CLASS – 12

WORKSHEET- ALTERNATING CURRENT

A. RMS AND AVERAGE VALUES OF ALTERNATING CURRENT EMF & POWER

(1 Mark Questions)

1. The instantaneous current and voltage of an ac circuit are given by: i = 10 sin 314t A and v = 50 sin 314t V. What is the power dissipation in the circuit?

Ans. $P_{av} = \frac{v_0 i_0}{2}\cos\phi = \frac{50\times10}{2}\cos 0° = 250W$.

2. If the rms current in a 50 Hz ac circuit is 5 A, the value of the current 1/300 seconds after its value becomes zero is
(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A (c) 5/6 A (d) $5/\sqrt{2}$ A

Sol. (b)
As given that v = 50Hz, I_{rms} = 5A, t = 1/300s
As we know that $I_{rms} = I_0/\sqrt{2}$
I_0 = peak value = $\sqrt{2}.I_{rms} = \sqrt{2}\times5 = 5\sqrt{2}A$

At t = 1/300sec, $I = I_0 \sin\omega t = 5\sqrt{2}\sin 2\pi\times50\times1/300$

$I = 5\sqrt{2}\sin\pi/3 = 5\sqrt{2}\times\sqrt{3}/2 = 5\sqrt{(3/2)}$ Amp (Since $\sin\pi/3 = \sqrt{3}/2$)
$I = 5\sqrt{3/2}$ A

3. Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?

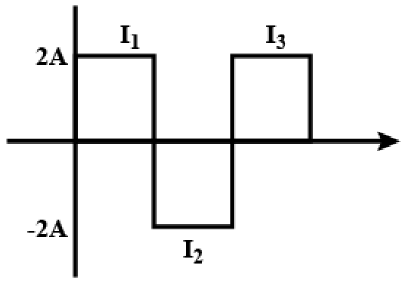
Sol. Yes, the instantaneous power output of an ac source can be negative. No, the average power output cannot be negative.

(2 Marks Questions)

4. The electric mains in a house are marked 220V, 50Hz. Write down the equation for instantaneous voltage.

Sol. Here ε_{rms} = 220V, f = 50Hz
Instantaneous voltage is given by
$\varepsilon = \varepsilon_0\sin\omega t = \sqrt{2}\,\varepsilon_{rms}\sin 2\pi ft = 1.414\times220\sin(2\times3.14\times50t) = 311\sin 314t$ volt.

5. Calculate the rms value of the alternating current shown in figure.



Sol. $I_{rms} = \sqrt{\frac{I_1^2+I_2^2+I_3^2}{3}} = \sqrt{\frac{2^2+(-2)^2+2^2}{3}} = 2A.$

6. Define peak value and root mean value of an alternating current. Derive an expression for the root mean square value of alternating current.

Sol. Rot mean square value of an alternating current: It is defined as that value of a steady voltage that produces the same amount of heat in a given resistance as is produced by the given alternating emf when applied to the same resistance for the same time. It is also called virtual or effective value of the alternating emf. It is denoted by ε_{rms} or ε_{eff} or ε_v.

Relation:

$\varepsilon = \varepsilon_0 \sin \omega t$

Heat produced in a small time dt will be $dH = \frac{\varepsilon_0^2}{R}dt = \frac{\varepsilon_0^2}{R}\sin^2\omega t dt$

$= \frac{\varepsilon_0^2}{R}\int_0^T \frac{(1-\cos 2\omega t)}{2} dt = \frac{\varepsilon_0^2}{2R}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$

$= \frac{\varepsilon_0^2}{2R}\left[(T-0) - \frac{1}{2\omega}\left|\sin\frac{4\pi}{T}t\right|_0^T\right]$

$= \frac{\varepsilon_0^2}{2R}\left[T - \frac{1}{2\omega}\sin(4\pi - \sin 0)\right]$

Or $H = \frac{\varepsilon_0^2}{2R} = [T-0] = \frac{\varepsilon_0^2 T}{2R}$

If ε_{rms} is the root mean value of the alternating current then the amount of heat produced by it in the same resistance R in time T will be $\frac{\varepsilon_{rms}^2 T}{R}$

From the above two equations we get $\frac{\varepsilon_{rms}^2 T}{R} = \frac{\varepsilon_0^2 T}{2R}$

Or $\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}} = 0.707\varepsilon_0$

7. (a) The peak voltage of an ac supply is 300 V. What is the rms voltage?
(b) The rms value of current in an ac circuit is 10 A. What is the peak current?

[Ans. 212.1 V, 141.14A]

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8. Explain the significance of phasor diagram.

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(3 Marks Questions)

9. Derive the average power in ac circuit and explain the term power factor.

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B. RESISTIVE CIRCUIT

(1 Mark Questions)

1. What is the maximum value of power factor? When does it occur?

Sol. One. For a purely resistive circuit, $\phi = 0$. Therefore power factor, $\cos\phi = \cos 0 = 1$.

(2 Marks Questions)

2. An alternating voltage given by V = 140 sin314t is connected across a pure resistor of 50Ω. Find (i) the frequency of the source (ii) the rms current through the resistor.

Sol. (i) $2\pi\upsilon = 314$ rad s^{-1} $\Rightarrow \upsilon = 50$Hz

(ii) $i_{rms} = \frac{V_{rms}}{R}$ where $V_{rms} = \frac{V}{\sqrt{2}} = \frac{140}{\sqrt{2}\times 50} = 1.98A = 2A$.

3. The peak value of an alternating voltage applied to a 50Ω resistance is 10V. Find the rms current, if the voltage frequency is 100Hz, write the equation for the instantaneous current.

Sol. Here R = 50Ω, $\varepsilon_0 = 10V$, f = 100Hz

$I_0 = \frac{\varepsilon_0}{R} = \frac{10}{50} = \frac{1}{5}A = 200mA$

$I_{rms} = 0.707\, I_0 = 0.707 \times 200 = 141.4$ mA

The instantaneous current is given by $I = I_0 \sin 2\pi ft = 200 \sin 200\,\pi t$ mA.

4. A 100 Ω resistor is connected to a 220 V, 50 Hz ac supply.
(a) What is the rms value of current in the circuit?
(b) What is the net power consumed over a full cycle?

Sol. (a) $I_{rms} = \frac{\varepsilon_{rms}}{R} = \frac{220}{100} = 2.20A$

(b) $P_{av} = \varepsilon_{rms} I_{rms} = 220 \times 2.2 = 484W$

(3 Marks Questions)

5. (a) For a given ac, $i = i_m \sin \omega t$, show that the average power dissipated in a resistor R over a complete cycle is ½ $i^2_m R$. (b) A light bulb is rated at 100W for a 220V ac supply. Calculate the resistance of the bulb.

Sol. (a) Average power in one cycle, $P = \frac{W}{t} = \frac{\int_0^T Vidt}{\int_0^T dt}$ where current and voltage are in same phase across resistance R.

If $i = i_m \sin\omega t$ then $V = V_m \sin\omega t$

Hence, $P = \frac{V_m i_m \int_0^T \sin^2\omega t dt}{\int_0^T dt}$

$P = \frac{V_m i_m}{T} \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right) dt$

$P = \frac{V_m i_m}{2T}\left[\int_0^T dt - \int_0^T \cos 2\omega t dt\right]$

$P = \frac{V_m i_m}{2T}[T-0] = \frac{V_m i_m}{2}$

Also, $i_m = V_m/R$

So, $P = \frac{i_m^2 R}{2}$

(b) $P = \frac{V^2}{R}$

$100 = \frac{(220)^2}{R} \Rightarrow R = \frac{220 \times 220}{100} = 484\Omega.$

(5 Marks Questions)

6. A resistance of 40Ω is connected to an ac source of 220V, 50Hz. Find (i) the rms current (ii) the maximum instantaneous current in the resistor and (iii) the time taken by the current to change from its maximum value to the rms value.

Sol. (i) $\varepsilon_{rms} = 220V$, R = 40W

Therefore $I_{rms} = \frac{\varepsilon_{rms}}{R} = \frac{220}{40} = 5.5A$

(ii) Maximum instantaneous current, $I_0 = \sqrt{2}\, I_{rms} = 1.414 \times 505 = 7.8A$.

(iii) Let the alternating current be given by $I = I_0 \sin \omega t$

Le the ac take its maximum and rms value at instants t_1 and t_2 respectively. Then

$I_0 = I_0 \sin \omega t_1$, which implies that $\omega t_1 = \pi/2$ and $I_{rms} = I_0/\sqrt{2} = I_0 \sin \omega t_2$, which implies that $\omega t_2 = \pi/2 + \pi/4$.

Therefore $t_2 - t_1 = \frac{\pi}{4\omega} = \frac{\pi}{4\times2\pi f} = \frac{\pi}{4\times2\pi\times50} = \frac{1}{400}$ s = 2.5ms.

C. CAPACITIVE CIRCUIT

(1 Mark Questions)

1. What is the impedance of a capacitor of capacitance C in an ac circuit using source of frequency ν Hz?

Sol. Impedance of a capacitor, $Z = X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C}$, where n is the frequency of source.

2. Define capacitive reactance. Write its SI units.

Sol. Capacitive reactance is the resistance offered by a capacitor to the flow of ac through it. It is denoted by X_C.

Mathematically, $X_c = \frac{1}{2\pi\upsilon C}$ where υ = frequency of the source, C = capacitance of the capacitor. Ohm (Ω) is the SI unit of capacitive reactance.

3. What is the minimum value of power factor? When does it occur?

Sol. Zero. For a purely inductive or capacitive circuit, $\phi = \pm\pi/2$. Therefore power factor, $\cos\phi = \cos(\pm\pi/2) = 0$.

4. How much average power, over a complete cycle, does an a.c. source supply to a capacitor?

Sol. $P_{av} = V_{rms} I_{rms} \cos(-\pi/2) = 0$.

(2 Marks Questions)

5. An ac source of emf $V = V_0 \sin \omega t$ is connected to a capacitor of capacitance C, Deduce the expression for the current (I) flowing in it. Plot the graph of (i) V vs ωt, and (ii) I vs ωt.

Sol. $V = V_0 \sin\omega t$

$C = q/V$, $q = CV_0 \sin\omega t$

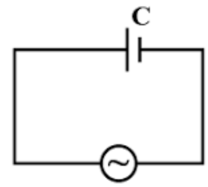
$i = \frac{dq}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$

$= \omega C v_0 \sin\omega t = \frac{V_0}{1/\omega C} \cos\omega t$

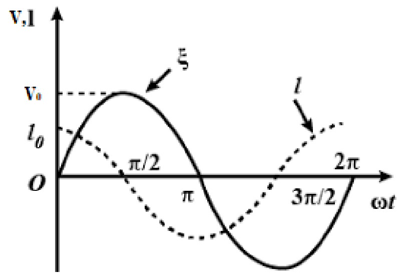
$i = \frac{V_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$ or $I = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

In pure capacitive circuit current leads voltage by π/2.

