CLASS – 11

WORKSHEET- GRAVITATION

A. KEPLER'S LAW

(1 Mark Questions)

1. Kepler's second law is a consequence of
   (a) conservation of energy (b) conservation of linear momentum
   (c) conservation of angular momentum (d) conservation of mass

Ans. (c)

2. When will the Kepler's law be applicable on the planets?

Sol. Kepler's first law means that planets move around the Sun in elliptical orbits. An ellipse is a shape that resembles a flattened circle. How much the circle is flattened is expressed by its eccentricity. The eccentricity is a number between 0 and 1.

(3 Marks Questions)

3. State and explain the Kepler's laws of planetary motion.

Sol. In astronomy, **Kepler's laws of planetary motion**, published by Johannes Kepler between 1609 and 1619, describe the orbits of planets around the Sun. The laws modified the heliocentric theory of Nicolaus Copernicus, replacing its circular orbits and epicycles with elliptical trajectories, and explaining how planetary velocities vary. The three laws state that:

1. The orbit of a planet is an ellipse with the Sun at one of the two foci.
2. A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time.
3. The square of a planet's orbital period is proportional to the cube of the length of the semi-major axis of its orbit.

The elliptical orbits of planets were indicated by calculations of the orbit of Mars. From this, Kepler inferred that other bodies in the Solar System, including those farther away from the Sun, also have elliptical orbits. The second law helps to establish that when a planet is closer to the Sun, it travels faster. The third law expresses that the farther a planet is from the Sun, the slower its orbital speed, and vice versa.

4. State and explain Kepler's laws of planetary motion. Name the physical quantities which remain constant during the planetary motion.

Sol. Same as 3. The physical quantities that remain constant during the planetary motion are angular momentum, mass and total energy.

5. (a) According to Kepler's second law, the radius vector to a planet from the sun sweeps out equal areas in equal interval and time. The law is consequence of which conservation law.

(b) State Kepler's third law.

Sol. (a) According to Kepler's second law, the radial vector to a planet from the Sun sweeps out equal areas in equal intervals of time. This law is a consequence of conservation of angular momentum.

(b) Kepler's III Law (Law of Periods) : The square of the time period of any planet about the Sun is proportional to the cube of the semi-major axis of the elliptical orbit.

B. NEWTON'S LAW OF GRAVITATION

(1 Mark Questions)

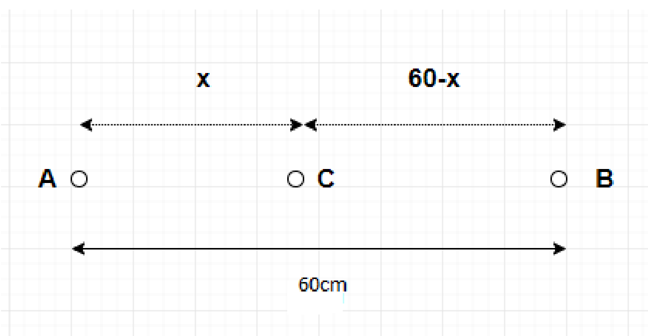
1. What are the dimensions of gravitational constant?

Sol. The gravitational constant is dimensionally represented as $M^{-1} L^3 T^{-2}$.

(2 Marks Questions)

2. Two bodies of masses 4kg and 9kg are separated by a distance of 60cm. A 1kg mass is placed in between these two masses. What is its distance from 4kg mass, if the net force on 1kg is zero?

Sol.



As per question, the net force on 1kg mass is equal to zero

Therefore, $G\frac{4\times1}{x^3} = G\frac{9\times1}{(60-x)^2}$ or $\frac{2}{3} = \frac{x}{(60-x)}$ or x = 24cm.

3. Calculate the force of attraction between two balls each of mass 1kg, when their centres are 10cm apart. Given $G = 6.67\times10^{-11}$ $Nm^2\ kg^{-2}$.

Sol. Here $m_1 = m_2 = 1kg$, r = 10cm = 0.10m

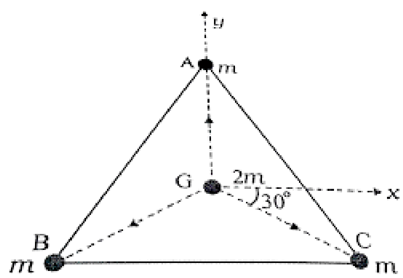
$\therefore F = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times1\times1}{(0.10)^2} = 6.67\times10^{-9}N.$

4. State Newton's law of gravitation. Hence define universal gravitational constant. Give the value and dimensions of G.

Sol. Newton's law of universal gravitation:- "According to this law each particle in the universe attracts every other particle in the universe. The force of attraction between them is directly proportional to the square of the distance between them". Consider two particles of masses m1 and m2 separated by a distance r. The force of attraction between them: F ∝ m1m2/r2 or, F = Gm1m2/r2 Where G = 6.67 x 10-11 Nm2 kg-2, is the universal gravitational constant.

