in the two cases
(c) work done in changing to the state of elongation.

Sol. Force constant is given by k, let the displacement be x, then force |F| = kx
(a) If elongation in the spring is tripled, then final force, F = k(3x)
Therefore their ration, $\frac{F'}{F} = \frac{k(3x)}{kx} = \frac{3}{1}$
(b) Initial elastic energy, E = ½ kx^2, final elastic energy, E' = ½ $k(3x)^2$
Their ratio, $\frac{E'}{E} = \frac{9}{1}$
(c) Work done in changing state = E' – E = ½ × $(9kx)^2$ – ½ kx^2 = $4kx^2$

9. The potential energy function for a particle executing linear simple harmonic motion is given by V (x) – $kx^2/2$, where k is the force constant of the oscillator. For k = 0.5 Nm^{-1} , the graph of V (x) versus x is shown in Fig. Show that a particle of total energy 1 J moving under this potential must 'turn back' when it reaches x = ± 2 m.



Sol. At any instant, the energy of the oscillator is partly kinetic and partly potential. Its total energy is E = K +V or E = ½ mv^2 + ½ kx^2.
An oscillating particle turns back at the point where its instantaneous velocity is zero i.e., the particle will turn back at such a point x where v = 0.
∴ E = 0 + ½ kx^2
But E = 1J, k = 0.2 Nm^{-1}
∴ 1 = ½ × 0.5 × x^2 or x^2 = 4 or x = ±2.

E. CONSERVATION OF MECHANICAL ENERGY

(1 Mark Questions)

1. A spark is produced, when two stones are struck against each other. Why?

Sol. The spark is produced because this collision is an inelastic collision and its mechanical energy is lost in form of light energy which appears as the spark.

2. Can kinetic energy be negative? What about potential energy?

Sol. Kinetic energy can only be zero or positive; it can never be negative. This is because kinetic energy is defined as half an object's mass multiplied by the object's velocity squared. Mass can never be negative because it is a measure of matter, and velocity is always positive because it is squared. Potential energy can be negative.

3. A body is falling freely under the action of gravity alone in vacuum. Which of the following quantities remain constant during the fall?

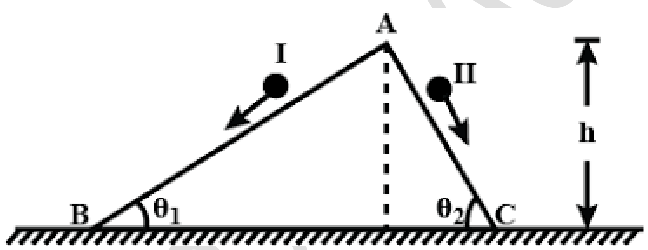
(a) Kinetic energy (b) Potential energy

(c) Total mechanical energy (d) Total linear momentum

Sol. (c)

As the body is falling freely under gravity and no external force act on body in vacuum so law of conservation, the potential energy deceases and kinetic energy increases because total mechanical energy (PE + KE) of the body and earth system will remain constant.

4. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track as shown in Figure. Which of the following statement is correct?



(a) Both the stones reach the bottom at the same time but not with the same speed.

(b) Both the stones reach the bottom with the same speed and stone I reaches the bottom earlier than stone II.

(c) Both the stones reach the bottom with the same speed and stone II reaches the bottom earlier than stone I.

(d) Both the stones reach the bottom at different times and with different speeds.

Sol. (c)

As the (inclined surface) are frictionless, hence, mechanical energy will be conserved. As both the tracks having common height, h (and no external force acts on system).

KE & PE of stone I at top = KE + PE of bottom of I.

From conservation of mechanical energy, 0 + 182 mv_1^2 = mgh + 0

$\Rightarrow v_1 = \sqrt{2gh}$ similarly $v_2 = \sqrt{2gh}$

Hence speed is same for both stones. For stone I, acceleration along inclined plane $a_1 = g \sin\theta_1$. Similarly for stone II $a_2 = g \sin\theta_2$.

$\sin\theta_1 < \sin\theta_2$. Thus $\theta_2 > \theta_1$ hence $a_2 > a_1$. a_2 is greater than a_1 and both length for track II is also less hence, stone II reaches earlier than stone I

5. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2m.

Sol. The mass of the car is 400 kg and the distance moved is 2m. Hence, the work done by the car against the gravity is zero.

(2 Marks Questions)

6. How high must a body be lifted to gain an amount of potential energy equal to the kinetic energy it has when moving at speed 20m/s? The value of acceleration due to gravity at a place is g = 9.8 m/s^2.

Sol. Here mgh = ½ mv^2

So, $h = \frac{v^2}{2g} = \frac{20\times 20}{2\times 9.8}$ = 20.2m.

7. Can there be a solution in which $E - U < 0$?

Sol. No, As E = K + U or K = E – U. But kinetic energy E cannot be negative. So E – U is never less than zero.

8. Define work, power and energy and give their SI units.

Sol. Work is defined as a force causing the movement or displacement of an object. The SI unit of work is the joule (J). Power is the rate at which work is done or energy is transferred in a unit of time. The SI unit of power is Watts(W). Energy is defined as the ability to do work. The SI unit of energy is joule (J).

9. A ball is dropped vertically from rest at a height of 12m. After striking the ground it bounces at a height of 1m. What fraction of kinetic energy does it loose on striking the ground?

