CLASS – 11

WORKSHEET- MECHANICAL PROPERTIES OF SOLIDS

A. Elastic Behaviour of Solids

(1 Mark Questions)

1. Out of the following the most plastic material is
(a) iron (b) wood (c) rubber (d) plasticine
Ans. (d)

2. Substances which can be stretched to cause large strains are called
(a) isomers (b) plastomers (c) elastomers (d) polymers
Ans. (c)

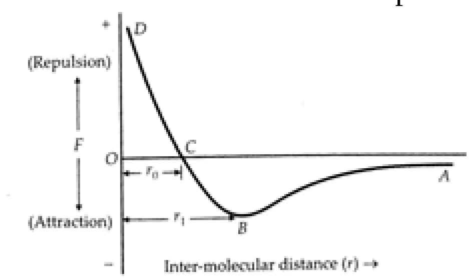
3. In which year did Robert Hooke presented his law of elasticity?
(a) 1672 (b) 1674 (c) 1676 (d) 1678
Ans. (c)

(2 marks questions)

4. What is perfectly elastic body? Give an example in which is close to perfectly elastic.
Sol. Perfectly elastic body: A body which regains its original configuration immediately and completely after the removal of deforming force from it, is called perfectly elastic body. Quartz and phosphor bronze are the examples of nearly perfectly elastic bodies.

(3 marks questions)

5. In the diagram a graph between the intermolecular force F acting between the molecules of a solid and the distance r between them is shown. Explain the graph.



Sol. (i) As the intermolecular distance r decreases, the force of attraction between the molecules increases. (ii) When the distance decreases to r_1, the force of attraction is maximum. (iii) As the distance further decreases, the attractive force goes on decreasing and when the distance decreases to r_0, the force becomes zero. When the distance decreases below r_0, the molecules begin to repel and the repulsive force increases rapidly.

B. Stress and Strain

(1 mark questions)

1. Fluids can develop

(a) longitudinal strains only (b) longitudinal and shearing strain

(c) longitudinal, shearing and volumetric strain (d) volumetric strain only

Ans. (d)

Fluids can develop volumetric strain only.

2. A steel cable with a radius 2cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 N/m^2, the maximum load the cable can support is

(a) $4\pi\times10^5$ N (b) $4\pi\times10^4$ N (c) $2\pi\times10^5$ N (d) $2\pi\times10^4$ N

Ans. (b)

3. Stress is a ______ quantity.

(a) scalar (b) vector (c) tensor (d) dimensionless

Ans. (c)

Stress is a tensor quantity

4. Stress and pressure are both force per unit area. Then in what respect does stress differ from pressure?

Sol. Pressure is external force per unit area, while stress is the internal force that comes into play from within a strain body acting transversely per unit area of the body.

5. The ratio stress/strain remains constant for small deformation. What will be the effect on this ratio when the deformation made is very large?

Sol. The ratio of stress to strain will decrease. Beyond the elastic limit, the body loses its ability to restore completely when subjected to stress.

(3 marks question)

6. Define the term strain. Why it has no units and dimensions? What are different types of strain?

Sol. Strain is the ratio of change in dimensions of the body to the original dimensions. Because it is a ratio, it is a dimensionless quantity. Different types of strain are : (i) Longitudinal strain (ii) Tangential strain or shearing strain (iii) Volumetric strain.

7. Define the term stress. Give its units and dimensions. Describe the different types of stress.

Sol. This restoring force will be equal in magnitude and opposite in direction to the applied deforming force. The measure of this restoring force generated per unit area of the material is called stress. Thus, Stress is defined as “The restoring force per unit area of the material”. It is a tensor quantity.

(a) Longitudinal stress: The stressing force acting per unit area along the length of the body is called longitudinal stress.

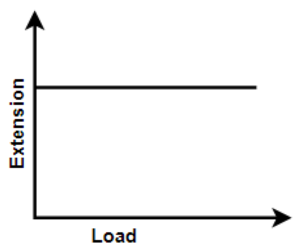
(b) Tensile stress: When a cylinder is stretched by two equal forces applied normal to its cross sectional area, the restoring force per unit area developed in this case is called tensile stress.