(5 Marks Questions)

5. Three equal mass of m kg each are fixed at the vertices of an equilateral triangle ABC as shown in figure.

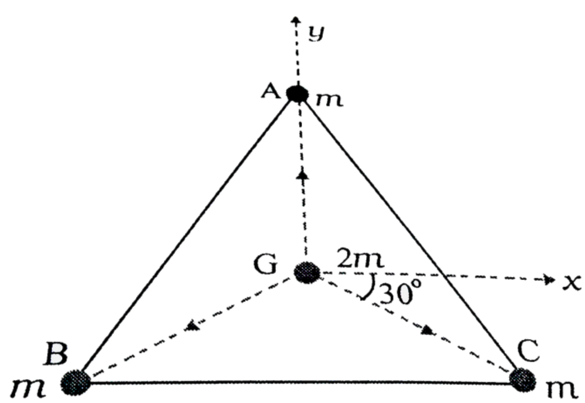


(a) What is the force acting on a mass 2m placed at the centroid G of the triangle?

(b) What is the force if the mass at the vertex A is doubled?

Take AG = BG = CG = 1cm.

Sol. We choose the coordinate axes as shown in figure.



(a) Angle between GC and positive x axis = 30°

Angle between GB and negative x axis = 30°

Force on mass at G due to mass at A = $\vec{F}_{GA} = \frac{Gm(2m)}{1^2}\hat{j}$

Force on mass at G due to mass at B = $\vec{F}_{GB} = \frac{Gm(2m)}{1^2}(-\hat{i}\cos30° - \hat{j}\sin30°)$

Force on mass at G due to mass at C = $\vec{F}_{GC} = \frac{Gn(2m)}{1^2}(+\hat{i}\cos30° - \hat{j}\sin30°)$

By the principle of superposition, the resultant force $\vec{F}_R$ om mass 2m is

$\vec{F}_R = \vec{F}_{GA} + \vec{F}_{GB} + \vec{F}_{GC}$

$= 2Gm^2[\hat{j} + (-\hat{i}\cos 30° - \hat{j}\sin 30°) + (+\hat{i}\cos 30° - \hat{j}\sin 30°)]$
$= 2Gm^2[\hat{j} - 2\hat{j}\ \sin 30°] = 2Gm^2(\hat{j} - \hat{j}) = 0$
The resultant force on mass 2m = 0.
(b) Now the mass at the vertex A is 2m. Then
$\vec{F}_{GA} = \frac{Gm(2m)(2m)}{1^2}\hat{j} = 4Gm^2\hat{j}$
$\vec{F}_{GB}$ and $\vec{F}_{GC}$ remain same as above.
Therefore $\vec{F}_R = \vec{F}_{GA} + \vec{F}_{GB} + \vec{F}_{GC}$
$= 2Gm^2[2\hat{j} + (-\hat{i}\cos 30° - \hat{j}\sin 30°) + (+\hat{i}\cos 30° - \hat{j}\sin 30°)]$
$= 2Gm^2[\hat{j} - 2\hat{j}\ \sin 30°] = 2Gm^2(2\hat{j} - \hat{j}) = 2Gm^2\hat{j}$

C. GRAVITATIONAL FIELD AND POTENTIAL

(1 Mark Questions)

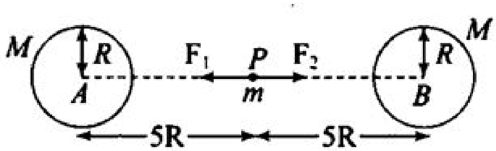
1. Is it possible to shield a body from gravitational effects?

Sol. No, it is not possible to shield a body from gravitational effect because the gravitational force between any two bodies is independent of the intervening medium or the presence of other bodies in their way.

(3 Marks Questions)

2. Two identical heavy spheres are separated by a distance 10 times their radii. Determine the potential at the midpoint. Is an object placed at that point in stable or unstable equilibrium?

Sol. Situation is shown in figure



P is the midpoint of AB.
$V_P = -\frac{GM}{5R} - \frac{GM}{5R} = \frac{-2}{5}\frac{GM}{R}$
The object placed at midpoint is in unstable equilibrium as once displaced from that point, it will not come back to that point.

3. Two bodies of mass 10kg and 1000kg are at a distance 1m apart. At which point on the line joining them will the gravitational field intensity be zero?

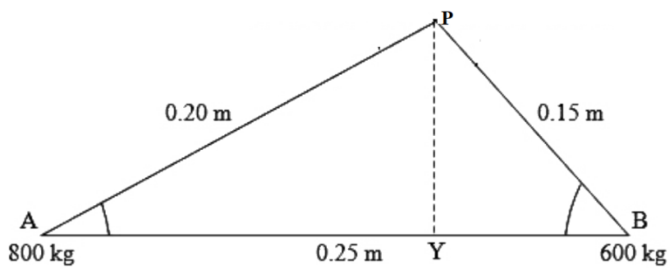
Sol. Let the resultant gravitational intensity be zero at a distance x from the mass of 10jkg on the line joining the centres of the two bodies. At this point, the gravitational intensities due to the two bodies must be equal and opposite.

$\therefore \frac{G\times10}{x^2} = \frac{G\times1000}{(1-x)^2}$

Or $100x^2 = (1 - x)^2$ or $10x = 1 - x$ or $11x = 1$ or $x = 1/11$m.

4. Two masses, 800kg and 600kg are at a distance 0.25m apart. Compute the magnitude of the intensity of the gravitational field at a point distant 0.20m from the 800kg mass and 0.15m from the 600kg mass.

Sol.



Let A and B be the positions of the two masses and P the point at which the intensity of the gravitational field is to be computed.