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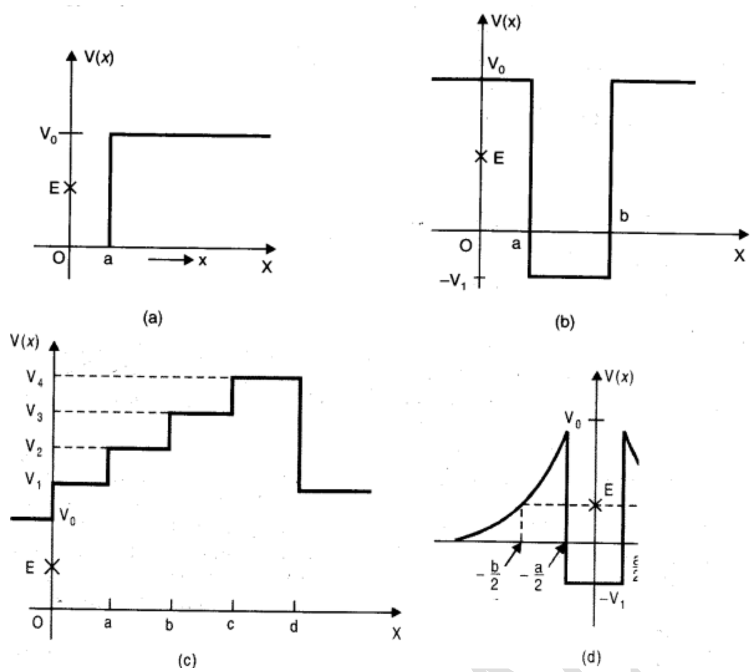
10. Given figures are examples of some potential energy functions in one dimension. The total energy of the particle is indicated by a cross on the ordinate axis. In each case, specify the regions, if any, in which the particle cannot be found for the given energy. Also, indicate the minimum total energy the particle must have in each case. Think of some physical contexts for which these potential energy shapes are relevant.



Sol. Total energy, E = KE + PE

∴ KE = E – PE

The particle can exist in such a region in which its KE is positive

(a) For x > a, PE (V_0) > E.

∴ KE is negative. The particle cannot exist in the region x > a. Here E_{min} = 0.

(b) In every region of the graph, PE (V) > E.

∴ KE is negative. The particle cannot be found in any region. Here E_{min} = V_1.

(c) For x < a and x > b. PE (V_0) > E.

∴ KE is negative. The particuel cannot be found in the region x <a and x > b. Here E_{min} = - V_1.

(d) For – b/2 < x < - a/2 and a/2 < x < b/2, PE (V) > E.

∴ KE is negative. The particle cannot be present in tehse regions. Hence E_{min} = - V_1.

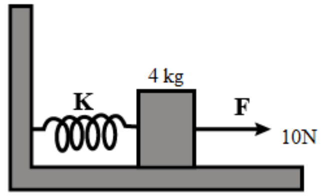
(3 Marks Questions)

11. A block of mass 2kg is dropped from a height of 40cm on a spring whose force-constant is 1960 Nm^{-1}. What will be the maximum distance x through which the spring is compressed?

Sol. Here mg(h +x) = 1/2 kx^2

$2 \times 9.8 \times (0.4 + x) = 980x^2 = 7.84 + 19.6x = 980\,x^2 \Rightarrow x = 0.1m = 10cm.$

12. The spring shown in figure has a force constant 24 n/m. The mass of the block attached to the spring is 4kg. Initially the block is at rest and spring is unstretched. The horizontal force of 10N is applied on the block, then what is the speed of the block when it has been moved through a distance of 0.5m?



Sol. Here $k = 24\ Nm^{-1}$, m = 4kg, x = 0.5m, F = 10N

By the law of conservation of energy, Work done on the spring = Gain in KE + Gain in PE

Or $Fx = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

Or $10 \times 0.5 = \frac{1}{2} \times 4 \times v^2 + \frac{1}{2} \times 24 \times (0.5)^2$

Or $5 = 2v^2 + 3$ or $v^2 = 1$

Therefore $v = 1ms^{-1}$.

13. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV, and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds, (electron mass = 9.11×10^{-31} kg, proton mass = 1.67×10^{-27} kg, 1 eV= 1.60×10^{19}J).

Sol. KE of the electron = $\frac{1}{2} m_e v_e^2$ = 10keV, KE of the proton = $\frac{1}{2} m_p v_p^2$ = 100 keV.

$\therefore \frac{\frac{1}{2}m_e v_e^2}{\frac{1}{2}m_p v_p^2} = \frac{10\ keV}{100\ keV} = \frac{1}{10}$

Or $\frac{9.11\times10^{-31}\times v_e^2}{1.67\times10^{-27}\times v_p^2} = \frac{1}{10}$

Or $\frac{v_e^2}{v_p^2} = \frac{1670}{9.11} = 183.3$

Or $\frac{v_e}{v_p} = 13.53$

14. A raindrop of radius 2 mm falls from a height of 500 m above the ground. It falls with decreasing acceleration (due to viscous resistance of the air) until at half its original height, it attains its maximum (terminal) speed, and moves with uniform speed thereafter. What is the work done by the gravitational force on the drop in the first and second half of its journey? What is the work done by the resistive force in the entire journey if its speed on reaching the ground is 10 ms^{-1}?

Sol. Whether the rain drop falls with decreasing acceleration or with uniform speed, the work done by gravitational force on the drop remains same.

Here $r = 2mm = 2 \times 10^{-3}m$, distance moved in each half journey, h = 500/2 = 250m, density of water, $\rho = 10^3 kgm^{-3}$

Mass of rain drop, m = Volume × density = $\frac{4}{3}\pi \times (2 \times 10^{-3})^3 \times 10^3 = \frac{32\pi}{3} \times 10^{-6}$ kg

Work done by the gravitational force on the rain drop in each journey, W = F × s = mgh

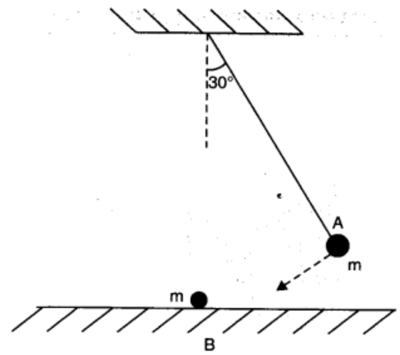
$= \frac{32\pi}{3} \times 10^{-6} \times 9.8 \times 250 = 0.082J$.