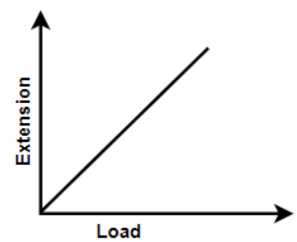
(c) Compressive stress: When a cylinder compressed under the action of deforming force, the restoring force per unit area is called compressive stress.

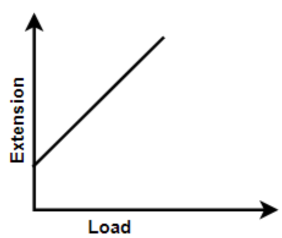
C. Hooke’s law

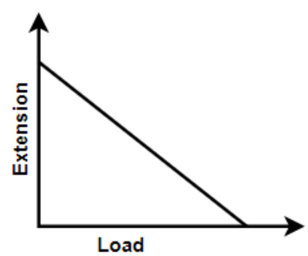
(1 marks questions)

1. Within elastic limit, which of the following graphs correctly represents the variation of extension in the length of a wire with the external load?

(a) 

(b) 

(c) 

(d) 

Ans. (b)

2. According to Hooke’s law of elasticity, the ratio of stress to strain

(a) decreases (b) increases (c) becomes zero (d) remains constant

Ans. (d)

The ratio of stress to strain is always constant.

3. The reciprocal of force constant is known as

(a) conductance (b) compliance (c) admittance (d) reactance

Ans. (c)

4. Write Hooke's law.

Sol. Mathematically, Hooke's law states that the applied force F equals a constant k times the displacement or change in length x, or F = kx. The value of k depends not only on the kind of elastic material under consideration but also on its dimensions and shape.

5. Is Hooke's law is applicable to all materials?

Sol. Hooke's law is accurate only for solid bodies if the forces and deformations are small. Hooke's law isn't a universal principle and only applies to the materials as long as they aren't stretched way past their capacity.

D. Stress Strain Curve

(1 mark questions)

1. Solids which break above the elastic limit are

(a) brittle (b) ductile (c) malleable (d) elastic

Ans. (a)

2. The breaking stress for a wire of unit cross section is called

(a) yield point (b) elastic fatigue (c) tensile strength (d) Young's modulus

Ans. (c)

The breaking stress of a wire of unit cross-section is called a tensile strength.

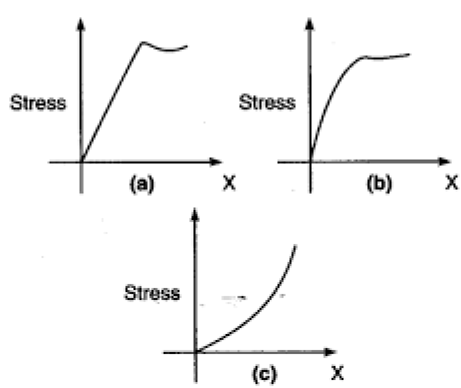
3. The breaking stress of a wire depends upon

(a) length of the wire (b) radius of the wire

(c) material of the wire (d) shape of the cross-section.

Ans. (c)

4. Following are the graphs of elastic materials. Which one corresponds to that of brittle material?



Sol. Graph (a) represents a brittle material as it indicates a very small plastic range of extension.

5. What is elastic fatigue?

Sol. The property of an elastic body by virtue of which its behaviour becomes less elastic under the action of repeated alternating deforming force is called elastic fatigue.

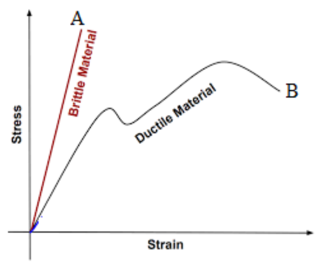
6. What do you mean by 'permanent set' in a body?

Sol. The amount by which a material stressed beyond its elastic limit fails to return to its original size or shape when the load is removed is called 'permanent set' in a body.

(3 marks questions)

7. On the basis of stress-strain curves, distinguish between ductile and brittle materials

Sol.