Gravitational intensity at P due to mass at A, $E_A = \frac{GM}{r^2} = G\frac{800}{(0.20)^2}$ = 20000G along PA

Gravitational intensity at P due to mass at B, $E_b = G\frac{600}{(0.15)^2} = \frac{80000}{3}$ G along PB

In ΔAPB, $PA^2 + PB^2 = AB^2$, so ∠APB = 90°.

Hence the magnitude of resultant gravitational intensity at P is

$E = \sqrt{E_A^2 + E_B^2} = G\sqrt{(20000)^2 + \left(\frac{80000}{3}\right)^2} = 6.66 \times 10^{-11} \times \frac{10000}{3} = 2.22 \times 10^{-6}$N.

D. ACCELERATION DUE TO GRAVITY

(1 Mark Questions)

1. What would be the weight of the body inside the earth if it were a hollow sphere?

Sol. The value of g is equal to zero in a hollow sphere, hence, the weight (mg) will also be zero.

2. Why does the weight of a body become zero at the centre of the earth?

Sol. The weight of a body at the centre of earth is zero because value of g is zero. As we move a body closer to the centre of the earth, the mass of the earth between the centre of the earth and the body keeps decreasing. This causes the force acting from the centre of the earth on the body to decrease.

3. The mass and diameter of the planet are twice those of the earth. What will be the time period of the pendulum on this planet, which is a second's pendulum on the earth?

Sol. Initially $g = Gm/r^2$ and $T = 2\pi\sqrt{\frac{l}{g}}=2s$ (for a second's pendulum)

When M and R are doubled, $g' = \frac{G(2M)}{(2R)^2} = \frac{g}{2}$ and $T' = 2\pi\sqrt{\frac{l}{g/2}} = \sqrt{2}T = 2\sqrt{2}s$.

4. If the change in the value of 'g' at a height 'h' above the surface of the earth is same as that at a depth 'x' below it (both x and h being much smaller than the radius of earth), then how are x and h related to each other?

Sol. Acceleration due to gravity at height h, $g_h = g\left(1 - \frac{2h}{R}\right)$

Acceleration due to gravity at depth d, $g_x = g\left(1 - \frac{x}{R}\right)$

So, $g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{x}{R}\right)$ or $\frac{2h}{R} = \frac{x}{R}$ or $x = 2h$

5. If the radius of the earth shrinks by 1%, its mass remaining the same by what percentage will the acceleration due to gravity on its surface change?

Sol. If the radius of the earth were to shrink by one percent, mass remaining the same, the value of g will increase by 2%.

6. What is weightlessness?

Sol. Weightlessness is condition experienced while in free-fall, in which the effect of gravity is canceled by the inertial (e.g., centrifugal) force resulting from orbital flight. The term *zero gravity* is often used to describe such a condition.

(2 Marks Questions)

7. A person sitting in a satellite of Earth feels weightlessness but a person standing on Moon has weight though Moon is also a satellite of Earth. Why?

Sol. When a person is sitting in an artificial satellite of earth, the gravitational attraction on him due to earth (i.e. his weight n earth) provides the necessary centripetal force. Since the net force acting on him is zero, the person feels weightless. But when he is standing on moon, the gravitational attraction acting on him due to moon is left unbalanced, which accounts for his weight on moon.

8. The acceleration due to gravity at the moon's surface is 1.67 ms^{-2}. If the radius of the moon is 1.74×10^6m, calculate the mass of the moon. Use the known value of G.

Sol. Here $G = 6.67\times10^{-11}\ Nm^2kg^{-2}$, $g = 1.67\ ms^{-2}$, $R = 1.74\times10^6$m

$M = \frac{gR^2}{G} = \frac{1.67\times(1.74\times10^6)^2}{6.67\times10^{-11}} = 7.58\times10^{22}$ kg

9. Prove that acceleration due to gravity on the surface of the earth is given by g = 4/3 π ρ GR, where G is gravitational constant, ρ is mean density and R is the radius of the earth.

Sol. Mass of the earth, $M = \frac{4}{3}\pi R^3 \rho$

Acceleration due to gravity on the earth's surface,

$g = \frac{GM}{R^2} = \frac{G}{R^2} \times \frac{4}{3}\pi R^3 \rho = \frac{4}{3}\pi\rho GR.$

10. Why is the weight of the body at the poles more than the weight at the equator? Explain.

Sol. As $g = GM/R^2$ and the value of R at the poles is less than that at the equator, so g at poles is greater than g at the equator. Now as $g_p > g_e$. Hence $mg_p > mg_e$ i.e., the weight of a body at the poles is more than the weight at the equator.

11. Define acceleration due to gravity. Show that the value of 'g' decreases with altitude or height.

Sol. The value of acceleration due to gravity at a height 'h' above the ground gh is given by , gh=g[1−Re2h] where , Re is radius of the Earth . Therefore gh decreases with an increasing in height.

(3 Marks Questions)

12. How much above the earth surface does the acceleration due to gravity reduces by 36% if its value on the earth surface? Take the value of radius of earth 6400km.

Sol. As we know that acceleration due to gravity at height h from the earth surface

$g' = g\left(1 - \frac{2h}{R}\right)$

From question, f' = 36% of g = 0.36g, R = 6400km = 6400×10^3m

Therefore, $0.36g = g\left(1 - \frac{2h}{R}\right)$

$\Rightarrow 0.36 = 1 - \frac{2h}{R} \Rightarrow \frac{2h}{R} = 1 - 0.36 = 0.64$

$\Rightarrow h = 0.32R \Rightarrow h = 0.32\times6400\times10^3$

$H = 2048\times10^3$m = 2048km.