(i)



(ii)



6. What is the inductive reactance of a coil if current through it is 800mA and the voltage across it is 40V?

Sol. $X_L = \frac{\varepsilon_{eff}}{I_{eff}} = \frac{40}{800\times10^{-3}} = 50\Omega$.

7. A 60 μF capacitor is connected to a 110 V, 60 Hz ac supply. Determine the rms value of the current in the circuit.

Sol. ε_{rms} = 110V, f = 60Hz

Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\times3.14\times60\times60\times10^{-6}} = 44.2\Omega$

$I_{rms} = \frac{\varepsilon_{rms}}{X_C} = \frac{110}{44.2} = 2.49A$

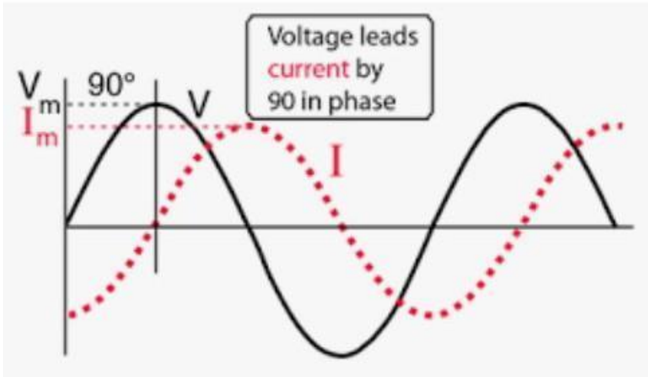
D. INDUCTIVE CIRCUIT

(1 Mark Questions)

1. When an ac source is connected across an inductor, show on graph, the nature of variation of the voltage and the current over one complete cycle.

Sol. $V = V_0 \sin \omega t$

$I = I_0\left(\omega t - \frac{\pi}{2}\right)$



2. Define Inductive Reactance.

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3. Draw the variation of Inductive Reactance with frequency of EMF source.

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(2 Marks Questions)

4. Prove that an ideal inductor does not dissipate power to an ac circuit.

Sol. Average power associated with an inductor: When ac is supplied to an ideal inductor, current lags behind the voltage in phase by π/2 radian. So we can write the instantaneous values of voltage and current as follows: V = V==sinωt and I = $I_0\sin\left(\omega t-\frac{\pi}{2}\right)=-I_0\sin\left(\frac{\pi}{2}-\omega t\right)$ = - I_0 cos ωt

Work done in small time dt is dW = P dt =- V_0I_0 sin ωt cos ωt dt = $-\frac{V_0}{I_0}\sin\omega t dt$

The average power dissipated per cycle in the inductor is

$P_{av}=\frac{W}{T}=\frac{1}{T}\int_0^T dW$

$=-\frac{V_0}{I_0}\int_0^T \sin 2\omega t dt$

$=+\frac{V_0I_0}{2T}\left[\frac{\cos 2\omega t}{2\omega}\right]_0^T=\frac{V_0I_0}{4T\omega}\left[\cos\frac{4\pi}{T}t\right]_0^T$

$=\frac{V_0I_0}{4T\omega}[\mathrm{Cos}4\pi-\cos 0]=\frac{V_0I_0}{4T\omega}[1-1]$

= 0

Thus the average power dissipated per cycle in an inductor is zero.

5. A 44 mH inductor is connected to 220 V, 50 Hz ac supply. Determine the rms value of the current in the circuit.

Sol. Here L = 44mH = 44 × 10^{-3}H, ε_{rms} = 220V, f = 50Hz

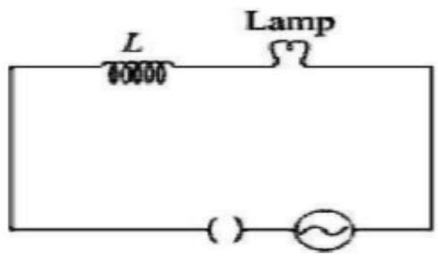
Reactance, $X_L = 2\pi fL = 2\pi \times 50 \times 44 \times 10^{-3}\ \Omega$

Current, $I_{rms} = \frac{\varepsilon_{rms}}{X_L} = \frac{220}{2\pi\times50\times44\times10^{-3}} = 15.9A$

(5 Marks Questions)

6. Show that an ideal inductor does not dissipate power in an ac circuit.

Sol.



As $P_{av} = V_{rnms}I_{rms}\cos\phi$

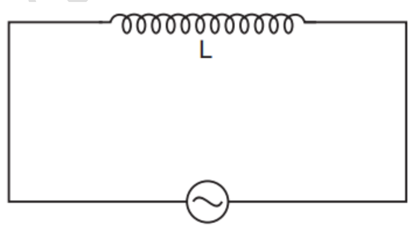
In ideal inductor, current I_{rms} lags behind applied voltage V_{rms} by $\pi/2$

Therefore $\phi = \pi/2$ so, $P_{av} = V_{rms}I_{rms}\cos\pi/2$ or $P_{av} = V_{rms}I_{rms} \times 0$ or $P_{av} = 0$.

7. Show that in an ac circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase.

Sol. The instantaneous ac potential difference across the ends of an inductor of inductance L is $V = V_0\sin\omega t$ …(i)

If I is the instantaneous current through L at instant, $V = L\frac{dI}{dt}$ or $dI = \frac{V_0}{L}\sin\omega t dt$



Integrating both sides, $I = \frac{V_0}{L}\int_0^t \sin\omega t dt = \frac{V_0}{L}\left[\frac{-\cos\omega t}{\omega}\right]_0^t$

Or $I = \frac{-V_0}{\omega L}\cos\omega t$ or $I = \frac{V_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$

$I = I_0\sin\left(\omega t - \frac{\pi}{2}\right)$ …(ii)

Where $I_0 = \frac{V_0}{\omega L}$ is the amplitude of the current

From equations (i) and (ii) it is clear that in an ac circuit, containing inductance, current lags voltage by $\pi/2$.

,E. SERIES LCR CIRCUIT AND RESONANCE

(1 Mark Questions)

1. The selectivity of a series LCR a.c. current is large, when
   (a) L is large and R is large (b) L is small and R is small
   (c) L is large and R is small (d) LR

Sol. (c)

2. The power factor of a series LCR circuit at resonance will be
   (a) 1 (b) 0 (c) ½ (d) $1/\sqrt{2}$

Sol. (a)

3. Answer the following questions. [1 mark each]

(a) In any ac circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for rms voltage?

Sol. (i) Yes, because the voltage variations across each element will follow the variations of the supply voltage oat all instants. (ii) No, the same is not true for rms voltage because voltages across different elements may not be in phase.

(b) A capacitor is used in the primary circuit of an induction coil.

Sol. When the primary circuit of the induction coil is broken, high voltage is induced which gets used in charging the capacitor. This avoids sparking in the circuit.

(c) An applied voltage signal consists of a superposition of a dc voltage and an ac voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the dc signal will appear across C and the ac signal across L.

Sol. Inductive reactance, $X_L = 2\pi fL$ i.e. $X_L \propto f$
Conductive reactance $X_C = 1/2\pi fC$ i.e. $X_C = 1/F$
For dc, f = 0, reactance of L is zero and that of C is infinite, so the dc signal appears across C. For high frequency ac, reactance of L is high and that of C is low. So the ac signal appears across L.

(d) A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

Sol. For a dc, $X_L = 0$. Inductance L has no effect even if it is increased by inserting iron core. But for ac, the lamp will shine dimly because of the impedance offered by the choke. When the iron core is inserted, impedance of the choke further increases and the lamp will dim further.

(e) Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Sol. If a fluorescent tube is connected directly across a 220V source, it would draw large current which would damage the tube. With the use of choke coil, the voltage is reduced to an appropriate value, without wasting any power. A resistor would waste a large amount of electrical energy as heat. So an ordinary resistor cannot be used instead of a choke coil.

(2 Marks Questions)

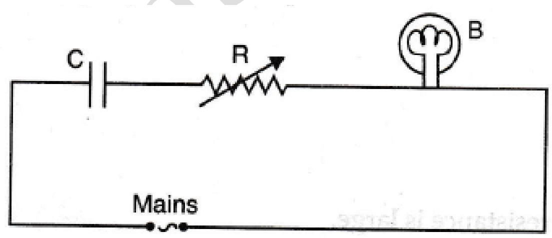
4. Explain the term 'sharpness of resonance' in ac circuit.

Sol. Sharpness of resonance: It is defined as the ratio of the voltage developed across the inductance (L) or capacitor (C) at resonance to the voltage developed across the resistance (R).

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

It may also be defined as the ratio o resonant angular frequency to the bandwidth of the circuit.

5. A capacitor, 'C', a variable resistor 'R' and a bulb 'B' are connected in series to the ac mains in circuits as shown. The bulb glows with some brightness.



How will the glow of the bulb change if (i) a dielectric slab is introduced between the plates of the capacitor, keeping resistance R to be the same; (ii) the resistance R is increased keeping the same capacitance?

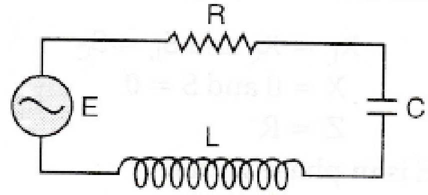
Sol. For the RC circuit, Impedance, $Z = \sqrt{R^2 + (\frac{1}{\omega C})^2}$,

Current $I = \frac{\varepsilon_0}{Z}$ …(i)

(i) When a dielectric slab is introduced between the plates of the capacitor, its capacitance increases. Hence, from equation (i), impedance of the circuit is decreased and the current through it is increased. So, brightness of the bulb will increase.

(ii) When the resistance R is increased and capacitance is same, then from equation (i), impedance of the circuit is increased and the current flowing through it is decreased. SO brightness of the bulb will decrease.

6. The figure shows a series LCR circuit connected to a variable frequency 200V source with L = 50mH, C = 80 μF and R = 40Ω. Determine



(i) the source frequency which derives the circuit in resonance; (ii) the quality factor (Q) of the circuit.