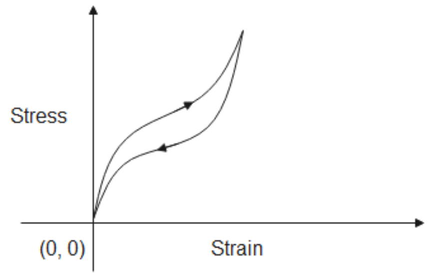


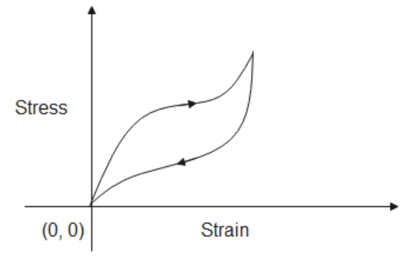
Stress strain curve of brittle and ductile materials can be shown in the given figure. In the figure curve A represents the brittle material and B represents ductile material.

Ductile material	**Brittle material**
(i) Ductile materials can withstand large strain before specimen rupture	Brittle materials fracture at much lower strains.
(ii) Slope of stress strain curve is smaller for ductile materials within elastic limit.	Slope of stress strain curve is greater for brittle materials
(iii) Ductile materials exhibits large yielding.	Brittle materials fails suddenly without much warning.
(iv) Steel and aluminium falls under this class.	Glass and cast iron falls under this class.

(5 marks questions)

8. Two different types of rubber are found to have stress strain curves as shown in the figure.





(a) In which significant ways do these curves shown in the figure differ from the stress strain curve of a metal wire?

(b) A heavy machine is to be installed in a factory. To absorb variations of the machine, a block of rubber is placed between the machinery and the floor. Which of these two rubber A and B would you prefer to use for this purpose? Why?

(c) Which of the two rubber materials would you choose for a car tyre?

Sol. (a) The stress strain curves for rubber differ from the stress strain of a metal wire in the following respects: (i) Hooke's law is not obeyed even for small stresses. (ii) There is no permanent set (residual strain) even for large stresses. (iii) There is large elastic regain for both types of rubber. (iv) Neither metal retraces the curve during unloading. Thus both materials exhibit elastic hysteresis.

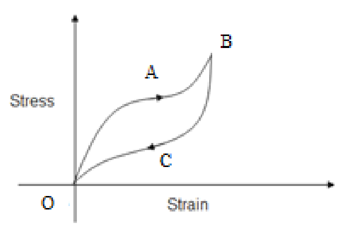
(b) The area of the hysteresis loop is proportional to the energy dissipated by the material as heat when the material undergoes loading and unloading. A material for which the hysteresis loop has larger area would absorb more energy when subjected to vibrations. Therefore to absorb vibrations, we would prefer rubber B.

(c) In car tyre, the energy dissipation must be minimized t avoid excessive heating of the car tyre. As rubber A has smaller hysteresis loop area (and hence smaller energy loss), so it is preferred to B for a car tyre.

9. (a) Describe elastic hysteresis. Mention its two applications.

(b) What is elastic after effect?

Sol.



(a) Elastic hysteresis shows the stress strain curve for a rubber sample when loaded and then unloaded. For increasing load, the stress strain curve is OAB and for decreasing load, the curve is BCO. The fact that the stress strain curve is not retraced on reversing the strain is known as elastic hysteresis.

The area under the curve OAB represents the work done per unit volume in stretching the energy given up by the rubber on unloading. S the area OABC of the hysteresis loop represents the energy lost as heat during the loading-unloading cycle.

Application of elastic hysteresis: (i) Car tyres are made with synthetic rubbers having small area hysteresis loops because a car tyre of such a rubber will not get excessively heated during the journey. (ii) A padding of vulcanized rubber having large area hysteresis loop is used in shock absorbers between the vibrating system and the flat board. As the rubber is compressed and released during each vibration, it dissipates a large amount of vibrational energy.

(b) If elastic limit is not exceeded, the solids such as quartz, phosphor bronze, silver and gold recover their original condition almost immediately after the deforming force is removed. However, solids in general, take appreciable time to recover their original condition completely. This delay in the recovery on removal of the deforming is called elastic after effect.