For entire journey, Work done by gravitational force + Work done by resistive force = Gain in KE

$2 \times 0.082 + W_r = \frac{1}{2} mv^2$

Or $W_r = \frac{1}{2} \times \frac{32\pi \times 10^{-6} \times (10)^2}{3} - 0.164$ = 0.0017 – 0.164 = – 0.01623 J.

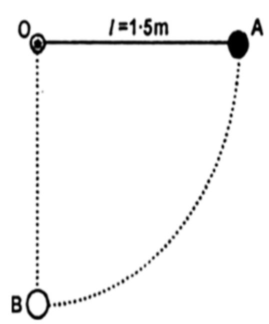
15. The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in Fig. How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic.



Sol. When the bob A hits the bob B on the table, I transfers its entire KE to the bob B because the collision is elastic. The bob A comes to rest at the location of B while the bob B begins to move with velocity of A.

16. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance?

Sol.



Here h = 1.5m, v = ?, PE of the bob at A = mgh, KE of the bob at B = ½ mv^2

As 5% of the PE is dissipated against air resistance, so

½ mv^2 = 95% of mgh

Or ½ $mv^2 = \frac{95}{100} \times mgh$

Or $v = \sqrt{\frac{2\times95\times gh}{100}} = \sqrt{\frac{2\times95\times9.8\times1.5}{100}} = \sqrt{27.93}$ = 5.3 ms^{-1}.

17. A trolley of mass 300 kg carrying a sandbag of 25 kg is moving uniformly with a speed of 27 km/h on a friction less track. After a while, sand starts leaking out of a hole on the trolley's floor at the rate of 0.05 kg s^{-1}. What is the speed of the trolley after the entire sand bag is empty?

Sol. As the trolley carrying the sandbag is moving with uniform speed of 27 km/h, so on external force is acting on the trolley + sandbag system. When the sand leaks out, it does not cause any external force to act on the system. Hence the speed of the trolley remains unchanged even after the sandbag becomes empty.

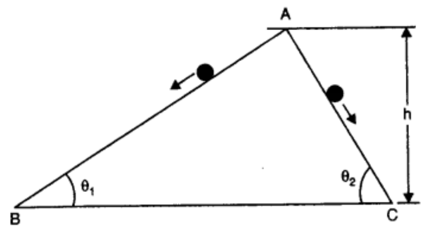
18. A person trying to lose weight (dieter) lifts a 10 kg mass, one thousand times, to a height of 0.5 m each time. Assume that the potential energy lost each time she lowers the mass is dissipated, (a) How much work does she do against the gravitational force? (b) Fat supplies 3.8 x 10^7J of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up?

Sol. (a) Here m = 10kg, h = 0.5m, n = 1000, g = 9.8 ms^{-2}.
Work done against the gravitational force, W = n × mgh = 1000 × 10 × 9.8 × 0.5
= 49000J
(b) Mechanical energy supplied by 1kg of fat = 20% of 3.8 × 10^7J
$= \frac{20\times3.8\times10^7}{100} = 76\times10^5$ J
Therefore fat consumed fro 76 × 10^5 J o f energy = 1kg
Fat consumed for 49000 J of energy $= \frac{1\times49000}{76\times10^5} = 6.45 \times 10^{-3}$ kg.

19. A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceiling by thin wire. Calculate the height to which the block rises. Also, estimate the amount of heat produced in the block.

Sol. Her mass of the bullet, m_1 = 0.012kg, mass of the block, m_2 = 0.4kg, initial velocity of the bullet, u_1 = 70 ms^{-1}, initial velocity of the block, u_2 = 0
Since on striking the wooden block the bullet comes to rest w.r.t. the block of wood.
Let v be the velocity acquired by the combination. Applying principle of conservation of linear momentum, $(m_1\ m_2)v = m_1u_1 + m_2u_2 = m_1u_1$
$v = \frac{m_1u_1}{m_1+m_2} = \frac{0.012\times70}{0.012+0.4} = \frac{0.84}{0.412} = 2.04\ ms^{-1}$
Let the block rise to a height h, $\frac{1}{2}(m_1 + m_2)v^2 = (m_1+ m_2)gh$
$h = \frac{v^2}{2g} = \frac{(2.04)^2}{2\times9.8} = 21.2$cm
The loss in kinetic energy of the system will appear as heat
Therefore heat produced = Initial KE of bullet – final KE of combination
$= \frac{1}{2} m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$
$= \frac{1}{2} \times 0.012 \times (70)^2 - \frac{1}{2}(0.412)(2.04)^2 = 29.4 - 0.86 = 28.54$J

20. Two inclined frictionless tracks, one gradual and the other steep meet at A from where two stones are allowed to slide down from rest, one on each track (Fig). Will the stones reach the bottom at the same time? Will they reach there at the same speed? Explain. Given θ_1 = 30°, θ_2 = 60°, and h = 10 m, what are the speeds and times taken by the two stones?