Sol. (i) $L = 50 \times 10^{-3}$ H, $C = 80 \times 10^{-6}$F, R = 40Ω,

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times10^{-3}\times80\times10^{-6}}}$

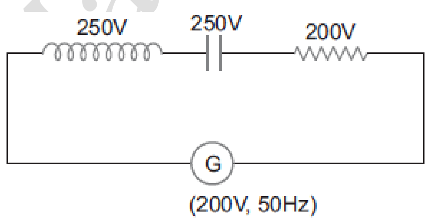
$\omega = \frac{10^3}{2} = 500 \text{ rad s}^{-1} = \upsilon = \frac{500}{2\pi} = 80\text{Hz}$

(ii) $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{40}\sqrt{\frac{50\times10^{-3}}{80\times10^{-6}}} = \frac{1}{40} \times \sqrt{625} = 0.625$

7. A series LCR circuit is connected to an ac source (200V, 50Hz). The voltages across the resistor, capacitor and inductor are respectively 200V, 250V and 250V.

(i) The algebraic sum of the voltages across the three elements is greater than the voltage of the source. How is this paradox resolved? (ii) Given the value of the resistance of R is 40Ω, calculate the current in the circuit.

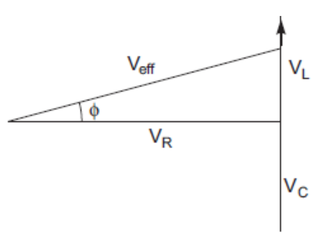
Sol. (i)



From the parameter, V_R = 200V, V_L = 250V and V_C = 250V

V_{ef} should be given as $V_{eff} = V_R + V_L + V_C$ = 200V + 250V + 250V = 700V

However V_{eff} > 200V of the ac source. This paradox can be solved only using phasor diagram as given below:



$(V_{eff}) = \sqrt{V_R^2 + (V_L - V_C)^2}$

Since $V_L = V_C$

(ii) Given R = 40Ω, so current in the LCR circuit,

$I_{eff} = \frac{V_{eff}}{R} = \frac{200}{40} = 5A$ [$X_L = X_C$ or $Z = R$]

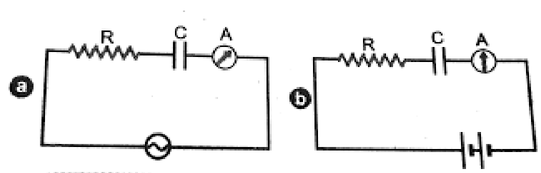
8. In a series LCR circuit, $V_L = V_C \neq V_R$. What is the value of power factor for this circuit?

Sol. Power factor, $\cos\phi = \frac{R}{Z}$

Since $V_L = V_C \Rightarrow X_L = X_C$, so, $Z = R \Rightarrow \cos\phi = 1$

9. Voltage across L and C in series are 180° out of phase. Comment.

10. The hot wire ammeter in Fig (a) shows some deflection but not in fig (b). Why?



11. Obtain the resonant frequency ω_r of a series LCR circuit with L = 2.0H, C = 32 μF and R = 10 Ω. What is the Q-value of this circuit?

Sol. Here L = 2.0H, C = 32μF = 32×10^{-6}F, R = 10Ω.

Resonant frequency, $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}} = \frac{1000}{8}$ = 125 rad s^{-1}

Q value= $\frac{\omega L}{R} = \frac{125 \times 2.0}{10}$ = 25

12. A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?

Sol. Here C = 30×10^{-6}F, L = 27×10^{-3}H

The angular frequency of free oscillations of the LC circuit is

$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{1}{9} \times 10^4$ rad s^{-1} = 1.1×10^3 rad s^{-1}

13. Suppose the initial charge on the capacitor in Question 12 is 6 μC. What is the total energy stored in the circuit initially? What is the total energy at a later time?

Sol. Here $C = 30 \times 10^{-6}F$, $q_0 = 6 \times 10^{-3}C$

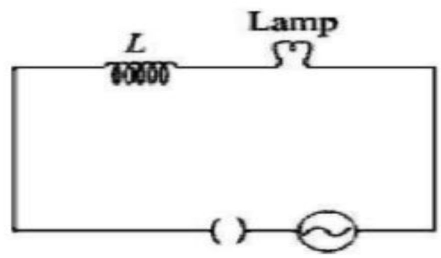
Total energy stored in the inductor initially,

$U = U_E^{max} = \frac{1}{2}\frac{q_0^2}{C} = \frac{1}{2}.\frac{(6\times10^{-3})^2}{30\times10^{-6}} = \frac{36}{60} = 0.6J$

(3 Marks Questions)

14. (i) When an AC source is connected to an ideal inductor show that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with an inductor and an AC source. What happen in the brightness of the lamp when the key is plugged in and an iron rod is inserted inside the inductor? Explain.



Sol. (i) As $P_{av} = V_{rnms}I_{rms}\cos\phi$

In ideal inductor, current I_{rms} lags behind applied voltage V_{rms} by $\pi/2$

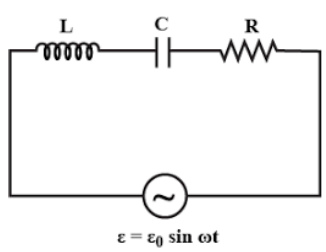
Therefore $\phi = \pi/2$ so, $P_{av} = V_{rms}I_{rms}\cos \pi/2$ or $P_{av} = V_{rms}I_{rms} \times 0$ or $P_{av} = 0$.

(ii) Brightness of the lamp decreases. It is because when iron rod is inserted inside the inductor, its inductance L increases, thereby increasing its inductive resistance X_L and hence impedance Z of the circuit. As $I_{rms} = V_{rms}/Z$, so this decreases the current I_{rms}. In the circuit and hence the brightness of the lamp.

15. A resistor R and an inductor L are connected in series to a source $V = V_0\sin \omega t$. Find the (a) peak value of the voltage drops across R and across L. (b) phase difference between the applied voltage and current. Which of them is ahead?

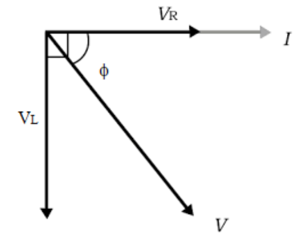
Sol. (a) (i) Peak voltage across R, $V_R = I_0R$.

$V_R = \frac{V_0}{\sqrt{R^2+X_L^2}}R$



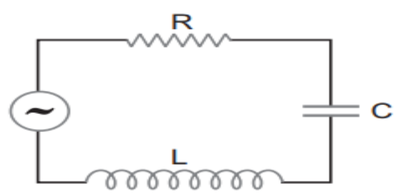
(ii) Peak value across L, $V_L = \frac{V_0X_L}{\sqrt{R^2+X_L^2}}$

(b)



Phase difference $\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)$

16. The figure shows a series LCR circuit with L = 10.0H, C = 40 μF, R = 60Ω connected to a variable frequency 240V source, calculate (i) the angular frequency of the source which derives the circuit at resonance. (ii) the current at the resonating frequency, (iii) the rms potential drop across the inductor at resonance.



Sol. Here L = 10.0Hm, C = 40 μF = 40 × 10^{-6}F, R = 60Ω, V_{rms} = 240V

(i) At resonance the angular frequency v of the source is $\omega_r = \frac{1}{\sqrt{(10.0)(40\times10^{-6})}} = \frac{1}{2\times10^{-2}} =$ 50 rads^{-1}.

(ii) At resonating frequency, Impedance Z = R (since $X_L = X_C$)

The rms current at resonance

Therefore $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} = \frac{240V}{60\Omega}$ = 4A

(iii) The inductive reactance is $X_L = \omega_r L$ = 50×10.0 = 500Ω. The rms potential drop across inductor at resonace, $(V_{rms})_L = I_{rms} \times X_L$ = (4A)(500Ω) = 2000V

17. A voltage $V = V_0\sin\omega t$ is applied to a series LCR circuit. Derive the expression for the average power dissipated over a cycle. Under what condition is (i) no power dissipated even though the current flows through the circuit (ii) maximum power dissipated in the circuit?

Sol. The rate at which electric energy in consumed in an electric circuit is called its power.

Suppose in an ac circuit, voltage and current are having a phase difference ϕ.

$V = V_0\sin \omega t$, $I = I_0\sin(\omega t - \phi)$

Work done by source of emnf in a small time dt with negligible change in current,

dW = Vidt

$dW = V_0I_0\sin\omega t \sin(\omega t - \phi)dt$ where $\sin(\omega t - \phi) = \sin \omega t \cos\phi - \cos \omega t \sin\phi$.

$dW = V_0I_0[\sin^2 wt \cos f - \sin\omega t \cos \omega t \sin\phi]dt$

$dW = V_0I_0 \left[\left(\frac{2-\cos2\omega t}{2}\right)\cos\phi - \frac{\sin2\omega t}{2}\sin\phi\right] dt$

Now total work done in a complete cycle W = $\frac{V_0 I_0}{2} \times \left[\int_0^T \cos\phi dt - \cos\phi \int_0^T \cos 2\omega t dt - \sin\phi \int_0^T \sin 2\omega t dt\right]$

We can solve $\int_0^T \cos 2\omega t dt = \int_0^T \sin 2\omega t dt = 0$

W = $\frac{V_0 I_0}{2} \int_0^T \cos\phi dt = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi T$

Thus power consumed over a cycle, P = W/T = $V_{rms} I_{rms} \cos\phi$

(i) Minimum power: IN an ac circuit containing pure L only, current I lags behind the applied voltage V by phase angle $\pi/2$. So average power consumed by pure inductor L in complete cycle of ac is then given by, P = $V_{rms} I_{rms} \cos \pi/2 = 0$.

(ii) Maximum power: IN ac circuit containing R only, both applied voltage V and current I are in same phase, os average power consumed by resistor R in complete cycle is then given by P = $V_{rms} I_{rms} \cos 0° = V_{rms} I_{rms}$ or P = $\frac{V_{rms}^2}{R}$

18. A bulb of resistance 10Ω connected to an inductor of inductance L, is in series with an ac source marked 100V, 50Hz. If the phase angle between the voltage and current is $\pi/4$ radian, calculate the value of L.

Sol. Here R = 10Ω, f = 50Hz, $\phi = \pi/4$ rad

As $\tan\phi = \frac{X_L}{R} = \frac{2\pi f L}{R}$

Therefore L = $\frac{R\tan\phi}{2\pi f} = \frac{10 \times \tan\pi/4}{2 \times 3.142 \times 50}$ = 0.0318H.

19. A circuit of a resistance of 10Ω and a capacitance of 0.1 μF. If an alternating emf of 100V, 50Hz is applied, find the current in the circuit.