E. Elastic Moduli

(1 mark questions)

1. For a perfectly rigid body

(a) Young's modulus is infinite and bulk modulus is zero.

(b) Young's modulus is zero and bulk modulus is infinite.

(c) Young's modulus is infinite and bulk modulus is also infinite.

(d) Young's modulus is zero and bulk modulus is also zero.

Ans. (c)

2. The ratio of shearing stress to the shearing strain is define as

(a) Young's modulus (b) bulk modulus (c) shear modulus (d) compressibility

Ans. (c)

The ratio of shearing stress to the corresponding shearing strain is called shear modulus.

3. For an ideal liquid

(a) bulk modulus is infinite and shear modulus is zero

(b) bulk modulus is zero and shear modulus is infinite

(c) bulk modulus is infinite and shear modulus is also infinite

(d) bulk modulus is zero and shear modulus is also zero

Ans. (b)

4. Which is more elastic rubber or copper?

Sol. Rubber is an elastic material because it retains its original shape after being compressed or stretched. Coefficient of elasticity is a quantity that measures an object's resistance to being deformed elastically. We are given Copper, Steel, Glass and Rubber. Out of these obviously rubber is the more elastic object.

5. Define compressibility of a material.

Sol. Compressibility can be defined as the proportional reduction in the thickness of a material under prescribed conditions of increased pressure or compressive loading.

6. What does the slope of stress versus strain graph give?

Sol. The slope of stress-strain diagram gives the Young's modulus of a material. Young's modulus tells us about the strength of the material.

7. Write dimensionless formula of Young’s modulus.

Sol. Young’s modulus, $Y = N/m^2 = kgm/s^2m^2 = kg/ms^2 = ML^{-1}T^{-2}$.

8. What is the value of bulk modulus for an incompressible liquid?

Sol. Bulk modulus for an incompressible liquid is infinity.

9. For solids with elastic modulus of rigidity, the shearing force is proportional to shear strain. On what factor does it depend in case of fluids?

Sol. In case of fluids the factor is Rate of Shear Strain.

10. Why steel is more elastic than rubber?

Sol. The strain produced in rubber is much larger compared to that in steel. This means that steel has a larger value of Young's modulus of elasticity and hence, steel has more elasticity than rubber.

11. A beam of metal supported at the two ends is loaded at the centre. The depression at the centre is proportional to

(a) Y^2 (b) Y (c) $1/Y$ (d) $1/Y^2$

Ans. (c)

12. Why are bridges declared unsafe after a long use?

Sol. A bridge during its use undergoes alternating strains for a large number of times each day, depending upon the movement of vehicles on it when a bridge is used for long time, it losses its elastic strength.

13. Why are electric poles given hollow structure?

Sol. Because of more strength and more durability electric poles are made of hollow cylinders.

14. Mention two applications of elasticity.

Sol. Applications of elasticity:

- Metallic part of machinery is never subjected to a stress beyond the elastic limit of the material.
- Metallic rope used in cranes to lift heavy weight is decided on the elastic limit of the material.
- In designing beam to support load (in construction of roofs and bridges).

(2 marks questions)

15. Define modulus of elasticity. Name its three components.

Sol. The ratio of stress and strain is called modulus of elasticity. Modulus of elasticity has three components: (a) Young's modulus (b) Shear modulus (c) Bulk modulus.

16. When the tension in a metal wire is T_1, its length is l_1. When the tension is T_2, its length is l_2. Find the natural length of wire.

Sol. $Y = \frac{Fl}{A\Delta l}$ where Y, l and A car constants,

As $F/\Delta l$ is constant or $\Delta l \propto F$

Therefore if tension is T_1, the $l_1 - l \propto T_1$…(i)

If tension is T_2, then $l_2 - l_1 \propto T_2$ …(ii)

Dividing (i) be (ii) we get, $\frac{l_1 - l}{l_2 - l} = \frac{T_1}{T_2}$ or $l_1T_2 = l_2T_1$ or $l(T_1 - T_2) = l_2T_1 - l_1T_2$

Or $l = \frac{l_2T_1 - l_1T_2}{T_1 - T_2} = \frac{l_1T_2 - l_2T_1}{T_1 - T_2}$

17. A square lead slab of side 50cm and thickness 5.0cm is subjected to a shearing force (on its narrow face) of magnitude 9.0×10^4N. The lower edge is riverted to the floor. How much is the upper edge displaced if the shear modulus of the lead is 5.6×10^9 N/m?