13. If the earth were made of lead of relative density 11.3, what then would be the value of acceleration due to gravity on the surface of the earth? Radius of the earth = 6.4×10^6m and $G = 6.67\times10^{-11}$ $Nm^2\ kg^{-2}$.

Sol. Density of the earth, ρ = Relative density × density of water = 11.3×10^3 kgm^{-3}

Acceleration due to gravity on the earth's surface,

$g = \frac{GM}{R^2} = \frac{G}{R^2} . \frac{4}{3}\pi R^3 \times \rho = \frac{4}{3}\pi G R \rho$

$= \frac{4}{3} \times \frac{22}{7} \times 6.67 \times 10^{-11} \times 6.4 \times 10^{6} \times 11.3 \times 10^{3} = 22.21\ ms^{-2}$

14. If the radius of the earth shrinks by 2.0%, mass remaining constant, then how would the value of acceleration due to gravity change?

Sol. Acceleration due to gravity on the surface of the earth is given by $g = \frac{GM}{R^2}$

Taking logarithm of both sides, we get, log g = log G + log M – 2 log R

As G and M are constants, so differentiation of the above equation gives

$\frac{dg}{g} = 0 + 1 - 2\frac{dR}{R}$

As radius of the earth decreases by 2%, so $\frac{dR}{R} = -\frac{2}{100}$

$\frac{dg}{g} \times 100 = -2\frac{dR}{R} \times 100 = -2 \times \left(-\frac{2}{100}\right) \times 100 = 4\%$

Thus the value of g increases by 4%.

15. A body weighs 90kg f on the surface of the earth. How much will it weight on the surface of Mars whose mass is 1/9 and the radius is ½ of that of the earth?

Sol. Weight of body on earth = 90kg, so, mass of body (m) = 90kg

Mass of mars = 1/9m, Radius = ½ of that of earth

Therefore, $F = mg = m\frac{GM}{R_e^2}$

$\frac{F_{mars}}{F_{earth}} = \frac{m[\frac{GM}{R_e^2}]mars}{m[\frac{GM}{R_e^2}]earth} \Rightarrow \frac{F_{mars}}{90kg} = \frac{(\frac{M_{earth}}{9})}{M_{earth}} \times \frac{(R_{earth})^2}{(\frac{R_{earth}}{2})^2} = \frac{4}{9}$

$F_{mars} = 90 \times \frac{4}{9} = 40kg.$

16. Determine the speed with which the earth would have to rotate on its axis so that a person on the equator would weigh $3/5^{th}$ as much as a t present. Take the equatorial radius as 6400 km.

Sol. Acceleration due to gravity at the equator is $g_e = g - R\omega^2$

Therefore $mg_e = mg - mR\omega^2$

Or $\frac{3}{5}mg = mg = mR\omega^2 \quad \left[\because mg_e = \frac{3}{5}mg\right]$

$\therefore \omega = \sqrt{\frac{2}{5}\frac{g}{R}} = \sqrt{\frac{2 \times 9.8}{5 \times 6400 \times 10^3}} = 7.8 \times 10^{-4}\ rad\ s^{-1}.$

(5 Marks Questions)

17. (a) Derive an expression showing variation of acceleration due to gravity with height.

(b) (i) A body weighs 6.3N on the surface of the earth. What is the geometrical force on it due to the earth at a height equal to half the radius of the earth?

(ii) Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250N on the surface?

Sol. (a) Acceleration due to gravity on the surface of earth at point A is given by $g = \frac{GM}{R^2}$ …(i)

(M – mass of earth, R – radius of earth)

Let g_h be the acceleration due to gravity at a point B at a height h above the surface of the earth. Then $g_h = \frac{GM}{(R+h)^2}$ …(ii)

Dividing (ii) by (i) we get

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM} = \frac{R^2}{(R+h)^2}$$

$$\frac{g_h}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2} \text{ or } g_h = g\left(1+\frac{h}{R}\right)^2$$

Using binomial theorem and neglecting the higher powers of h/R, (since R >>h), we get $g_h = g\left(1 - \frac{2h}{R}\right)$

(b) (i) Acceleration due to gravity g at height h is given by

$$g' = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{(R+\frac{R}{2})^2} = \frac{4}{9}g$$

Gravitational force on body at height h is , $F = mg' = m\frac{4}{9}g = mg\frac{4}{9} = 63\times\frac{4}{9} = 28N$(since mg = 63)

(ii) Variation of acceleration due to gravity g with depth d is $g' = g\left(1 - \frac{d}{R}\right) = 250\left(1 - \frac{R/2}{R}\right) = 125N$.

E. GRAVITATIONAL POTENTIAL ENERGY AND ESCAPE VELOCITY

(1 Mark Questions)

1. Why do different planets have different escape velocities?

Sol. Different planets have different escape velocities because they have different masses and sizes.

(2 Marks Questions)

2. The escape speed of a projectile on the earth's surface is 11.2 km/s. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Sol. Using law of conservation of energy, $\frac{1}{2} mv_0^2 = \frac{1}{2} mv^2 - \frac{1}{2} mv_e^2$

Here v_0 = speed of projectivle when far away from the earth, v = velocity fo projection of the body, v_e = escape velocity.