Sol. $X_C = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 0.1 \times 10^{-6}} = 3.2 \times 10^4 \Omega$.

$Z = \sqrt{R^2 + X_C^2} = \sqrt{100 + 10.24 \times 10^8} = 3.2 \times 10^4 \Omega$.

$I_{rms} = \frac{\varepsilon_{rms}}{Z} = \frac{100}{3.2 \times 10^4} = 3.14 \times 10^{-3}$A = 3.14 mA

20. A resistor of 50 ohm, an inductor of $(20/\pi)$ H and a capacitor of $(5/\pi)$μF are connected in series to a voltage source 230V, 50Hz. Find the impedance of the circuit.

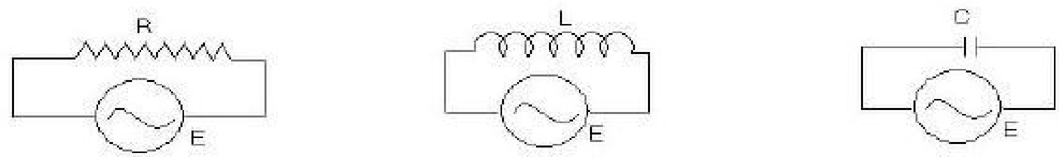
Sol. Here R = 50Ω, L = $\frac{20}{\pi}$H, C = $\frac{5}{\pi}$ μF, F = $\frac{5}{\pi} \times 10^{-6}$F, ε_{eff} = 230V, f = 50HZ

$X_L = 2\pi f = \frac{20}{\pi} \times 2 \times \pi \times 50 = 2000\Omega$

$X_C = \frac{1}{C \times 2\pi f} = \frac{1}{\frac{5}{\pi} \times 10^{-6} \times 2 \times \pi \times 50} = 2000\Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50)^2 + (2000 - 2000)^2} = \sqrt{2500}\Omega = 50\Omega$.

21. Figure (a), (b) and (c) show three a.c. circuits in which equal currents are flowing. If the frequency of emf be increased, how will the current be affected in these circuits? Give reasons for your answer.

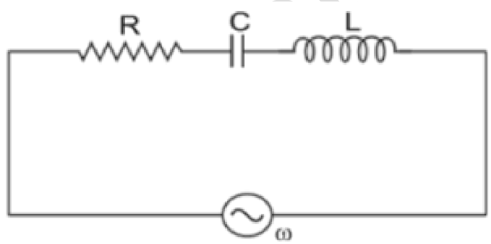


Sol. (a) R is not affected by frequency. So current does not change on increasing f.

(b) Inductive reactance, $X_L = 2\pi fL$. When the frequency f is increased, X_L increases and hence current in the circuit decreases.

(c) Capacitive reactance $X_C = \frac{1}{2\pi fC}$. As the frequency f increased, X_C decreases and hence current in the circuit increases.

22. In the circuit shown in fig, R represents an electric bulb. If the frequency of the supply is doubled, how should the values of C and L be changed so that the glow in the bulb remains unchanged?

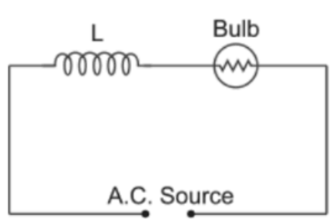


Sol. Current in the LCR circuit is given by

$$I_{eff} = \frac{\varepsilon_{eff}}{\sqrt{R^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}}$$

When the frequency f of the supply is doubled, both the values of L and C should be halved, so that the resistance $\left(2\pi fL - \frac{1}{2\pi fC}\right)$ remains unchanged and hence current n the circuit remains the same. Then the glow of the bulbs will remain unchanged.

23. An inductor 'L' of reactance X_L is connected in series with a bulb 'B' to an a.c. source as shown in figure.



Briefly explain how does the brightness of the bulb change, when (i) number of turns of the inductor is reduced and (ii) a capacitor of reactance $X_C = X_L$ is included in series in the same circuit.

Sol. (i) When the number of turns in the inductor is reduced, its resistance X_L decreases. The current in the circuit increases and hence brightness of the bulb increases.

(ii) With capacitor of reactance $X_C = X_L$ the impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$ becomes minimum. The current in the circuit becomes maximum. The bulb glows with maximum brightness.

24. A series LCR circuit with R = 20 Ω, L = 1.5 H and C = 35 μF is connected to a variable-frequency 200 V ac supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Sol. When the frequency of the ac source equals the natural frequency of the circuit, the impedance is Z = R = 20Ω

The average power dissipated per cycle, $P_{av} = \frac{\varepsilon_{rms}^2}{Z} = \frac{\varepsilon_{rms}^2}{R} = \frac{(200)^2}{20} = 2000W$.

25. A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

Sol. For tuning, the frequency of free LC oscillations should be equal to the frequency of the radio-wave. The value of this frequency is $f = \frac{1}{2\pi\sqrt{LC}}$ $or\ C = \frac{1}{4\pi^2 f^2 L}$

(i) For f = 800 kHz = 800 × 10^3Hz

$C = \frac{1}{4\pi^2 \times (800\times10^3)^2 \times 200\times10^{-6}} = 197.8 \times 10^{-12}F = 198pF$ [since 1pF = 10^{-12}F]

(ii) For f = 1200 kHz = 1200 × 10^3 Hz

$C = \frac{1}{4\pi^2 \times (1200\times10^3)^2 \times 200\times10^{-6}} = 87.9 \times 10^{-12}F = 88pF$

Thus the variable capacitor should have a range of about 88pF to 198pF.

26. A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V. 50 Hz ac supply.

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

Sol. For an LR circuit, if $V = V_0 \cos \omega t$, then $I = \frac{\mathbf{V}}{\sqrt{\mathbf{R^2+\omega^2L^2}}}\mathbf{\cos(\omega t - \phi)}$ where $\tan \phi = \frac{\omega L}{R}$.

(a) Maximum current in the coil is $I_0 = \frac{V_0}{\sqrt{R^2+\omega^2L^2}} = \frac{V_0}{\sqrt{R^2+4\pi^2f^2L^2}}$

Given L = 0.50H, R = 100Ω, V_{eff} = 240V and f = 50 Hz

So, $I_0 = \frac{\sqrt{2}\times240}{\sqrt{100^2+4\pi^2+(50)^2\times(0.50)^2}}A$ [Since $V_{eff} = V_0/\sqrt{2}$)

$= \frac{1.414\times240}{\sqrt{10000+24674}} = \frac{1.414\times240}{186.2} = 1.82A$

(b) V is maximum at t = 0, I is maximum at t = $\frac{\phi}{\omega}$ (i.e when $\omega t - \phi = 0$). If ϕ is positive, this means current maximum lags behind voltage maximum by time lag,

$\Delta t = \frac{\phi}{\omega}$

Now $\tan\phi = \frac{2\pi f L}{R} = \frac{2\pi\times 50\times 0.5}{100} = 1.571$

Therefore $\phi = \tan^{-1}(1.571) = 57.5° = \frac{57.5\pi}{180}$rad

Time lag, $\Delta t = \frac{\phi}{\omega} = \frac{57.5\pi}{180\times 2\pi\times 50} s = 3.19\times 10^{-3}$s = 3.2 ms.

27. Obtain the answers (a) and (b) in Q. 26, if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?

Sol. Here f = 10kHz = 10^4Hz, $\omega = 2\pi f = 2\pi\times 10^4$ rad s^{-1} , ε_{rms} = 240V

$I_0 = \frac{\varepsilon_0}{\sqrt{R^2+\omega^2L^2}} = \frac{\sqrt{2}\times 240}{\sqrt{10^4\times 4\pi^2\times 10^8\times 0.5^2}} = 1.08\times 10^{-2}$A.

Here the contribution of resistance R is negligible as compared to the reactance ωL.

Also, $\tan\phi = \frac{\omega L}{R} = \frac{2\pi\times 10^4\times 0.5}{100} = 100\pi$ which is very large. So ϕ is nearly equal to $\pi/2$ rad.

Thus we see that $I_{0=}$ is much smaller $(1.08\times 10^{-2}$A) than its value (1.82A) at high frequency. At high frequency, L nearly amounts to an open circuit, i.e., it offers very large resistance. In an dc circuit (after attaining steady state) $\omega = 0$, so I, acts like a pure conductor.

28. A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.
(a) What is the maximum current in the circuit?
(b) What is the time lag between the current maximum and the voltage maximum?

Sol. For a CR circuit, if $V = V_0\cos\omega t$, then $I = \frac{V_0}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\cos(\omega t+\phi)$ where $\tan\phi = \frac{1}{\omega CR}$

Here V_{eff} = 110V, $\omega = 2\pi f = 2\pi\times 60$ rad s^{-1}, R = 40Ω, C = 100μF = 10^{-4}F

(a) Maximum current in the circuit is $I_0 = \frac{V_0}{\sqrt{R^2+\frac{1}{\omega^2C^2}}} = \frac{\sqrt{2}V_{eff}}{\sqrt{R^2+\frac{1}{\omega^2C^2}}} = \frac{1.44\times 10}{\sqrt{40^2+\frac{1}{(2\pi\times 60\times 10^{-4})^2}}}$

$= \frac{1.414\times 110}{\sqrt{1600+703.62}} = \frac{155.54}{45} = 3.24$A

(b) The phase angle ϕ is given by $\tan\phi = \frac{1}{\omega CR} = \frac{1}{2\pi\times 60\times 10^{-4}\times 40} = 0.6631$

Therefore $\phi = 33.5° = \frac{33.5\pi}{180}$rad

Time lag, $\Delta t = \frac{\phi}{\omega} = \frac{33.5\pi}{180\times 2\pi\times 60} = 1.55\times 10^{-3}$ = 1.55 ms

Hence the voltage lags behind the current or the current leads the voltage.