Sol. The area of cross section of the face where the force is applied is $A = 50\times5 = 250\text{cm}^2$

$= 250\times10^{-4}\text{m}^2$; $\eta = \frac{F/A}{\theta}$

Therefore shear strain, $\theta = \frac{F}{A\eta} = \frac{9.0\times10^4}{250\times10^{-4}\times5.6\times10^9} = 6.4\times10^{-4}$

Therefore displacement of the upper edge is given by θ = length of one side

$= (6.4\times10^{-4}) \times 0.5 = 3.5 \times 10^{-4}$m

18. A wire of length l, area of cross section A and Young's modulus Y is stretched by an amount x. What is the work done in stretching the wire?

Sol. Restoring force in extension, $x > F = \frac{AYx}{L}$

Work done is stretching it by dx, dW= F.dx

$W = \int dW = \int_0^x Fdx; W = \int_0^x \frac{AYx}{L}.dx = \frac{1}{2}\frac{AYx^2}{L}$

19. Determine the volume contraction of a solid copper cube, 10cm on an edge, when subjected to a hydraulic pressure of 7.0×10^6 Pa. (Bulk modulus of Cu = 140510^9 Pa).

Sol. Here L = 10cm = 0.1m, B- bulk modulus of Cu = 140×10^9Pa, P = 7×10^6Pa

ΔV = volume contraction of solid copper cube= ?

So, $V = L^3 = (0.1)^3 = 0.001\ m^3$

From formula, $B = \frac{P}{(\frac{\Delta V}{V})}$, we get

$\Delta V = -\frac{PV}{B} = -\frac{7\times10^6\times0.001}{140\times10^9} = 0.05\times10^{-6}m^3 = -0.05\ cm^3$.

20. Given the following values for an elastic material: Young's modulus = $7\times10^{10}N\ m^{-2}$ and bulk modulus = $11\times10^{10}\ N\ m^{-2}$. Calculate the Poisson's ratio of the material.

Sol. $K = \frac{Y}{3(1-2\sigma)}$ or $11\times10^{10} = \frac{7\times10^{10}}{3(1-2\sigma)}$ or $\frac{7}{33} = 1-2\sigma$ or $2\sigma = 1-\frac{7}{33}$ or $\sigma = \frac{26}{60}$ = 0.39.

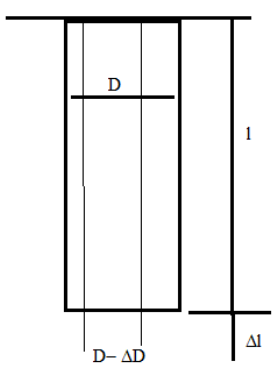
21. How is the knowledge of elasticity be used to estimate the maximum height of a mountain on earth?

Sol. The maximum height of mountain on earth can be estimated elastic behaviour of rocks i.e. by shear modulus of rocks. Also, pressure is less than elastic limit of earth's supporting material at lease of mountain.

(3 marks questions)

22. Define Poisson's ratio. Write an expression for it. What is the significance of negative sign in this expression?

Sol. Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.



Suppose the length of the loaded wire increases from l to l + Δl and its diameter decreases from D to D – ΔD.

Longitudinal strain = Δl/l

Lateral; strain = ΔD/D

Poisson's ratio is $\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{-\Delta D/D}{\Delta l/l}$ or $\sigma = \frac{-l}{D}.\frac{\Delta D}{\Delta l}$

The negative sign indicates tht longitudinal and lateral strains are in opposite directions.

23. (a) What is elastic potential energy?

(b) Derive an expression for the elastic potential energy stored in a stretched wire under stress.

(c) Prove that elastic energy density is equal to ½ × stress × strain.

Sol. (a) Energy stored in a material due to its elastic behaviours when some force is applied on it is called elastic potential energy.