Sol. Let a_1 be the acceleration of the stone 1 down the inclined track AB. Then $a_1 = g \sin\theta_1$
If the stone 1 takes time t_1 to slide down the trck AB, then
AB = 0 + ½ $a_1t_1^2$ [s= ut + ½ at^2]
$\frac{h}{\sin\theta_1} = \frac{1}{2} g\sin\theta_1 t_1^2$
Or $t_1^2 = \frac{2h}{g\sin^2\theta_1}$
Or $t_1 = \frac{1}{\sin\theta_1}\sqrt{\frac{2h}{g}}$
Similarly for stone 2 we can write, $t_2 = \frac{1}{\sin\theta_2}\sqrt{\frac{2h}{g}}$
For both stones, h is same. As $\theta_1 < \theta_2$, so $\sin\theta_1 < \sin\theta_2$.
Consequently $r_1 > t_2$
Thus the stone2 on the steeper plane AC reaches the bottom earlier than stone 1.
As both stones are initially at the same height h, so PE at A = KE at B or C
mgh = ½ mv^2, v = $\sqrt{2gh}$ i.e. both the stones will the bottom with the same speed.
Given $\theta_1 = 30°$, $\theta_2 = 60°$, h = 10m, g = 10 ms^{-2}
Therefore $t_1 = \frac{1}{\sin\theta_1}\sqrt{\frac{2h}{g}} = \frac{1}{\sin 30°}\sqrt{\frac{2\times10}{10}} = 2\sqrt{2}$s.
$t_2 = \frac{1}{\sin\theta_2}\sqrt{\frac{2h}{g}} = \frac{1}{\sin 60°}\sqrt{\frac{2\times10}{10}} = 2\sqrt{\frac{2}{3}}$s.
v = $\sqrt{2gh} = \sqrt{2\times10\times10} = \sqrt{2}\times10$ = 1.141 × 10 = 14.14 ms^{-1}.

21. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with a uniform speed of 7 ms^{-1}. It hits the floor of the elevator (length of elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary?

Sol. As the elevator is moving down with a uniform speed (a = 0), so the value of g remains the same.
Here m = 0.3kg, h = 3m, g = 9.8 ms^{-2}.
PE lost by the bolt = mgh = 0.3 × 9.8 × 3 = 8.82J
As the bolt does not rebound, the energy is converted into heat.
Therefore heat produced = 8.82J.
Even if the elevator wer stationary, the same amount of heat would have produced because the value of g is same in all inertial frames of reference.

(5 Marks Questions)

22. State the law of conservation of mechanical energy. Show that the total mechanical energy of a body falling freely under gravity is conserved. Show it graphically.

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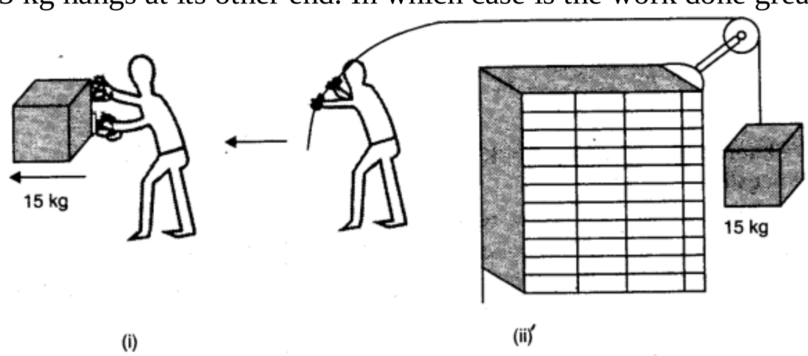
23. Answer the following:
(a) The casing of a rocket in flight bums up due to friction. At whose expense is the heat energy required for burning obtained? The rocket or the atmosphere?
(b) Comets move around the sun in highly elliptical orbits. The gravitational force on the comet due to the sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?
(c) An artificial satellite orbiting the earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?
(d) In Fig.(i), the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. (ii), he walks the same distance pulling the rope behind him. The rope goes over a pulley, and a mass of 15 kg hangs at its other end. In which case is the work done greater?



Sol. (a) Heat energy required for the burning of the casing of the rocket in flight is obtained from the rocket itself. It is obtained at the expense of the mass of the rocket and its kinetic and potential energies.
(b) The gravitational force acting on the corner is a conservative force. The work done by a conservative over any path is equal to the negative of the change in PE. Over a

complete orbit of any shape, there is no change in PE of the comet. Hence no work is done by the gravitational force on the comet.

(c) As the satellite comes closer to the earth, the potential energy decreases. As the sum of kinetic and potential energy remains constant, the kinetic energy and velocity of the satellite increase. But the total energy of the satellite goes on decreasing due to the loss of energy against friction.

(d) In case (i) no work is done against gravity because the displacement of 2m (horizontal) and the weight of 15kg (acting vertically downwards)(are perpendicular to each other. Work is done only against friction.

In case (ii) work has to be done against gravity (- mgh = 15 × 9.8 × 2 = 294J) in addition to the work to be done against friction while a distance of 2m. Thus the work done in case (ii) is greater than in case (i).

F. COLLISION

(1 Mark Questions)

1. What is the value of coefficient of restitution in (i) perfectly elastic collision and (ii) perfectly inelastic collision?

Sol. The coefficient of restitution for a perfectly elastic collision is e = 1. Inelastic collisions will have a coefficient of restitution between 0 and 1.

2. Point out the correct alternative:

(a) When a conservative force does positive work on a body, the potential energy of the body increases/decreases/remains unaltered.

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(b)Work done by a body against friction always results in a loss of its kinetic/potential energy.

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(c) The rate of change of total momentum of a many-particle system is proportional to the external force/sum of the internal forces of the system.

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(d) In an inelastic collision of two bodies, the quantities which do not change after the collision are the total kinetic energy/total linear momentum/total energy of the system of two bodies.

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3. State if each of the following statements is true or false. Give reasons for your answer.
(a) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

Sol. False. Total momentum and total energy of the entire system are conserved and not of individual bodies.

(b) Total energy of a system is always conserved, no matter what internal and external forces on the body are present.

Sol. False. The external forces acting on a body may change its energy.

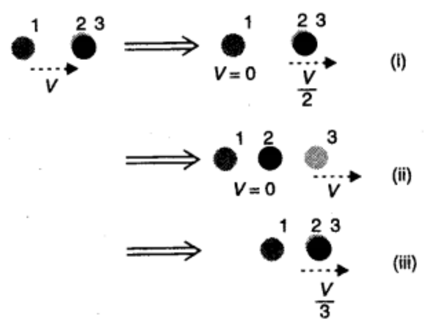
(c) Work done in the motion of a body over a closed loop is zero for every force in nature.