29. Obtain the answer to (a) and (b) in Q.28 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

Sol. Here R = 40Ω, C = 100μF = 10^{-4} F, ε_{rms} = 110V, f = 12 kHz = 12×10^3Hz

(a) $X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi\times12\times10^3\times10^{-4}} = 0.133\Omega$

$I_{rms} = \frac{\varepsilon_{rms}}{\sqrt{R^2+X_C^2}} = \frac{110}{\sqrt{40^2+(0.133)^2}} = 2.75A$

Therefore $I_0 = \sqrt{2}\, I_{rms} = 1.414\times2.75 = 3.89A$

(b) $\tan\phi = \frac{X_C}{R} = \frac{0.133}{40} = 0.0033$

Or $\phi = 0.2° = 0°$

Now in the absence of capacitor, $I_{rms} = \frac{\varepsilon_{rms}}{R} = \frac{110}{40} = 2.75A$

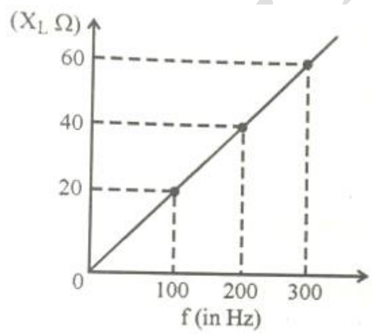
Hence at very high frequency (12 kHz) the current in the circuit is same both in the presence or absence of the capacitor. It follows that at high frequency capacitor acts like a conductor.

For a dc supply, f = 0, so $X_C = \frac{1}{2\pi fC} = \infty$

Hence in dc circuit, a capacitor amounts to an open circuit, i.e. it offers a very high resistance.

(5 Marks Questions)

30. The variation of inductive resistance (X_L) of an inductor with the frequency (f) of the ac source of 100V and variable frequency is shown in the fig.



(i) Calculate the self inductance of the inductor.

(ii) When this inductor is used in series with a capacitor of unknown value and a resistor of 10Ω at 300s^{-1}, maximum power dissipation occurs in the circuit. Calculate the capacitance of the capacitor.

Sol. (i) We know that $X_L = \omega L = 2\pi fL$

$\Rightarrow L = \frac{X_L}{2\pi F} = \frac{20}{2\times3.14\times100}$ \= 0.0318H = 31.8 mH

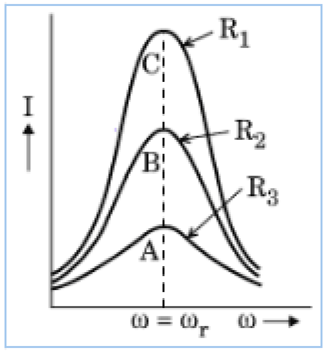
(ii) For maximum power dissipation, $X_L = X_C$

$2\pi fl = \frac{1}{2\pi fc} \Rightarrow 2\times3.14\times300\times31.8\times10^{-3} = \frac{1}{2\times3.14\times300C}$

$\Rightarrow C = 8.8\times10^{-6}$ F = 8.8μF.

31. (a) What do you understand by 'sharpness of resonance' for a series LCR resonant circuit? How it is related with the quality factor 'Q' of the circuit? Using the graphs given

in the diagram, explain the factors which affect it. For which graph is the resistance (R) minimum?



(b) A 2μF capacitor, 100Ω resistor and 8H inductor are connected in series with an ac source. Find the frequency of the ac source for which the current drawn in the circuit is maximum. If the peak value of emf of the source is 200V, calculate the (i) maximum current and, (ii) inductive and capacitive resistance of the circuit at resonance.

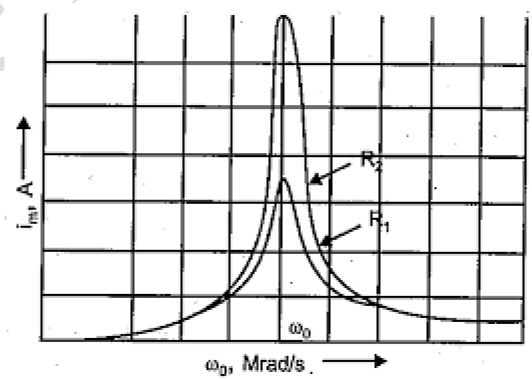
Sol. (a) Sharpness of resonance: It is defined as the ratio of the voltage developed across the inductance (L) or capacitor (C) at resonance to the voltage developed across the resistance (R).

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

It may also be defined as the ratio o resonant angular frequency to the bandwidth of the circuit.

Circuit becomes more selective of the resonance is more sharp, maximum current is more, the circuit is close to resonance for smaller range of ($2\Delta\omega$) of frequencies. Thus the tuning of the circuit will be good.

Figure shows the variation of i_m with ω in a LCR series circuit for tow values of resistnce R_1 and R_2 ($R_1 > R_2$).



The condition for resonance in the LCR circuit is $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

When see that the current amplitude is maximum at the resonant frequency, Since $i_m = V_m/R$ at resonance, the current amplitude for case R_2 is sharper to that for case R_1.

Quality factor or simply the Q factor of resonant LCR circuit is defined as the ratio of voltage drop across the resistance at resonance.

$Q = \frac{V_L}{V_R} = \frac{\omega L}{R}$. Thus finally $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

The Q factor determines the sharpness at resonance as for higher value of Q factor the tuning of the circuit and its sensitivity to accept resonating frequency signals will be much higher. At resonance, current in an ac series LCR circuit is maximum, and depends only on the ohmic resistance R of the circuit. Thus if the ohmic resistance R of series LCR circuit is low, then large current flows in circuit at resonance. So graph C i.e., resistance R_1 has minimum value.

(b) To draw maximum current from a series LCR circuit, the circuit at particular frequency $X_L = X_C$.

The frequency of the series will be $\upsilon = \frac{1}{2\times 3.14\sqrt{8\times 2\times 10^{-6}}}$ = 39.80 Hz

This frequency is known as the seris resonance frequency.

(i) $I_0 = E_0/R = 200/100 = 2A$

(ii) Inductive reactance $X_L = \omega L = 2\pi\upsilon L = 2\times 3.14\times 39.80\times 8 = 2000\Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} = \frac{1}{2\times 3.14\times 39.80\times 2\times 10^{-6}} = 2000\Omega$

32. Explain (i) Resistance (ii) Reactance and (iii) Impedance (iv) Admittance.

Sol (i) Resistance: The property due to which a conductor resists the flow of electrons through it, is called ersistance of the conductor. It is measured by the ratio of potential difference between the ends of the conductor to the current flowing through it. If an alternating current is passed through a resistor, the current and voltage are in the same phase.

(ii) Reactance: The opposition offered by an inductor or a capacitor or both to the flow of ac through it, is called reactance. It is of two types:

(a) Capacitive reactance (X_C): $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} \Rightarrow X_C \propto \frac{1}{\upsilon}$

(b) Inductive reactance (X_L): $X_L = \omega L = \omega\upsilon L$ [Here υ - frequency of ac, L – inductance of the inductor]

So, $X_L \propto \upsilon$

(iii) Impedance: The total opposition offered by LCR circuit to the flow of alternating current is called impedance. It is denoted by Z and is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$

The impedance of an ac circuit plays the same role as resistance in dc circuit.

(iv) Admittance: Admittance is a measure of how easily a circuit or device will allow a current to flow. It is defined as the reciprocal of impedance, analogous to how conductance & resistance are defined. The SI unit of admittance is the siemens (symbol S)

33. A resistance of 2ohms, a coil of inductance 0.01H are connected in series with a capacitor, and put across a 200volt, 50Hz supply. Calculate: (i) the capacitance of the

capacitor so that the circuit resonates. (ii) the current and voltage across the capacitor at resonance (take $\pi = 3$)

Sol. Here $R = 2\Omega$, $L = 0.01H$, $\varepsilon_{eff} = 200V$, $f = 50$ Hz

(i) Resonance frequency, $f = \frac{1}{2\pi\sqrt{LC}}$

So, $C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\times(3)^2\times(50)^2\times(0.01)} = \frac{1}{4\times9\times2500\times0.01} = \frac{1}{900}$

$= 0.0011F = 11 \times 10^{-4}F$.

(ii) $I_{eff} = \frac{\varepsilon_{eff}}{R} = \frac{200}{2} = 100A$

Therefore, $V_C = I_{eff}\, X_C = I_{eff}\frac{1}{2\pi fC} = \frac{1}{2\times3\times50\times11\times10^{-4}} = \frac{100\times10^4}{3300} = 303.03V$

34. An inductor 200mH, capacitor 500 μF, resistor 10Ω are connected in series with a 100V, variable frequency a.c. source. Calculate the (i) frequency at which the power factor of the circuit is unity. (ii) current amplitude at this frequency, (iii) Q factor.

Sol. (i) Power dissipated will be unity at resonance, because then Z = R and $\cos\phi = R/Z = 1$

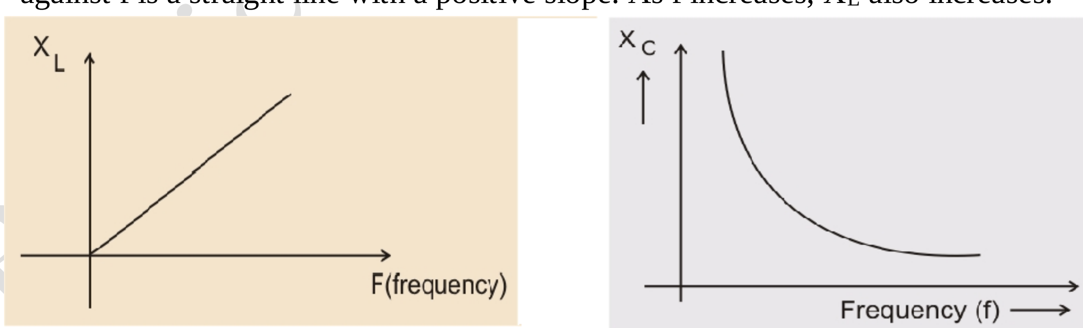
Therefore $f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{200\times10^{-3}\times500\times10^{-6}}}$ Hz $= \frac{1}{2\pi\times10^{-2}} = \frac{50}{\pi}$ Hz

(ii) $I_0 = \frac{\varepsilon_0}{R} = \frac{\sqrt{2}\varepsilon_{rms}}{R} = \frac{1.414\times100}{10} = 14.14A$

(iii) Q factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{200\times10^{-3}}{500\times10^{-6}}} = \frac{20}{10} = 2$

35. (i) Draw the graphs showing variation of inductive reactance and capacitive reactance with frequency of applied ac source (ii) Can the voltage drop across the inductor or the capacitor in a series LCR circuit be greater than the applied voltage of the a.c. source? Justify your answer.

Sol. (i) Inductive reactance $X_L = 2\pi f L$ i.e., $X_L \propto f$. As shown in figure (a), graph of X_L against f is a straight line with a positive slope. As f increases, X_L also increases.



(a) (b)

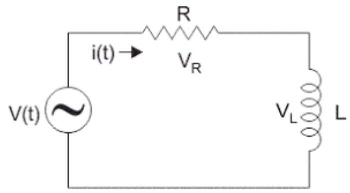
Capacitive reactance, $X_C = \frac{1}{2\pi fC}$ i.e. $X_c \propto 1/f$

Figure (b) show the variation of X_C with f. As f increases, X_c decreases.