(b) Hooke's Law, F = -kx, where F is the force and x is the elongation.

The work done = energy stored in stretched string = F.dx

The energy stored can be found from integrating by substituting for force,

and we find,

The energy stored = $kx^2/2$, where x is the final elongation.

The energy density = energy/volume = $(kx^2/2)/(AL)$

=1/2(kx/A)(x/L) = 1/2(F/A)(x/L) = 1/2(stress)(strain)

(c) Since elastic potential energy density is given by $\frac{E}{V} = \frac{1}{2} Y\alpha^2 = \frac{1}{2} \times Y \times (\text{strain})^2$

Since, $Y = \frac{\text{stress}}{\text{strain}}$

Therefore, $\frac{E}{V} = \frac{1}{2} \times \frac{\text{stress}}{\text{strain}} \times (\text{strain})^2$ = ½ × stress × strain.

24. A wire of area of cross section 3.0 mm^2, and natural length 50cm, is fixed at one end and a mass of 2.1kg is hung from the other end. Determine the elastic potential energy stored in the wire in the steady state. (Given: Young's modulus of the material of the wire = 2.0×10^{11} Nm^{-2} and g = 10 ms^{-2}).

Sol. Young's modulus of the two wires is given by

$Y_1 = \frac{F_1 \times l_1}{A_1 \times \Delta l_1}$ and $Y_2 = \frac{F_2 \times l_2}{A_2 \times \Delta l_2}$

$\Delta l_1 = \frac{F_1 l_1}{A_1 Y_1}$ and $\Delta l_2 = \frac{F_2 l_2}{A_2 Y_2}$

Taking the ratio of Δl_1 and Δl_2, $\frac{\Delta l_1}{\Delta l_2} = \left(\frac{F_1}{F_2}\right)\left(\frac{l_1}{l_2}\right)\left(\frac{A_2}{A_1}\right)\left(\frac{Y_2}{Y_1}\right)$

Since $F_1 = F_2$ and $Y_1 = Y_2$ (given)

So, $\frac{\Delta l_1}{\Delta l_2} = \left(\frac{l_1}{l_2}\right)\left(\frac{A_2}{A_1}\right) = \left(\frac{l_1}{l_2}\right)\left(\frac{D_2^2}{D_1^2}\right)$

$l_1 = \frac{1}{p} l_2$ and $D_1 =$ nD2

So, $\frac{\Delta l_1}{\Delta l_2} = \frac{1}{pn^2}$

25. A box shaped piece of gelatin dessert has a top area of 15cm^2 and a height of 3cm. When a shearing force of 0.50N is applied to the upper surface, the upper surface displaces 4mm relative to the bottom surface. What are the shearing stress, shearing strain and the shear modulus for the gelatin?

Sol. $A = 15\times10^{-4}m^2$, $l = 3\times10^{-2}m$, $F = 0.50N$, $\Delta l = 4\times10^{-3}m$

Shearing stress = $\frac{F}{A} = \frac{0.50}{15\times10^{-4}} Nm^{-2}$ = 333.3 Nm^{-2}

Shearing strain = $\frac{\Delta l}{l} = \frac{4\times10^{-3}}{3\times10^{-2}}$ = 0.133

Shear modulus = $\frac{stress}{strain}$ = 2500 Nm^{-2}

26. A structural steel rod has a radius of 10mm and a length 1m. A 100kN force F stretches it along the length. Calculate (a) the stress (b) elongation, and (c) strain on the rod. Given that the Young's modulus of the structural steel is 2.0×10^{11} Nm^{-2}.

Sol. Here r= 10mm = 0.01m, l = 1m, F = 100kM = 10^5N, Y = $2.0\times10^{11}Nm^{-2}$

(a) Stress = $\frac{F}{A} = \frac{F}{\pi r^2} = \frac{10^5}{(3.14)\times(0.01)^2}$ = 3.18×10^8 Nm^{-2}

(b) As Y = $\frac{F}{A}.\frac{l}{\Delta l}$

Therefore, elongation, $\Delta l = \frac{F}{A}.\frac{l}{Y} = \frac{3.18\times10^8\times1}{2.0\times10^{11}}$ = 1.59×10^{-3}m = 1.59mm

(c) Strain = $\frac{\Delta l}{l} = \frac{1.59\times10^{-3}m}{1m}$ = 1.59×10^{-3} = 0.16%.