Sol. False. In case of a non conservative force like friction, the work done in the motion of a body over a closed loop is not zero.

(d) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system.

Sol. True. In an elastic collision, a part of the initial KE of the system always changes into some other form of energy.

4. Two identical ball bearings in contact with each other and resting on a friction less table are hit head-on by another ball bearing of the same mass moving initially with a speed V. If the collision is elastic, which of the following (Fig.) is a possible result after collision?



Sol. The system consists of three identical ball bearings marked 1, 2 and 3. Let m be the mass of each ball bearing.

Total kinetic energy of the system before collision = ½ mv^2 + 0 + 0 = ½ mv^2

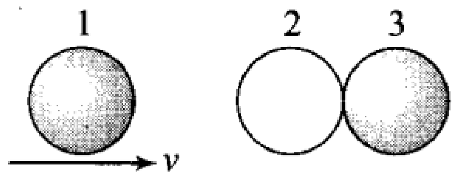
Case (i) KE of the system after collision = $0 + \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{4}mv^2$

Case (ii) KE of the system after collision = 0 + ½ mv^2 = ½ mv^2

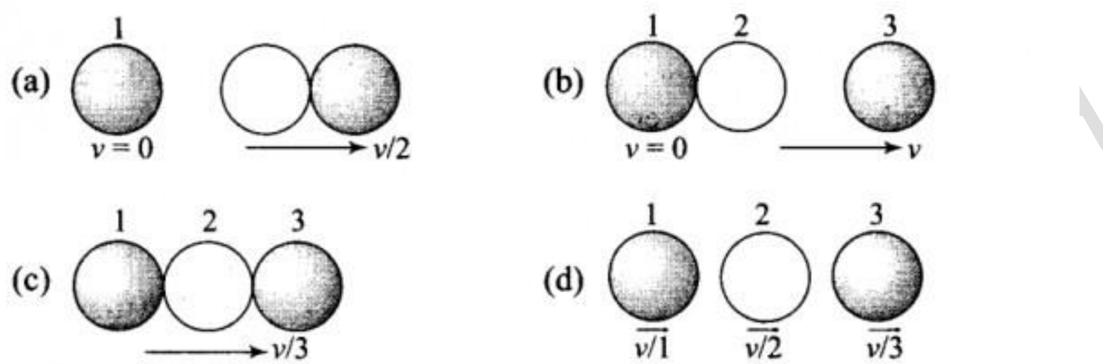
Case (iii) KE of the system after collision = $\frac{1}{2}(3m)\left(\frac{v}{3}\right)^2 = \frac{1}{6}mv^2$

Because in an elastic collision, the kinetic energy of the system remains unchanged. Hence case (ii) is the only possible result of the collision.

5. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed V as shown in Figure.



If the collision is elastic, which of the following (Figure) is a possible result after collision?



Sol. (b)

If two bodies of equal masses collides elastically, their velocities are interchanged.
When ball 1 collides with ball 2 the velocity of ball 1, v_1 becomes zero and velocity of ball 2, v_2 becomes v, i.e. similarly then its own all momentum is mV.
So, $v_1 = 0 \Rightarrow v_2 = v$, $P_1 = 0$, $P_2 = mV$
Now ball 2 collides to ball 3 and its transfer its momentum is mV to ball 3 and itself comes to rest.
So, $v_2 = 0 \Rightarrow v_3 = v$, $P_2 = 0$, $P_3 = mV$
So, ball 1 and ball 2, become in rest and ball 3 move with velocity v in forward direction

6. In an elastic collision of two billiard balls, which of the following quantities remain conserved during the short time of collision of the balls (i.e., when they are in contact). (a) Kinetic energy. (b) Total linear momentum? Give reason for your answer in each case.

Sol. (a) In the given elastic collision, kinetic energy of the ball is not conserved during the short time of collision. However, the total kinetic energy before collision of the balls and after collision are same. At the time of collision, the kinetic energy of the ball gets converted into potential energy. (b) Yes, the total linear momentum conserved during the short time of an elastic collision of two balls. Linear momentum of a particle is always conserved whatever the cause may be.

(3 Marks Questions)

7. A 10kg ball and 20kg ball approach each other with velocities 20 ms^{-1} and 10 ms^{-1} respectively. What are their velocities after collision if the collision is perfectly elastic?

Sol. Here m_1 = 10kg, m_2 = 20kg, u_1 = 20 ms^{-1}, u_2 = - 10 ms^{-1}

$v_1 = \frac{m_1-m_2}{m_1+m_2}.u_1 + \frac{2m_2}{m_1+m_2}.u_2 = \frac{10-20}{10+20} \times 20 + \frac{2\times20}{10+20} \times (-10)$

$= -\frac{20}{3} - \frac{40}{3} = -\frac{60}{3}$ = - 20 ms^{-1}

$v_2 = \frac{2m_1}{m_1+m_2}.u_1 + \frac{m_2-m_1}{m_1+m_2}.u_2 = \frac{2\times10}{10+20} \times 20 + \frac{20-10}{10+20} \times (-10)$

$= \frac{40}{3} - \frac{10}{3} = \frac{30}{3}$ = 10 ms^{-1}.

8. A ball of 0.1kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1kg ball rebounds at one-third of its original speed, what is the mass of the other ball?

Sol. Here m_1 = 0.1kg, m_2 = ?, u_1 = u (say), v_1 = u/3. u_2 = 0

Now, $v_1 = \frac{m_1-m_2}{m_1+m_2}.u_1 + \frac{2m_2}{m_1+m_2}.u_2$

$\therefore -\frac{u}{3} = \frac{0.1-m_2}{0.1+m_2}.u + 0$

Or $-\frac{1}{3} = \frac{0.1-m_2}{0.1+m_2}$

Or – 0.1 – m_2 = 0.3 – $3m_2$ or $2m_2$= 0.4

$\therefore$ m_2 = 0.2 kg

9. What is the meaning of 'Collision' in physics? Differentiate between elastic and inelastic collision. Give one example each.