(ii) Yes the voltage drop across the inductor or the capacitor in a series circuit can be greater than the applied voltage. These two voltages ae not in same phase, hence they cannot be added like ordinary numbers.

36. Derive an expression for the impedance of an ac circuit with an inductor L and a resistor R in series. Also obtain the expression for average power in the circuit.

Sol. AC circuit containing L and R in series: As shown in figure, consider a resistance R and inductance L connected in series to a source of alternating emf ε given by $\varepsilon = \varepsilon_0 \sin\omega t$.



Let I be the current through the series circuit at any instant. Then

1. Voltage $\vec{V}_R = R\vec{I}$ across the resistance R will be in phase with current $\vec{I}$. So phasors $\vec{V}_R$ and $\vec{I}$ are in same direction. The amplitude of $\vec{V}_R$ is $V_0^R = I_0 R$

2. Voltage $\vec{V}_L = X_L\vec{I}$ across the inductance L is ahead of current $\vec{I}$ in phase by π/2 rad. So phasor $\vec{V}_L$ lies π/2 rad anticlockwise w.r.t. the phasor $\vec{I}$. Its amplitude is $V_0^L = I_0 X_L$ where X_L is the inductive reactance.

By parallelogram law of vector addition, $\vec{V}_R + \vec{V}_L = \vec{\varepsilon}$

Using Pythagorean theorem, we get $\varepsilon_0^2 = (V_0^R)^2 + (V_0^L)^2 - (I_0R)^2 + (I_0X_L)^2 = I_0^2(R^2 + X_L^2)$

Or $I_0 = \frac{\varepsilon_0}{\sqrt{R^2+X_L^2}}$

Clearly, $\sqrt{R^2 + X_L^2}$ is the effective resitane of the series LR circuit which opposes or impedes the flow of ac through it. It is called impedance and is denoted by Z. Thus

$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$ [since $X_L = \omega L$]

The phase angle ϕ between the resultant voltage and current is given by

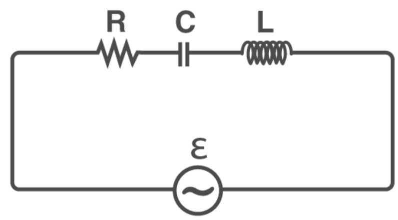
$\tan\phi = \frac{V_0^L}{V_0^R} = \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R} = \frac{\omega L}{R}$

It is obvious from the phasor diagram that the current lags behind the emf of phase angle ϕ so the instantaneous value of current is given by $I = I_0 \sin(\omega t - \phi)$.

$P_{av} = \varepsilon_{rms}.I_{rms}\frac{R}{Z} = \varepsilon_{rms}.I_{rms}.\frac{R}{\sqrt{R^2+\omega^2L^2}}$.

37. A series LCR circuit is connected to an ac source having voltage $V = V_m \sin\omega t$. Derive the expression for the instantaneous current I and its phase relationship to the applied voltage. Obtain the condition for resonance to occur. Define 'power factor'. State the conditions under which it is (i) maximum and (ii) minimum.

38. Figure below shows a series LCR circuit connected to a variable frequency 230 V source. L = 5.0 H, C = 80μF, R = 40 Ω.



(a) Determine the source frequency which drives the circuit in resonance.
(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.
(c) Determine the RMS potential drops across the three elements of the circuit. Show that the potential drop across the LC combination is zero at the resonating frequency.

[Ans. (a)50 rad s^{-1},(b) 40Ω, 8.1A, (c) 230V]

39. An LC circuit contains a 20 mH inductor and a 50 μF capacitor with initial charge of 10 mC. The resistance of the circuit in negligible. Let the instant the circuit is closed be t = 0.
(a) What is the total energy stored initially? Is it conserved during LC oscillations?
(b) What is the natural frequency of the circuit?
(c) At what time is the energy stored

- completely electrical (i.e., stored in the capacitor)?
- completely magnetic (i.e., stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and capacitor?
(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Sol. Her L = 20mH = 20 × 10^{-3}H, C = 50μF = 50 × 10^{-6}F, Initial charge on capacitor, q_0 = 10mC = 10 × 10^{-3}C

(a) Total energy stored initially= $\frac{q_0^2}{2C} = \frac{(10^{-2})^2}{2\times50\times10^{-6}}$J = 1J

Yes, the total energy is conserved in LC oscillations because the resistance of the LC circuit is negligible.

(b) The natural frequency of the circuit is f = $\frac{1}{2\pi\sqrt{LC}}$ = $\frac{1}{2\times3.14\times\sqrt{20\times10^{-3}\times50\times10^{-6}}}$Hz

Or f = $\frac{1}{2\times3.14\times10^{-3}}$ = 159.2 = 159 Hz.

(c) The charge of a capacitor at any instant during LC oscillations is q = q_0 cos ωt = q_0 cos $\frac{2\pi t}{T}$

(i) The energy stored wil be completely electrical when

q = $\pm$ q_0

or cos$\frac{2\pi t}{T}$ = ±1

or $\frac{2\pi t}{T}$ = nπ, where n is an integer

or t = $\frac{n}{2}$T or $t = 0, \frac{T}{2}, T, \frac{3T}{2}$, ...

(ii) The energy stored is completely magnetic when the electrical energy is zero or when q = $q_0$cos ωt = $q_0$cos$\frac{2\pi t}{T}$ = 0

Or $\frac{2\pi t}{T}$ = (2n+1) π/2

Or t = (2n +1) $\frac{T}{4}$ or t = $\frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}$, ...

In both cases, T = 1/f = 1/159s = 6.28 ×= 6.3ms.

(d) Total energy = $\frac{1}{2}\frac{q_0^2}{C}$.

Let q be the charge on the capacitor at the instants when the energy of capacitor becomes half of the total energy. At these instants energy of the capacitor = $\frac{1}{2}\frac{q^2}{C}$

∴ $\frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\left(\frac{1}{2}\frac{q_0^2}{C}\right)$ or q = $\pm\frac{q_0}{\sqrt{2}}$

But q = q_0 cos ωt = $q_0$cos $\frac{2\pi t}{T}$

∴ $\pm\frac{q_0}{\sqrt{2}} = q_0\cos\frac{2\pi t}{T}$

Or cos$\frac{2\pi t}{T} = \pm\frac{1}{\sqrt{2}} = \pm\cos\frac{\pi}{4} = \cos\frac{\pi}{4}$ or $\cos\frac{3\pi}{4}$

Or $\frac{2\pi t}{T}$ = nπ + $\left(\frac{\pi}{4} \text{or } \frac{3\pi}{4}\right)$

Or t = (4π +1) $\frac{T}{8}$ or (4π + 3) $\frac{T}{8}$

Or t = $\frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \frac{7T}{8}$, ...

(e) R camps out the LC oscillations eventually. The whole of the initial energy = 1J is finally lost as heat.

40. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance of the circuit is negligible.
(a) Obtain the current amplitude and rms values.
(b) Obtain the rms values of potential drops across each element.
(c) What is the average power transferred to the inductor?
(d) What is the average power transferred to the capacitor.
(e) What is the total average power absorbed by the circuit? ['Average 'implies' averaged over one cycle'].

Sol. Here L = 80mH = 80×10^{-3}H, C = 60μF = 60×10^{-6}F, V_{rms} = 230V, f = 50 Hz

(a) Reactance of the circuit = $\left|\boldsymbol{\omega L - \frac{1}{\omega C}}\right| = \left|\boldsymbol{2\pi f L - \frac{1}{2\pi f C}}\right|$

= $\left|2\pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}\right|$

= |25.13 – 53.05|Ω = 27.92Ω

$I_{rms} = \frac{V_{rms}}{\text{Reactance}} = \frac{230}{27.92} A = 8.24A$

Current amplitude, $I_0 = \sqrt{2}\, I_{rms}$ = 1.414 × 8.24 = 11.653 = 11.7A

(b) Potential drop across L is $V_{rms}^{L} = I_{rms} \times \omega L$ = 8.24×25.13 = 207V

Potential drop across C is $V_{rms}^{C} = I_{rms} \times 1/\omega C$ = 8.24 × 53.05 = 437V

(c) In an inductor voltage leads the current by π/2, therefore average power transferred to the inductor per cycle is $P_{av} = V_{rms}I_{rms} \cos \pi/2 = 0$

(d) In the capacitor, voltage lags behind the current by π/2, therefore, average power transferred to the capacitor per cycle is $P_{av} = V_{rms}I_{rms} \cos\left(-\frac{\pi}{2}\right) = 0$

(e) Total average poer absorbed = 0.

41. Suppose the circuit in previous question has a resistance of 15 **Ω**. Obtain the average power transferred to each element of the circuit and the total power absorbed.

Sol. Here R = 15Ω,

Therefore Impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$= \sqrt{15^2 + \left(2\pi \times 50 \times 80 \times 10^{-3} - \frac{1}{2\pi \times 50 \times 60 \times 10^{-6}}\right)^2}$

$= \sqrt{225 + 779.5} = \sqrt{1004.5}$ = 31.7Ω

Therefore $I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{31.7}$ = 7.255A

Average power transferred to L = $V_{eff}I_{eff} \cos\frac{\pi}{2} = 0$

Average power transferred to C = $V_{eff}I_{eff}\cos\left(-\frac{\pi}{2}\right) = 0$

Average power transferred to R = $I_{rms}^2 = (7.255)^2 \times 15$ = 789.5W

F. TRANSFORMER

(1 Mark Questions)

1. Laminated iron sheets are used to minimize ______ currents in the core of a transformer.

Ans. Laminated iron sheets are used to minimize eddy currents in the core of a transformer.

2. What is the function of a step up transformer?

Sol. A step up transformer is used to convert a low voltage at high current into a high voltage at low current.

3. The output of a step-down transformer is measured to be 24 V when connected to a 12 watt light bulb. The value of the peak current is

(a) $1/\sqrt{2}$ A. (b) $\sqrt{2}$ A. (c) 2 A. (d) $2\sqrt{2}$ A

Sol. (a)

As given that, secondary voltage (V_S) is V_S = 24 volt

Power associated with secondary is P_S = 12 watt

As we know that $P_S = V_S I_S$

$I_S = P_S/V_S$ = 12/24 = ½ A = 0.5 Amp

Peak value of the current in the secondary

$I_0 = I_S\sqrt{2} = 0.5\sqrt{2} = 5/10.\sqrt{2}$ [$I_0 = 1/\sqrt{2}$ Amp]

(2 Marks Questions)

4. State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers?

Sol. A transformer is based on principle of mutual induction which states that due to continuous change in the current in the primary coil, an emf gets induced across the secondary coil.