$\therefore v_0 = \sqrt{v^2 - v_e^2} = \sqrt{(3v_e)^2 - v_e^2} = \sqrt{8}v_e$[given v = $3v_e$]

$\sqrt{8} \times 11.2 = 31.68$ km s^{-1}.

3. Determine the escape velocity of a body from the moon. Take the moon to be uniform sphere of radius 1.76×10^6m, and mass 7.36×10^{22}kg. Given G = 6.67×10^{-11} Nm^2 kg^{-2}.

Sol. Here R = 1.76×10^6m, M = 7.36×10^{22}kg

$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2\times6.67\times10^{-11}\times7.36\times10^{22}}{1.76\times10^6}}$ = 2375ms^{-1} = 2.375 kms^{-1}.

(3 Marks Questions)

4. A rocket is fired from the earth towards the sun. At what distance from the earth's centre is the gravitational force on the rocket zero? Mass of the sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg. Neglect the effect of other planets etc. (orbital radius = 1.5×10^{11} m).

Sol. Let d be the distance of a point from the earth where gravitational forces on the rocket due to the sun and the earth become equal and opposite. Then distance fo rocket from the sun = (r – d). If m is the mass of the rocket, then

$\frac{GM_Sm}{(r-d)^2} = \frac{GM_Em}{d^2}$ or $\frac{(r-d)^2}{d^2} = \frac{M_S}{M_E}$

Or $\frac{r-d}{d} = \sqrt{\frac{M_S}{M_E}}$

Given $M_S = 2 \times 10^{30}$kg, $M_E = 6 \times 10^{24}$kg, r = 1.5×10^{11}m

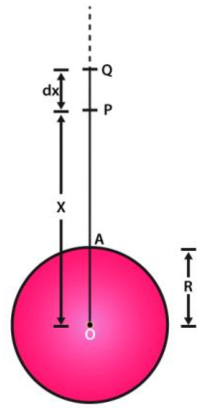
So, $\frac{r-d}{d} = \sqrt{\frac{2\times10^{30}}{6\times10^{24}}} = \frac{10^3}{\sqrt{3}}$

Or $\frac{r}{d} - 1 = \frac{10^3}{\sqrt{3}}$ or $\frac{r}{d} = 1 + \frac{10^3}{\sqrt{3}} = \frac{\sqrt{3}+10^3}{\sqrt{3}}$

Or d = $\frac{r\sqrt{3}}{\sqrt{3}+10^3} = \frac{1.5\times10^{11}\times\sqrt{3}}{\sqrt{3}+10^3}$ = 2.6×10^8m.

5. Define escape velocity. Derive and expression for the escape velocity of a body from the surface of the earth. Write any two significant features of this velocity.

Sol.



Escape velocity is the minimum velocity with which a body must be projected vertically upwards in order that it may just escape the gravitational field of the earth.

Consider the earth to be sphere of mass M and radius R with centre O. Suppose a body of mass m lies at point P at distance x from its centre, as shown in figure. The gravitational force of attraction on the body at P is $F = \frac{GMm}{x^2}dx$. The work done in moving the body from the surface of the earth (x = R) to a region beyond the gravitational field of the earth (x = ∞) will be

$W = \int dW = \int_R^\infty \frac{GMm}{x^2}dx = GMm\int_R^\infty x^{-2}dx = GMm\left[-\frac{1}{x}\right]_R^\infty = GMm\left[-\frac{1}{\infty}+\frac{1}{R}\right] = \frac{GMm}{R}$

If v_e is the escape velocity of the body, then the kinetic energy $\frac{1}{2}mv_e^2$ imparted to the body at the surface of the earth will be just sufficient to perform work W.

Therefore $\frac{1}{2}mv_e^2 = \frac{GMm}{R}$ or $v_e^2 = \frac{2GMm}{R}$

Escape velocity, $v_e = \sqrt{2GMm/R}$ …(i)

As $g = \frac{GMm}{r^2}$ or $v_e^2 = \frac{2GM}{R}$

Therefore, $v_e = \sqrt{2gR^2/R}$ or $v_e = \sqrt{2gR}$ …(ii)

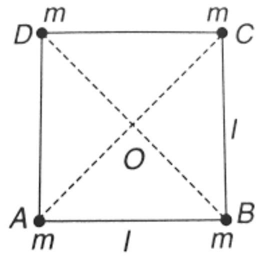
If ρ is the mean density of the earth, then $M = \frac{4}{3}\pi R^3\rho$

From (i), $v_e = \sqrt{\frac{2G}{R}\times\frac{4}{3}\pi R^3\rho} = \sqrt{\frac{8\pi\rho GR^2}{3}}$

Significant feature: The escape velocity, v_{esc}, is the necessary velocity to escape the Earth's gravitational attraction. In theory, this means that the initial kinetic energy plus the gravitational potential is equal to zero when the distance approaches infinity. In practice, the escape velocity is associated with the necessary velocity to send a rocket into Space.

6. Find the potential energy of a system of four particles, each of mass m, placed at the vertices of a square of side 1. Also obtain the potential at the centre of the square.

Sol.