Sol. A collision is said to occur between two bodies, either if they physically collide against each other or if the path of one is affected by the force exerted by the other.

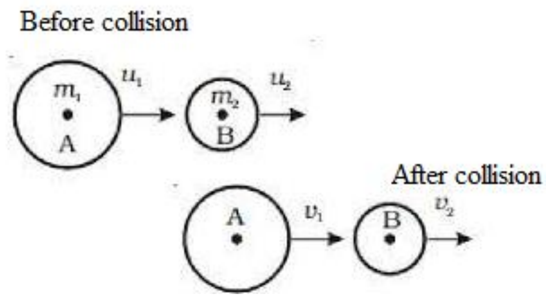
The collisions between particles may be elastic or inelastic.

Elastic collision: If there is no loss of kinetic energy during a collision. It is called an elastic collision. Examples: Collision between subatomic particles, collision between glass balls, etc.

Inelastic collision: If there is a loss of kinetic energy during a collision, it is called an inelastic collision. Examples: Collision between two vehicles, collision between a ball and floor.

10. Show that in case of one dimensional elastic collision of two bodies, the relative velocity of separation after the collision is equal to the relative velocity of approach before the collision.

Sol. Elastic collision in one dimension: As shown in figure consider two perfectly elastic bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2 respectively. Let $u_1 < u_2$. After some time, the two bodies collide with velocities v_1 and v_2 respectively. The two bodies will separate after the collision if $v_2 > v_1$.

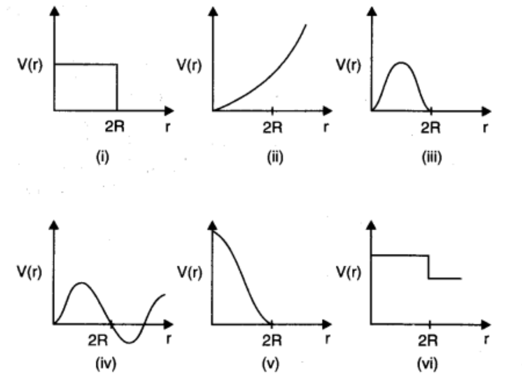


As linear momentum is conserved in any collision, so $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ …(1)
Or $m_1u_1 - m_1v_1 = m_2u_2 – m_2u_2$
Or $m_1(u_1 – v_1) = m_1(v_2 – u_2)$ …(2)
Since KE is also conserved n an elastic collision,
so $\frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2u_2^2 = \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2$
or $m_1u_1^2 - m_1v_1^2 = m_2u_2^2 – m_2u_2^2$
or $m_1(u_1 + v_1)(u_1 – v_1) = m_2(u_2 + v_2)(u_2 – v_2)$ …(3)
Dividing (3) by (2) we get
$u_1 + v_1 = v_2 + u_2$
or $u_1 – u_2 = v_2 – v_1$ …(4)
or relative velocity of A w.r.t. B before collision = relative velocity of B w.r.t. A after collision
or Relative velocity of approach = Relative velocity of separation
Thus in an elastic on dimensional collision, the relative velocity of approach before collision is equal to the relative velocity after the collision.

11. Prove that bodies of identical masses exchange their velocities after head on elastic collision.

Sol. Here $m_1 = m_2 = m$ (say), $u_1 = v$, $u_2 = - v$.
As the collision is perfectly elastic, velocities after the collision will be
$v_1 = \frac{m_1-m_2}{m_1+m_2}.u_1 + \frac{2m_2}{m_1+m_2}.u_2$
$= \frac{m-m}{m+m}.v + \frac{2m}{m+m}(-v)$
$= 0 – v = - v.$
$v_2 = \frac{2m_1}{m_1+m_2}.u_1 + \frac{m_2-m_1}{m_1+m_2}.u_2$
$= \frac{2m}{m+m}.v + \frac{m-m}{m+m}(-v)$
$= v + 0 = v.$
Thus two bodies bounce back with equal speeds after the collision.

12. Which of the following potential energy curves in Fig. cannot possibly describe the elastic collision of two billiard balls? Here r is distance between centres of the balls.



Sol. During the short time of collision, the kinetic energy gets converted into potential energy . But the PE of a system of two masses varies inversely as the distance between them i.e., $V \propto 1/r$. hence all the potential energy curves except one shown in (v) cannot describe an elastic collision.

(5 Marks Questions)

13. Define elastic collision and discuss it for two bodies in one dimension. Calculate the velocities of bodies after collision. Discuss special cases only.

Sol. Same as Q. 10

After collision: From equation (4) above

$V_2 = u_1 - u_2 + v_1$

Putting the value of v_2 in equation (1) , we get

$m_1u_1 + m_2u_2 = m_1v_1 + m_2 (u_1 - u_2 + v_1) = m_1 v_1 + m_2u_1 - m_2u_2 + m_2v_1$

or $(m_1 - m_2)u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

or $v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$ ...(5)

Interchanging the subscripts 1 and 2 in the above equation we get

$v_2 = = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1$ ...(6)

Special case:

(i) When two bodies of equal masses collide: Let $m_1 = m_2 = m$ (say

From equation (5), $v_1 = \frac{2mu_2}{2m} = u_2$ = velocity of body of mass m_2 before collision

From equation (6), $v_2 = \frac{2mu_1}{2m} = u_1$ = velocity of body of mass m_1 before collision

Hence when two bodies of equal masses suffer one dimensional elastic collision, their velocities get exchanged after the collision.

(ii) When a body collides against a stationary body of equal mass. Here $m_1 = m_2$ (say) and u_2 From (5), $v_1 = 0$ and from (6), $v_2 = u_1$

Hence when an elastic body collides against another elastic body of equal mass, initially at rest, after the collision the first body comes to rest while second body moves with the initial velocity of the first.