27. If the normal density of sea water is 1.00 g cm^{-3}, what will be its density at a depth of 3km? Given compressibility of water = 0.0005 per atmosphere, 1 atmosphere pressure = 10^6 dyne cm^{-2}, g = 980 cm s^{-2}.

Sol. B = 1/Compressibility = 1/0.0005

= 2×10^3 atm = $2\times10^3\times10^6$ dyne cm^{-2} = 2×10^9 dyne cm^{-2}

P = $h\rho g = 3\times10^5\times1\times980 = 294\times10^6$ dyne cm^{-2}

[since h = 3km = 3×10^5cm, ρ(water) = 1g cm^{-1}]

As B = $\frac{PV}{\Delta V}$

Therefore $\Delta V = \frac{PV}{B} = \frac{294\times10^6\times1}{2\times10^9}$ [since V = 1cm^3[= 0.147 cm^3

Volume of 1g of water at a depth of 3km,

V' = V – ΔV = 1 – 0.147 = 0.853 cm^3

Density = mass/volume = 1/0.853 = 1.1723 g cm^{-3}.

28. A steel wire has length 2m, radius 1mm and Y = 2×10^{11} N/m^2. A 1kg sphere is attached to one end of the wire and whirled in a vertical circle with an angular velocity of 2 revolutions per second. What is the elongation of the wire when the sphere is at the lowest point of the vertical circle?

Sol. $Y = \frac{[mg+ml\omega^2]t}{\pi r^2\Delta l}$ or $\Delta l = \frac{m[g+l\omega^2]t}{\pi r^2 Y}$

Or $\Delta l = \frac{1[10+2\times4\pi^2\times4]2}{\pi(1\times10^{-3})^2\times2\times10^{11}}$m

Or $\Delta l = \frac{[20+64\times9.88]7}{2\times22\times10^5}$m

$= \frac{4566.54}{44 \times 10^5} \times 10^3 \text{mm} = 1\text{mm}$

29. A cable is replaced by another cable of the same length and material but of half the diameter.

(a) How does this affect its elongation under a given load?

(b) How many times will be the maximum load it can now support without exceeding the elastic limit?

Sol. (a) Young's modulus, $Y = \frac{\text{Mgl}}{\pi r^2 \Delta l} = \frac{\text{Mgl}}{\pi\left(\frac{D}{2}\right)^2 . \Delta l} = \frac{4\text{Mgl}}{\Delta D^2 . \Delta l}$

Where D is the diameter of the wire

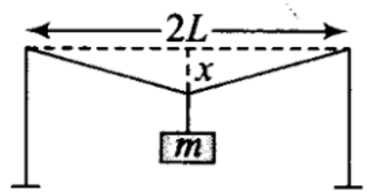
Elongation, $\Delta l = \frac{4\text{Mgl}}{\Delta D^2 Y}$ i.e., $\Delta l \propto \frac{1}{D^2}$

Clearly if the diameter becomes half, the elongation will increase four times.

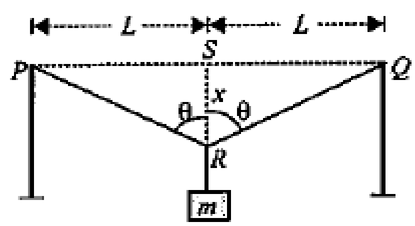
(b) Also load, $\text{Mg} = \frac{\Delta D^2 . \Delta l . Y}{4l}$ i.e. $\text{Mg} \propto D^2$

Clearly if the diameter becomes half, the wire can support ¼ times the original load.

30. A steel wire of length 2l and cross section area A is stretched within elastic limit as shown in figure. Calculate the strain in the wire.