In the figure, AB = BC = CD = DA = l

Therefore AC = BD = $\sqrt{l^2 + l^2} = \sqrt{2}l$

OA = OB = OC = OD =$\sqrt{2}l/l$ = $1/\sqrt{2}$

By the principle of superposition, total potential energy of the system of particles is

$U = U_{BA} + (U_{CB} + U_{CA}) + (U_{DA} + U_{DB} + U_{DC}) = 4U_{BA} + 2U_{DB}$

(since $U_{BA} = U_{DA} = U_{DC} = U_{CB}$; $U_{CA} = U_{DB}$)

$= 4\left(-\frac{Gmm}{l}\right) + 2\left(-\frac{Gmm}{l/\sqrt{2}}\right) = -\frac{2Gm^2}{l}\left[2 + \frac{1}{\sqrt{2}}\right] = -\frac{2Gm^2}{l}[2 + 0.707] = -\frac{5.41Gm^2}{l}$

Total gravitational potential at the centre, O

$V = V_A + V_B + V_C + V_D = 4V_A = 4\left(-\frac{Gm}{OA}\right) = 4\left(-\frac{Gm}{l/\sqrt{2}}\right) = -\frac{4\sqrt{2}Gm}{l}$

(5 Marks Questions)

7. Calculate the speed v with which a projectile should be launched from the surface of Earth so as to reach a height equal to one fourth of Earth's radius, R.

Sol. If the projectile of mass m is given a speed v from Earth's surface.

$K_1 = ½\ mv^2$, $U_1 = -\frac{GMm}{R}$

Thus $E_1 = K_1 + U_1 = ½\ mv^2 - \frac{GMm}{R}$ …(i)

When the particle comes to rest after attaining a height R/4, $K_f = 0$ and

$U_f = -\frac{GMm}{R+R/4} = -\frac{4GMm}{5R}$

Thus $E_f = K_f + U_f = -\frac{4GMm}{5R}$ …(ii)

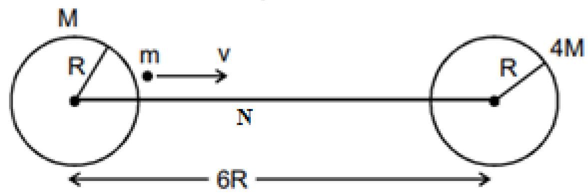
As $E_i = E_f$, from equations (i) and (ii).

$½\ mv^2 - \frac{GMm}{R} = -\frac{4GMm}{5R}$ or $½\ mv^2 = \frac{GMm}{R} - \frac{4GMm}{5R} = \frac{GMm}{5R}$

Or $v = \sqrt{\frac{2GM}{5R}} = \sqrt{\frac{2gR^2}{5R}} = \sqrt{\frac{2gR}{5}}$ $\left(\text{as } g = \frac{GM}{R^2}\right)$

8. Two uniform solid spheres of equal radii R , but mass M and 4M have a centre to centre separation 6R, as shown in the figure. The two spheres are held fixed. A projectile of mass m is projected from the surface of the sphere of mass M directly towards the centre

of the second sphere. Obtain an expression for the minimum speed v of the projectile so that it reaches the surface of the second sphere.



Sol. The two spheres exert gravitational forces on the projectile in mutually opposite directions. At the neutral point N, these two forces cancel each other. If ON = r, then

$\frac{GMm}{r^2} = \frac{G(4M)m}{(6R-r)^2}$

Or $(6R - r)^2 = 4r^2$ or $6R - r = \pm 2r$ or $r = 3R$ or $-6R$

The neutral point r = - 6R is inadmissible

Therefore ON = r = 2R

It will be sufficient to project m with a minimum sped v which enables it to reach the point N. Thereafter, the particle m gets attracted by the gravitational pull of 4M.

The total mechanical energy of m at surface of left sphere is

E_1 = KE of m + PE due to left sphere + PE due to right sphere

$= \frac{1}{2} mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R}$

At the neutral point, speed of the particle becomes zero. The energy is purely potential.

Therefore E_N = PE due to left sphere + PE due to right sphere

$= -\frac{GMm}{2R} - \frac{4GMm}{4R}$

By conversion of mechanical energy, $E_i = E_N$

Or $\frac{1}{2} mv^2 - \frac{GMm}{R} - \frac{4GMm}{5R} = -\frac{GMm}{2R} - \frac{4GMm}{4R}$

$v^2 = \frac{2GM}{R}\left(\frac{4}{5} - \frac{1}{2}\right) = \frac{3GM}{5R}$

So, $v = \sqrt{\frac{3GM}{5R}}$

F. PLANET AND SATELLITES

(1 Mark Questions)

1. The orbit of satellite is

(a) circular (b) elliptic (c) helical (d) both (a) and (b)

Ans. (d)

2. A satellite of small mass burns during its descent and not during ascent. Why?

Sol. The speed of the satellite during descent is much larger than that during its ascent. As the air resistance is directly proportional to velocity, so heat produced during descent is very large and the satellite burns up.

3. Why are space rockets usually launched from west to east in the equatorial plane?

Sol. Our earth rotates from west to east. So, when a rocket is propelled from west to east in the equatorial plane, the rocket gets added advantage of earth's rotational speed. A component of earth's rotational speed adds up with the projection speed of the rocket.

4. Two satellites are at different heights. Which would have greater velocity?

Sol. The satellite at the smaller height would have greater velocity. This is because $v_o \propto 1\sqrt{r}$.

5. What is parking orbit?

Sol. A parking orbit is a temporary orbit used during the launch of a spacecraft. A launch vehicle boosts into the parking orbit, then coasts for a while, then fires again to enter the final desired trajectory.