(iii) When a body collides against a massive stationary body: Here $m_1 << m_2$ and $u_2 = 0$. Neglecting m_1 in equation (5) we get

$v_1 = -\frac{m_2 u_1}{m_2} = -u_1$

From (6) $v_2 = 0$

Hence when a light body collides against massive body at rest, the light body rebounds after the collision with an equal and opposite velocity while the massive body practically at rest. A light ball on striking a wall rebounds almost with the same speed and the wall remains at rest.

(iv) When a massive body collides against a light body: Here $m_1 >> m_2$ and $u_2 = 0$

Neglecting m_2 in equation (5) we get

$v_1 = \frac{m_2 u_1}{m_2} = -u_1$ and $v_2 = \frac{2m_1 u_1}{m_1} = 2u_1$

Hence when a massive body collides against a light body at rest, the velocity of the massive body almost unchanged while the light body starts moving with twice the velocity of massive body.

14. A large mass 'M' moving with a velocity 'v' collides head on with a very small mass 'm' at rest. If the collision is elastic, obtain an expression for the energy lost by the large mass M (take $M + m \approx M$).

Sol. Here $m_1 = M$, $u_1 = v$, $m_2 = m$, $u_2 = 0$

Velocity of M after collision, $v_1 = \frac{m_1 - m_2}{m_1 + m_2} . u_1 + \frac{2m_2}{m_1 + m_2} . u_2$

$= \frac{M-m}{M+m} v + \frac{2m}{M+m} . 0$

Kinetic energy lost by mass M

$= \frac{1}{2} Mv^2 = \frac{1}{2} M\left(\frac{M-m}{M+m} v\right)^2$

$= \frac{1}{2} Mv^2 \left[1 - \left(\frac{M-m}{M+m}\right)^2\right]$

$= \frac{1}{2} Mv^2 . \frac{4mm}{(M+m)^2}$

$= \frac{1}{2} Mv^2 . \frac{4mm}{M^2} [M + m = M]$

$= 2mv^2$.

15. Answer carefully, with reasons:

(a) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls (i.e., when they are in contact)?

(b) Is the total linear momentum conserved during the short time of an elastic collision of two balls?

(c) What are the answers to (a) and (b) for an inelastic collision?

(d) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note, we are talking here of potential energy corresponding to the force during collision, not gravitational potential energy).

Sol. (a) During the short time of collision when the balls are in contact, the kinetic energy of the balls gets converted into potential energy. In an elastic collision, though the kinetic energy before collision is equal to the kinetic energy after the collision by kinetic energy is not conserved during the short time of collision.

(b) Yes, the total linear momentum is conserved during the short time of an elastic collision of two balls.

(c) In an inelastic collision, the total KE is not conserved during collision, as well as even after the collision. But the total linear momentum of the two balls is conserved.

(d) The collision is elastic because the forces involved are conservative.

G. CASE STUDY

1. Invariance: Newton's laws of motion are applicable in all inertial reference frames. Some physical quantities, when measured by observers in difference reference frames have exactly the same value. Such physical quantities are called invariant. In Newtonian mechanics mass, time and force are invariant quantities. On the other hand, some physical quantities, when measured by observers in difference reference frames, do not have the same value. Such physical quantities are called non invariant. In Newtonian mechanics displacement, velocity and work (which is the dot product of force and displacement) are not invariant. Also kinetic energy (= = ½ mv^2) is not invariant.

Physicists believe that all laws of physics are invariant in all inertial reference frames. From example, the work energy principle states that the change in kinetic energy of a particle is equal to the work done on it by the force. Although work and kinetic energy are not invariant in all reference frames, the work energy principle remains invariant. Thus even though different observers measured the motion of the same particle find different values of work and change in kinetic energy, they all find the work energy principle holds in their respective frames.

(i) Chose the invariant quantities from the following:

(a) mass (b) time (c) velocity (d) displacement

Sol. The correct choices are a and b

(ii) Which of the following quantities is/are not invariant

(a) work (b) kinetic energy (c) torque (d) displacement

Sol. All choices are correct

(iii) Choose the correct statement from the following:

(a) Kinetic energy is not invariant (b) Potential energy is not invariant

(c) Laws of conservation of energy and momentum are invariant

(d) All laws of physics are invariant.

Sol. The correct choices are c and d.

H. ASSERTION REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false
(d) If both assertion and reason are false
(e) If assertion is false but reason is true

1. Assertion: When a gas is allowed to expand, work done by gas is positive.
Reason: Force due to gaseous pressure and displacement (of piston) are in the same direction.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
Since the gaseous pressure and the displacement (of piston) are in the same direction. Therefore θ = 0°. Therefore work done = Fs cos θ = Fs = Positive. Thus during expansion of work done by gas is positive.

2. Assertion: A quick collision between two bodies is more violent than slow collision, even when initial and final velocities are identical.
Reason: The rate of change of momentum determine that force is small or large.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
In a quick collision, time t is small. As F × t = constant, therefore, force involved is large, i.e. collision is more violent in comparison to slow collision.

3. Assertion: If two protons are brought near one another, the potential energy of the system will increase.
Reason: The charge on the proton is + 1.6 × 10^{-19}C.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.
If two protons are brought near one another, work has to be done against electrostatic force (since same charge repel each other). The work done is stored as potential energy in the system.

4. Assertion: in case of bullet fired from gun, the ratio of kinetic energy of gun and bullet is equal to ratio of mass of bullet and gun.
Reason: In firing, momentum is conserved.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
From conservation of momentum, mv + MV = 0 or V = $-\frac{mV}{M}$. Negative sign show that the direction of V is opposite to the direction or v i.e., the gun recoils.
So that $\frac{v}{V} = \frac{M}{m}$.

$$\therefore \frac{Recoil\ energy\ of\ gun}{KE\ of\ bullet} = \frac{\frac{1}{2}MV^2}{\frac{1}{2}mv^2} = \frac{M}{m}\left(\frac{m}{M}\right)^2 = \frac{m}{M}.$$

5. Assertion: A race car travelling around a circular work have a non zero impulse.
Reason: The impulse is zero only when there is no net change in momentum.