Electric power generated at the power station is stepped up to very high voltages by means of a step up transformer and transmitted to a distinct place. At receiving end, it is stepped down by a step down transformer.

5. A transformer has 300 primary turns and 2400 secondary turns. If the primary supply voltage is 230V, what is the secondary voltage?

Sol. $\varepsilon_2 = \frac{N_2}{N_1}.\varepsilon_1 = \frac{2400}{300} \times 230$ = 1840V = 1.84kV.

6. What are the various energy losses in a transformer? How can they be reduced?

Sol. The main causes for energy loss in transformers are as follows:

1. Copper loss: Some energy is lost due to heating of copper wires used in the primary and secondary windings. The power loss (= I^2R) can be minimized by using thick copper wires of low resistance.

2. Eddy current loss: The alternating magnetic flux induces eddy currents in the iron core which leads to some energy loss in the form of heat. Thus loss can be reduced by using laminated iron core.

3. Hysteresis loss: The alternating current carries the iron core through cycles of magnetization and demagnetization. Work is done in each of these cycles and is lost as heat. This is called hysteresis loss and can be minimized by using core material having narrow hysteresis loop.

4. Flux leakage: The magnetic flux produced by the primary may not fully pass through the secondary. Some of the flux may leak into air. This los can be minimized by winding the primary and secondary coils over one another.

5. Humming loss: As the transformer works, the core lengthens and shortens each cycle of the alternating voltage due to a phenomenon called magnetostriction. This gives rise to a humming sound. So, some of the electrical energy is lost in the form of humming sound.

(3 Marks Questions)

7. Give two disadvantages of transmitting a.c. over long distances at low voltage and high current.

Sol. Following are the two disadvantages of transmitting electrical power at low voltage:

(1) Large lengths of transmission cables have sufficient resistance. Hence a large amount of energy (I^2Rt) will be lost as heat during transmission.

(2) Large voltage drop (IR) occurs along the line wire. Hence the voltage at the receiving station will be much smaller than that at the generating station.

8. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

Sol. Here ε_1 = 2300V, N_1 = 4000, ε_2 = 230V, N_2 = ?

As $\frac{\varepsilon_2}{\varepsilon_1} = \frac{N_2}{N_1}$

Therefore, $N_2 = N_1\frac{\varepsilon_2}{\varepsilon_1} = 4000 \times \frac{230}{2300}$ = 400 turns

9. At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is 100 m^3s^{-1}. If the turbine generator efficiency is 60%, estimate the electric power available from the plant (g = 9.8 ms^{-2}).

Sol. Hydroelectric power = $\frac{\text{Work}}{\text{Time}} = \frac{\text{force}\times\text{distance}}{\text{Time}}$ = Pressure × area × velocity

= $h \rho g \times A \times v = h \rho g \times \beta$

Where $\beta = Av$ = volume of wter flowing per second across a cross section.

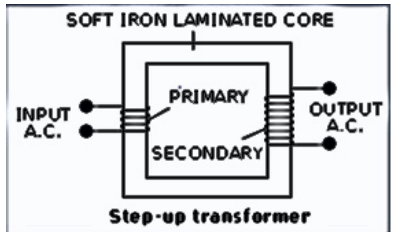
Electric power available = 60% of total hydroelectric power = 0.6 $h\rho g\beta$

= $0.6\times300\times10^3\times9.8\times100$W = 176.4×10^6W = 176 MW

(5 Marks Questions)

10. With the help of a labeled diagram, explain the working of a step up transformer. Give reasons to explain the following: (i) the core of the transformers is laminated (ii) thick copper wire is used in windings.

Sol.



Step up transformer (or transformer) is based on the principle of mutual induction.
An alternating potential (Vp) when applied to the primary coil induced an emf in it.

$\varepsilon_p = -N_p\frac{d\phi}{dt}$

If resistance of primary coil is low, $V_p = \varepsilon_p$, i.e $V_p = -N_p\frac{d\phi}{dt}$

At the same flux is linked with the secondary coil with the help of soft iron core due to mutual induction, emf is induced in it

$\varepsilon_s = -N_s\frac{d\phi}{dt}$

If output circuit is open $V_s = \varepsilon_s$

$V_s = -N_s\frac{d\phi}{dt}$

Thus $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

For an ideal transformer, $P_{out} = P_{in} \Rightarrow I_sV_s = I_pV_p$

$\therefore \frac{V_s}{V_p} = \frac{I_p}{I_s} = \frac{N_s}{N_p}$

For step up transformer, $\frac{N_s}{N_p} > 1$

In case of dc voltage, flux does not change. Thus no emf is induced in the circuit.
(i) The core of the transformer is laminated to reduce eddy current losses.
(ii) Thick copper wire is used in windings of transformers because of its low resistivity i.e. low resistance.

11. (a) Draw a labeled diagram of a step up transformer. Obtain the ratio of secondary to primary voltage in terms of number of turns and currents in the two coils. (b) A power transmission line feeds input power at 2200V to a step down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary to get the power output to 220V.

Sol. (a) Same as Q 11
(b) $N_p = 3000$, $V_p = 2200V$, $V_s = 220V$, $N_s = ?$

As $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ or $N_s = \frac{N_pV_s}{V_p} = \frac{3000\times220}{2200} = 300$.

G. CASE STUDY

1. 1. In essence of the simplest tuned radio frequency receiver is a simple crystal set. Desired frequency is tuned by a tuned coil/ capacitor combination, and then the signal is presented to a simple crystal or diode detector where the amplitude modulated signal, is demodulated. This is then passed straight to the headphones or speaker. In radio set there is an LC oscillator comprising of a variable capacitor (or sometimes a variable coupling coil) with a knob on the front panel to tune the receiver.

Capacitors used in old radio sets is gang capacitor. It consists of two sets of parallel circular plats one of which can rotate manually by means of a knob. The rotation causes overlapping areas of plats to change, thus changing its capacitance. Air gap between plates acts as dielectric.

The capacitor has to be tuned in tandem corresponding to the frequency of a station so that the LC combination of the radio set resonates at the frequency of the desired station.

When capacitive resistance (X_C) is equal to the inductive reactance (X_L), then the resonance occurs and the resonant frequency is given by $\omega_0 = \frac{1}{\sqrt{LC}}$ current amplitude becomes maximum at the resonant frequency. It is important to note that resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltages across L and C cancel each other (both being out of phase) and the Current amplitude is $\frac{V_m}{R}$, the total source voltage appearing across R.

This means that we cannot have resonance in a RL or RC circuit.

(i) Name the phenomenon involved in the tuning a radio set to a particular radio station.

(a) Stabilization (b) Rectification (c) Resonance (d) Reflection

Ans. (c)

Phenomenon involved in tuning a radio set to a particular radio station is resonance. The capacitor has to be tuned in tandem corresponding to the frequency of a station. So, that the LC combination of the radio set resonance at the frequency of the desired station.

(ii) Resonance may occur in

(a) RL circuit (b) RC circuit (c) LC circuit (d) circuit having resistor only

Ans. (c)

A simple radio receiver is a simple crystal set with a coil and capacitor combination. Desired frequency is tuned by tuning the coil capacitor combination. Tuning means to make capacitive reactance (X_C) equal to the inductance reactance (X_L), so that the resonance occurs.

(iii) Resonance frequency is equal to

(a) $\frac{1}{LC}$ (b) $\frac{1}{\sqrt{LC}}$ (c) $\sqrt{\frac{L}{C}}$ (d) $\sqrt{\frac{C}{L}}$

Ans. (b)

The resonant frequency is given by $\omega_0 = \frac{1}{\sqrt{LC}}$

(iv) Resonance occurs only when

(a) $X_C = R$ (b) $X_L = R$ (c) $X_L = X_C$ (d) $X_C > X_L$

Ans. (c)

At resonance, capacitive reactance (X_C) is equal to the inductive reactance (X_L). Circuit is totally resistive and the current amplitude becomes maximum.

(v) Capacitor used in radio set for tuning is a

(a) parallel plate capacitor (b) spherical capacitor

(c) paper capacitor (d) electrolytic capacitor

Ans. (a)

Capacitors used in old radio sets is gang capacitor. It consists of two sets of parallel circular plats one of which can rotate manually by means of a knob. The rotation causes overlapping areas of plats to change, thus changing its capacitance.

2. At power plant, a transformer increases eh voltage of generated power by thousands of volts so that it can be sent of long distances through high voltage transmission power lines. Transmission lines are bundles of wires that carry electric power from power plants to distant substations.

At substations, transformers lower the voltage of incoming power to make it acceptable for high volume delivery to nearby end users.

Electricity is sent at extremely high voltage because it limits so called line losses, Very good conductors of electricity also offer some resistance and this resistance becomes considerable over long distances causing considerable loss.

At generating station, normally voltage is stepped up to around thousands of volts. Power losses increase with the square of current. Therefore, keeping voltage high current becomes low and the loss is minimized.

Another option of minimizing loss is the use of wires of super conducting material. Super conducting materials are capable of conducting without resistance, they must be kept extremely cold, nearly absolute zero, and this requirement makes standard super conducting materials impractical to use. However, recent advances in super conducting matrials have decreased cooling requirement. In Germany recently 1knm super conducting cable have been installed connecting the generating station and the destination. It has eliminated the line loss and the cable is capable of sending five times more electricity than conventional cable. Using super conducting cables Germany has also get rid of the need of costly transformers.

Transformers generate waste heat when they are in operation and oil is the coolant of choice. It transfers the heat through convection to the transformer housing, which has cooling fins or radiator similar to heat exchangers on the outside.

Flush point is a very important parameter of transformer oil. Flash point of an oil is the temperature at which the coil ignites spontaneously. This must be as high as possible (not less than 160°C from the point of safety).

Fire point is the temperature at which the oil flashes and continuously burns. This must be very high for the chosen oil (not less than 200°C).

(i) Which of the following statement is true for long distance transmission of electricity?

(a) Step down transformer is used at generating station and step up transformer is used at destination substation.

(b) Step down transformers are used at generating station and destination substation.

(c) Step up transformers are sued at generating station and destination substation.

(d) None of the above.

Ans. (d)

At power plant, a transformer increases eh voltage of generated power by thousands of volts so that it can be sent of long distances through high voltage transmission power lines.