6. The height of a geostationary satellite is

(a) 1000 km (b) 32000 km (c) 36000 km (d) 850 km

Ans. (c)

Geostationary satellites are those which orbit around the earth and they are placed at an altitude of approximately 35,800 kilometers. Thus, we can conclude their height to be 36000 km.

7. Weightlessness in satellite is due to

(a) zero gravitational acceleration (b) zero acceleration

(c) zero mass (d) none of these

Ans. (d)

8. The artificial satellite does not have any fuel, but even it remains in its orbit around the earth. Why?

Sol. Once the satellite has acquired orbital velocity and is firmly in orbit around the Earth it does not require any energy of its own to move. This is because its being constantly acted upon by the gravitational force of the Earth which tries to pull it downwards but due to Earth's curvature it does not fall.

9. A satellite revolves close to the surface of a planet. How is its orbital velocity related with velocity of escape from that planet?

Sol. If orbital velocity increases, the escape velocity will also increase and vise-versa. If orbital velocity decreases, the escape velocity will also decrease and vise-versa.

10. What is (i) period of revolution and (ii) sense of rotation of a geostationary satellite?

Sol. (i) The time period of revolution of a geostationary satellite around earth is same as that rotation of earth about its own axis i.e. 24 hours.

(ii) Geostationary satellites revolve in the same direction as the Earth's rotation. As we all know, Earth rotates from west to east. Therefore, the geostationary satellite revolves from west to east in the equatorial plane.

11. What are the time period and height of a geostationary satellite above the surface of the earth?

Sol. A geostationary orbit can be achieved only at an altitude very close to 35,786 kilometres (22,236 miles) and directly above the equator. This equates to an orbital speed of 3.07 kilometres per second (1.91 miles per second) and an orbital period of 1,436 minutes, one sidereal day.

(2 Marks Questions)

12. What is the direction of areal velocity of the earth around the sun?

Sol. Areal velocity is given by $\frac{\Delta\vec{A}}{\Delta t} = \frac{1}{2}\vec{r} \times \frac{\Delta\vec{r}}{\Delta t} = \frac{1}{2}(\vec{r} \times \vec{v})$

Direction of $\frac{d\vec{A}}{dt}$ is perpendicular to the plane containing $\vec{r}$ and $\vec{v}$. In other sense, areal velocity is perpendicular to the plane containing the earth and the sun.

13. What are the necessary conditions for a satellite to appear stationary?

Sol. Necessary conditions for a geostationary satellite. These are as follows:

(i) It should revolve in an orbit concentric, and coplanar with the equatorial plane of the earth.

(ii) Its sense of rotation should be same as that of the earth i.e. from west to east.

(iii) Its period of revolution around the earth should be exactly same as that of the earth about its own axis, i.e., 24 hours.

(iv) It should revolve at a height of nearly 36000km above the earth's surface.

14. A geo-stationary satellite orbits around the earth in a circular orbit of radius 36,000km. Then what will be the time period of a spy satellite orbiting a few hundred km above the earth's surface (R_{earth} = 6,400km)?

Sol. As $T^2 \propto R^2$ and R_e = 6400km, therefore

$\frac{T^2}{(24)^2} = \left(\frac{6400}{36000}\right)^3$ or T = 1.7h

For the spy satellite, R is slightly greater than R_e. So, $T_s > T$ or T_s = 2h.

(3 Marks Questions)

15. The mean orbital radius of the earth around the sun is 1.5×10^8 km. Calculate the mass of the sun if $G = 6.67\times10^{-11}$ Nm^2 kg^{-2}.

Sol. Here $r = 1.5\times10^8$ km $= 1.5\times10^{11}$m, T = 365 days = 365×24×3600s

Therefore centripetal force required = Force of gravitation between the earth and the sun

So, $\frac{mv^2}{r} = \frac{Gmm}{r^2}$ or $\frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{Gmm}{r^2}$ or $M = \frac{4\pi^2 r^3}{GT^2}$

$= \frac{4\times9.87\times(1.5\times10^{11})^3}{6.67\times10^{-11}\times(365\times24\times3600)^2} = 2.01\times10^{30}$kg.

16. An earth's satellite makes a circle around the earth in 90 minutes. Calculate the height of the satellite above the earth's surface. Given radius of the earth is 6400 km and g = 980 cms^{-2}.

Sol. Here T = 90 min = 5400s, R = 6400km = 6.4×10^6m, g = 980cms^{-2} = 9.8ms^{-2}

As $T = 2\pi\sqrt{\frac{(R+h)^2}{gR^2}}$

$\therefore R + h = \left[\frac{gR^2T^2}{4\pi^2}\right]^{1/3} = \left[\frac{9.8\times(6.4\times10^6)^2\times(5400)^2}{4\times9.87}\right]^{1/3}$ = 6.668×10^6m = 6668km

Hence h = 6668 – R = 6668 – 6400 = 268km.

17. Define the term orbital speed, Establish a relation for orbital speed of a satellite orbiting very close to the surface of the earth. Find the ratio of this orbital speed and escape speed.

Sol. Orbital velocity is the minimum velocity that is needed to put the satellite into a given orbit around Earth. Consider a satellite of mass 'm' moving around in an orbit at height 'h' above the ground. Let M be mass of Earth, R be the radius of earth and the orbital velocity be Vo.