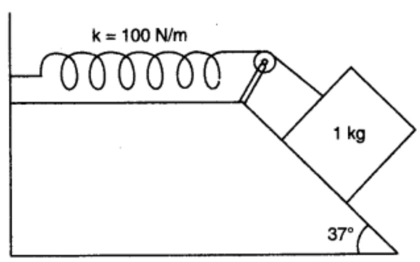
Ans. (d) Both assertion and reason are false.

For the movement around a short portion of the circular track, the direction of impulse vector is same as that of change in momentum. The direction of the vector represents the change in momentum is the same as the direction vector representing the change in velocity. For a particle uniform circular motion, we know that the direction of vector representing the change in velocity is that of acceleration which is towards the centre of circle. Thus the impulse vector is directed towards centre of circle.

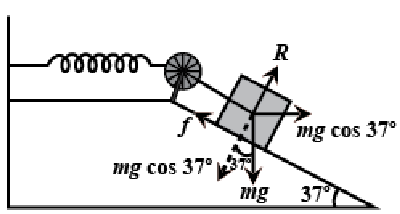
If the car makes one rotation around the track, there is not net change in momentum, so there must be zero net impulse. The force on the car causing the circular motion is the friction between the tires and the roadway and this force is always directed toward the centre fo the circle. For every location of the car on the track, another point of its motion is diametrically opposed across the circle. Thus, as we add up the vector impulse around the circle, they cancel in pairs, total impulse is zero.

I. CHALLENGING PROBLEMS

1. A 1 kg block situated on a rough incline is connected to a spring with spring constant 100 Nm^{-1} as shown in Figure. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has negligible mass and the pulley is frictionless.



Sol. Here m = 1kg, k = 100 Nm^{-1}, g = 10 ms^{-2}.



Clearly from the figure, we have R = mg cos θ, f = μR = μ mg cos θ

Net force on the block down the incline = mg sin θ – f = mg sin θ – μ mg cos θ

= mg (sin θ – μ cos θ)

Distance moved, x = 10cm = 0.2m

In equilibrium, Work done = PE of stretched spring

= mg (sin θ – μ cos θ)x = ½ kx^2

Or 2 = mg (sin θ – μ cos θ) = kx

Or 2 × 1 × 10 (sin 37° - μ cos 37°) = 100 × 0.1
Or 20 (0.601 – μ × 0.798) = 10
Therefore, μ = 0.126.

2. A trolley of mass 200 kg moves with a uniform speed of 36 km h^{-1} on a friction less track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to the trolley's motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

Sol. The child gives an impulse to the trolley at the start and then runs with a constant relative velocity of 4 ms^{-1} with respect to new velocity.

$P_i = (m_1 + m_2)u_1 = (20 + 200) \times \frac{36\times5}{18} = 2200$ kg ms^{-1}

Let the new velocity of the trolley = v_2

Child's velocity relative to the trolley in opposite direction = 4 ms^{-1}.

Therefore child's actual velocity (relative to ground) = $v_2 - 4$

Total final momentum, $Pf = m_1v_1 + m_2v_2 = 20(v_2 - 4) + 200\ v_2 = 220v_2 - 80$

By conservation of linear momentum, $P_f = P_i$

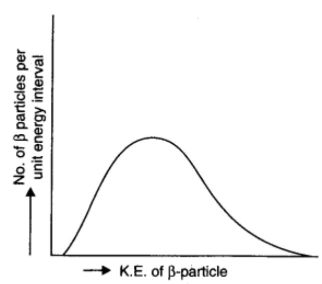
$200v_2 - 80 = 2200$

So, $v_2 = 2280/220 = 10.36$ ms^{-1}

Time taken by the child to cover the length of the trolley = $\frac{10\text{m}}{4\ \text{ms}^{-1}}$ = 2.5s

Distance travelled by the trolley in 2.5s = 10.36 × 2.5 = 25.9 m.

3. Consider the decay of a free neutron at rest: n > p + e^-. Show that the two body decay of this type must necessarily give an electron of fixed energy, and therefore, cannot account for the observed continuous energy distribution in the β -decay of a neutron or a nucleus, Fig.



Sol. If the decay of the neutron (inside the nucleus) into proton and electron is according to the given reaction, then the energy released in the decay must be carried by the electrons coming out of the nucleus. By mass energy conservation, these electrons must have a definite value of energy. However, the given graph shows that the emitted electron can have any value of energy between zero and the maximum value. Hence the given decay mode cannot account for the observed continuous energy spectrum in the β-decay.

4. The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t? (b) What is the kinetic energy of the air? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km/h and the density of air is 1.2 kg m^{-3}. What is the electrical power produced?

Sol. (a) Volume of the air passing through the windmill in time t = Area of circle × distance covered by wind in time t = A × vt = Avt

Mass of air passing through the windmill in time t, m = Density × volume = ρAvt

(b) Kinetic energy of the air is, $K = \frac{1}{2} mv^2 = \frac{1}{2} \rho Avt \times v^2 = \frac{1}{2} \rho Av^3 t$.

(c) KE of air converted into electrical energy in time t

$K' = 25\% \text{ of } K = \frac{25}{100} \times \frac{1}{2}\rho Av^3 t = \frac{1}{8}\rho Av^3 t$

Electrical power produced = $\frac{K'}{t} = \frac{1}{8}\rho Av^3 = \frac{1}{8} \times 1.2 \times 30 \times (10)^3$ [since $v = 36\ kmh^{-1} = 10\ ms^{-1}$]

$= 4.5 \times 10^3 W = 4.5\ kW$.

SPACE FOR ROUGH WORK

SPACE FOR NOTES