At substations, transformers lower the voltage of incoming power to make it acceptable for high volume delivery to nearby end users.

(ii) Super conducting transmission line has the following disadvantages:

(a) Resistance being zero, there in not PR loss.

(b) There is no requirement of costly step up and step down transformers.

(c) Cable is capable of sending more electricity.

(d) All of the above.

Ans. (d)

Super conducting materials are capable of conducting without resistance. So, this eliminates the line loss and the cable is capable of sending more electricity than conventional cable. Using super conducting cables, one can get rid of the need of costly transformers

(iii) Why does stepping up voltages reduce power loss?

(a) Since resistance of conductor decreases with increase in voltage.

(b) Since current decreases with increase of voltage

(c) Both of the above (d) None of the above

Ans. (b)

At generating station, normally voltage is stepped up to around thousands of volts. Power losses increase with the square of current. Therefore, keeping voltage high current becomes low and the loss is minimized.

(iv) Oil transfers heat from transformer winding by the process of

(a) convection (b) conduction (c) radiation (d) all of these

Ans. (a)

Transformers generate waste heat when they are in operation and oil is the coolant of choice. It transfers the heat through convection to the transformer housing.

(v) Flush point of an oil is

(a) the temperature at which the oil flashes and continuously biurns.

(b) The temperature at which the oil ignites spontaneously

(c) the temperature at which the oil starts boiling.
(d) the temperature at which the oil forms fumes.

Ans. (b)
Flush point is a very important parameter of transformer oil. Flash point of an oil is the temperature at which the coil ignites spontaneously. This must be as high as possible (not less than 160°C from the point of safety).

H. ASSERTION REASON TYPE QUESTIONS:

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true.

1. Assertion: An electric lamp connected in series with a variable capacitor and A.C. source, its brightness increases with increase in capacitance.
Reason: Capacitive resistance decreases with increase in capacitance of capacitor.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
Capacitive resistance $X_{C=}= \frac{1}{\omega C}$. When capacitance (C) increases, the capacitive reactance decreases. Due to decrease in its values, the current in the circuit will increase $\left(I = \frac{E}{\sqrt{R^2+X^2}}\right)$ and hence brightness of source (or electric lamp) will also increases.

2. Assertion: Choke coil is preferred over a resistor to adjust current in an A.C. circuit.
Reason: Power factor for inductance is zero.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
If resistor is used in controlling AC supply, electric energy will be wasted in the form of heat energy across the resistance wire. However, AC supply can be controlled with choke without any wastage of energy. This is because power factor (cos θ) for resistance is one an it is zero for an inductance.

3. Assertion: The core of transformer is made laminated in order to increase the eddy currents.
Reason: The sensitivity of transformer increases with increase in eddy current.

Ans. (d) Both assertion and reason are false.
Eddy currents is produced in the iron core due to induced emf since resistance of the iron core is quite small, the magnitude of eddy currents is quite large. As a result, large amount of heat is produced. To avoid it a laminated core is used in transformer. In laminated core iron stripes are quite thin and each strip possesses very large resistance, the magnitude of eddy currents produced is quite small and hence only a small amount of heat is produced.

4. Assertion: The working of dynamo is based on the principle of self induction.
Reason: Self induction of a coil is numerically equal to the magnetic flux linked to the coil, when a unit current flow through it.

Ans. (e) Assertion is false but reason is true.

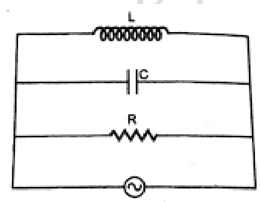
In a dynamo, a a coil is rotated with a fixed frequency in a given magnetic field. As a result of this, magnetic flux linked with the coil changes continuously with respect to time at a constant rate and therefore induced current is produced continuously in the coil. The dynamo is based on the principle of electromagnetic induction.

5. Assertion: A capacitor suitable capacitance can be used in an A.C. circuit in place of the choke coil.
Reason: A capacitor blocks D.C. and allows A.C. only.
Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.
We can use a capacitor of suitable capacitance as a choke coil, because average power consumed in an ideal capacitor is zero. Therefore, like a choke coil, a condenser can reduce A.C. without power dissipation.

I. CHALLENGING PROBLEMS

1. Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L, C and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements of frequency. Source has emf 230 V and L = 5.0 H, C = 80 μF, ff = 40 Ω.

Sol. The effective impedance of the parallel LCR combination is given by



$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{\frac{1}{j\omega C}}$$

$$\text{Or } \frac{1}{Z} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$\text{Or} \frac{1}{|Z|} = \sqrt{\frac{1}{R^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

Where |Z| is the modulus of the complex impedance Z. Obviously 1/|Z| is minimum when $\omega = \omega_r$ when ωc = 1/ωL', so that |Z| is minimum and the total current amplitude is minimum. Hence at resonance the current in the parallel LCXR circuit is minimum.

At resonance, Z = R, for total current in the circuit, $I_{rms} = \frac{V_{rms}}{R} = \frac{230}{40} = 5.75A$

The rms current in the R branch is $I_{rms}{}^{R} = \frac{V_{rms}}{R} = \frac{230}{40} = 5.75A$

The rms current in the L branch is $I_{rms}{}^{L} = \frac{V_{rms}}{\omega_{rL}} = \frac{230}{50\times5.0} = 0.92A$

The rms current in the C branch is $I_{rms}^{C} = \frac{V_{rms}}{I/\omega_{rC}} = V_{rms} \times \omega_{rC} = 230\times50\times80\times10^{-6}A =$ 0.92A

Not the a the total current in the circuit is the same as that in the R branch. This is because the currents in L and C branches are 180° out of phase and add up to zero at every instant of the cycle.

2. Obtain the resonant frequency and Q-factor of a series LCR circuit with L = 3.0 H, C= 27 μF, and R = 7.4 fl. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Sol. Here L = 3.0H, C = 27 μF = 27 × 10^{-6}F, R = 7.4 Ω

Resonant frequency, $\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3.0\times27\times10^{-6}}}$ = 111 rad s^{-1}

Q factor of the circuit, $Q = \frac{\omega_r L}{R} = \frac{111\times3.0}{7.4}$ = 45

To improve sharpness of resonance by a factor of 2, Q should be doubled. To double Q without changing ω_r, R should be reduced to half i.e. to 3.7Ω.

3. A series LCR circuit with L = 0.12 H, C = 480 μF, R = 23 Ω is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies?

(d) What is the Q-factor of the given circuit?

Sol. Here L = 0.12H, C = 480nF = 480 × 10^{-9}F, R = 23Ω, V_{rms} = 230V

(a) Current amplitude is maximum at resonant angular frequency

$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.12\times480\times10^{-9}}}$rad s^{-1} = 4167 rad s^{-1}

Resonant frequency $f_r = \frac{\omega_r}{2\pi} = \frac{4167}{2\pi}$ = 663 Hz

The maximum value of current amplitude is $I_0^{max} = \frac{V_0}{R} = \frac{\sqrt{2}\times230}{23}$ = 1.41A

(b) The power absorbed in maximum at the same resonant frequency (663Hz) for which I_0 is maximum.

Therefore $P_{av}^{max} = \frac{1}{2}(I_0^{max})^2 R = \frac{1}{2}\frac{V_0^2}{R} = \frac{1}{2}.\frac{(\sqrt{2}\times230)^2}{23}$ = 2300W

(c) The two angular frequencies for which the power transferred to the circuit is half the power at the resonant frequency, are $\omega = \omega_r + \Delta\omega = \omega_r \pm \frac{R}{L}$

The corresponding frequencies will be $f = f_r + \Delta f = f_r + \frac{\Delta\omega}{2\pi}$

Now $\frac{\Delta\omega}{2\pi} = \frac{1}{2\pi}\times\frac{R}{2L} = \frac{1}{2\pi}\times\frac{23}{2\times0.12}$ = 15.25Hz = 15 Hz

Therefore required values of f = 663 ± 15 648 Hz or 678 Hz

At these frequencies, power absorbed = ½ P_{max}. As $P \propto I^2$, the current amplitude oat these half power points $= \frac{1}{\sqrt{2}} I_0^{max} = 1\frac{4.1}{\sqrt{2}}$ = 9.97A = 10A

(d) The Q factor of the circuit is, $Q = \frac{\omega_r L}{R} = \frac{4167\times0.12}{23}$ = 21.7.

4. A small town with a demand of 800 kW of 1 electric power at 220 V is situated 15 km away from an electric plant generating power at 440V. The resistance of the two wire line carrying power is 0.5 Q per km. The town gets 1 power from the line through a 4000-220 V step- down transformer at a sub station in the town.
(a) Estimate the line power loss in the form of heat.
(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?
(c) Characterize the step up transformer at the plant.

Sol. Line resistance = Length of two wire line × Resistance per unit length
$= 2 \times 15\text{km} \times 0.5\ \Omega\ \text{km}^{-1} = 15\Omega$
Voltage at which power is sent through the line = 4000V
Power supplied to town substation = 800 kW = 800×10^3W
So rms value of current in the line = $\frac{\text{Power}}{\text{Voltage}} = \frac{800\times10^3}{4000}$A = 200A
(a) Line power loss = $I^2R = (200)^2 \times 15\text{W} = 600$ kW
(b) Power supplied by the plant = Power received at substation + line power loss
= 800+ 600 = 1400 kW
(c) Voltage drop on the line = IR = 200 × 15 = 3000V
Voltage output of the step up transformer at the plant = 4000+ 3000 = 7000V
Hence the step up transformer at the plant is 440 – 7000 V

5. Repeat the same exercise as in the previous question with the replacement of the earlier transformer by a 40,000-220 V step down transformer. (Neglect, as before, leakage losses through this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?

Sol. The rms current in the two wire line = $\frac{800\times10^3\text{W}}{40{,}000\text{V}}$ = 20A
(a) Line power loss = $I^2 R = (20)^2 \times 15 = 6000\text{W} = 6$ kW
(b) Power supplied by the plant = 800 + 6 = 806 kW
(c) Voltage drop on the line = IR = 20 × 15 = 300V
Voltage output of the step up transformer at the plant = 40,000 + 300 = 40, 300V
Therefore, the step up transformer at the plane is 440V – 40,300V.
Power loss in last question = $\frac{600}{1400} \times 100$ = 43%
Power loss in this question, $\frac{6}{806} \times 100$ = 0.74%
Thus the percentage power loss is greatly reduced by high voltage transmission. At high voltage transmission, a small current flows and hence power loss is less ($P \propto I^2$).

SPACE FOR ROUGH WORK

SPACE FOR NOTES