We know that, F_g = G M m/(R + h) ^{2} and centrifugal force F_C = G V_0^2 /(R + h)

In equilibrium i.e., while rotating around the orbit

GMm/$(R+h)^2$= m V_0^2 /(R + h)

$V_0 = \sqrt{\frac{Gm}{(R+h)}} = \sqrt{g\frac{R^2}{(R+h)}}$ or $V_0 \cong \sqrt{gR}$ (Since h <<R; R + h = R)

$\frac{V_0}{V_{esc}} = \frac{\sqrt{gR}}{\sqrt{2gR}} = \frac{1}{\sqrt{2}}$

(5 Marks Questions)

18. (a) Define Orbital Velocity and establish an expression for it.

(b) Calculate the value of orbital velocity for an artificial satellite of earth orbiting at a height of 1000 km.

Given: Mass of earth = 6×10^{24} kg, Radius of earth = 6400 km

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G. CASE STUDY

1. Earth satellite: A body moving in an orbit around the earth is called earth's satellite. The moon is the natural satellite of the earth. The first artificial (or man made) satellite was put into earth's orbit in 1956. Artificial satellites are put into orbit at an altitude of a few hundred kilometers. The satellite is carried in a rocket which is launched from the earth with a velocity greater than escape velocity. The escape velocity is the velocity with which a body must be projected in order that it may escape the gravitational pull of the earth. When the rocket has achieved the desired height, the satellite is released horizontally by imparting to it a very high speed so that it remains moving in a nearly circular orbit around the earth. The velocity is called the orbital velocity which is about 8 km/s for a satellite a few hundred kilometers above the earth.

(i) The escape velocity of a rocket fired from the earth depends upon

(a) the mass of the rocket (b) the volume of fuel in the rocket

(c) the acceleration due to the gravity of the earth

(d) the direction in which the rocket is fired

Ans. (c)

(ii) The centripetal force necessary to keep a satellite in a circular orbit around the earth is provided by

(a) a continuous ejection of hot gases by the satellite.

(b) the gravitational attraction between the earth and the satellite

(c) the gravitational pull of the sun exerted on the satellite

(d) the weightlessness of the satellite

Ans. (b)

(iii) A satellite of mass m is in a stable circular orbit around the earth at an altitude of about 100 km. If M is the mass of the earth, R its radius and g is the acceleration due to gravity, the time period T of the revolution of the satellite is given by

(a) $T = 2\pi\sqrt{\frac{R}{g}}$ (b) $T = 2\pi\sqrt{\frac{g}{R}}$ (c) $T = 2\pi\sqrt{\frac{MR}{mg}}$ (d) $T = 2\pi\sqrt{\frac{mR}{Mg}}$

Ans. (a)

(iv) An artificial satellite is orbiting the earth at an altitude of 500km. A bomb is released from the satellite. The bomb will

(a) explode due to the heat generated by the friction of air

(b) fall freely on the earth (c) escape into outer space

(d) orbit the earth along with the satellite.

Ans. (d)

H. ASSERTION REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false **(d) If both assertion and reason are false**

(e) If assertion is false but reason is true

1. Assertion: The value of acceleration due to gravity does not depend upon mass of the body.

Reason: Acceleration due to gravity is a constant quantity.

Ans. (c) Assertion is true but reason is false

Acceleration due to gravity is given by $g = GM/R^2$. Thus it does not depend on mass of body on which it is acting. Also it is not a constant quantity changes with change n value of both M and R (distance between two bodies). Even for earth it is a constant only near the earth's surface.

2. Assertion: Gravitational potential of earth at every place on it is negative.

Reason: Everybody on earth is bound by the attraction of earth.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.

The gravitational potential at a point in the gravitational field of a body is defined as the work done in bringing unit mass from infinity to earth, it is attracted by the gravitational field, so work done on the body, so the gravitational potential is negative.

3. Assertion: If an earth satellite moves to a lower orbit, there is some dissipation of energy but the satellite speed increases.

Reason: The speed of satellite is constant quality.

Ans. (c) Assertion is true but reason is false

Due to resistance force of atmosphere, the satellite revolving around the earth losses kinetic energy. Therefore in a particular orbit the gravitational attraction of earth on satellite becomes greater than that required for circular orbit thee. Therefore satellite moves down to a lower orbit. In the lower orbit as the potential energy becomes more negative (U = GM/r), hence kinetic energy $\left(E_K = \frac{GMm}{2r}\right)$ increases, hence speed of satellite increases.

4. Assertion: For a satellite revolving very near to earth's surface the time period of revolution is given by 1 hour 24 minutes.

Reason: The period of revolution of a satellite depends only upon its height above the earth's surface.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.

The time period of a satellite at a height h above the earth surface, T = $2\pi\sqrt{\frac{(R_e+h)^3}{gR_e^2}}$

For a satellite revolving near to earth's surface, h = 0

Therefore T = $\sqrt{\frac{R_e}{g}} = 2 \times 3.14\sqrt{\frac{6.37\times10^6}{9.8}} = 5063sec$ = 84 minutes ≅ 1 hr 24 min.

5. Assertion: The periodic time of communication satellite is 18 hours.

Reason: The time of revolution of satellite depends upon its distance from earth surface.

Ans. (e) Assertion is false but reason is true

A communication satellite completes one revolution in the same time as that of earth about its axis. So the times period of revolution of revolution of communication satellite is 24 hours.