Sol. Consider the given diagram.



so, change in length $\Delta L = (A_0 + B_0) - (AC + CB) = 2BO - 2AC$

$= 2[BO - AC)$ [since AO = BO, AC = CB)

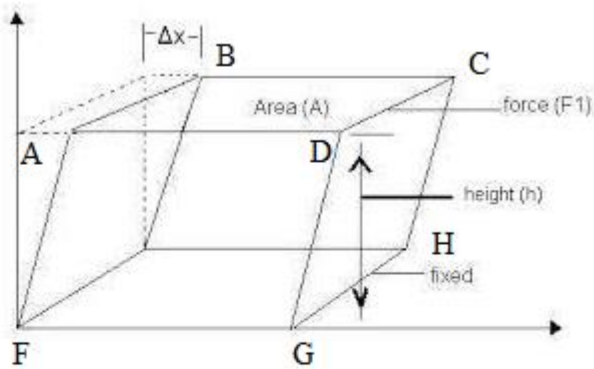
$= 2[(x^2 + L^2]^{1/2} - L] = 2L[(1 + (x^2/L^2))^{1/2} - 1]$

$\Delta L = 2L [1 + ½ (x^2/L^2) - 1] = x^2/L$ (Since $x << L$)

strain $= \Delta L/L = (x^2/L)/2L = x^2/2L^2$

31. Define shear modulus. With the help of a diagram, explain how shear modulus can be calculated.

Sol. Shear modulus, also known as Modulus of rigidity, is the measure of the rigidity of the body, given by the ratio of shear stress to shear strain.



Consider a rectangular block whose lower face EFGH is fixed. Let a rectangular force F be applied on the upper face ANCD of the block in direction shown. The force provides a shear stress which displaces the upper face through a small distance AA' = ΔL.

If AF = L, and area of the upper face is A, then tangential stress = F/A

Shear strain= θ = tan θ = $\frac{AA'}{AF} = \frac{\Delta L}{L}$

$\therefore\ \eta = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$ or $\eta = \frac{FL}{A\Delta L}$

32. A steel wire of length 4.7m and cross section area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5m and cross sectional area of 4.0×10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Sol. Let the Young's Modulus of steel and copper be Y_s and Y_c respectively.

Length of steel wire, L_s = 4.7 m, Length of copper wire, L_c = 3.5 m

Area of cross-section of steel wire, $A_s = 3 \times 10^{-5}$ m^2

Area of cross-section of copper wire, $A_c = 4 \times 10^{-5}$ m^2

Since change in lengths are equal.

$\therefore \Delta L_s = \Delta L_c = \Delta L$.

As the load is same, $F_s = F_c = F$

We have, Y = (F/A x L/ΔL)

$\therefore \frac{Y_s}{Y_c} = \frac{L_s A_c}{A_s L_c} = \frac{4\times4.7\times10^{-5}}{3\times3.5510^{-5}} = 1.79.$

The ratio of Young's Modulus of steel to that of copper is ≃ 1.8 :1.

33. (a) Define modulus of rigidity.

(b) A steel cable with a radius of 1.5cm supports a chairlift at a ski area. If the maximum stress is not to exceed 10^8 n/m^2, what is the maximum load, the cable can support?

Sol. (a) Within elastic limit, the ratio of tangential stress to shear strain is called modulus of rigidity.

(b) Here maximum stress = 10^8 Nm^{-2}, radius of the cable, r = 1.5cm = 0.015m,

Therefore area of cross section of the cable, $A = \pi r^2 = \pi \times (0.015)^2 = 2.25 \times 10^{-4}\ \pi\ m^2$.

The maximum load, the cable can support, F = maximum stress × area of cross setion

$= 10^5 \times 2.25 \times 10^{-4}\pi = 7.07 \times 10^4$N.

34. (a) Which is more elastic, rubber or steel. Give reason.

(b) What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03×10^3 kg m^{-3}? Compressibility of water is 45.8×10^{-11} Pa^{-1}.

Sol. (a) Steel is more elastic than rubber because steel comes back to its original shape faster than rubber when the deforming forces are removed.

(b) Here P = 80.0atm = $80.0\times1.013\times 10^5$pa; Compressibility, $1/B = 45.8\times10^{-11}Pa^{-1}$

Density of water at surface, $\rho = 1.03\times10^3$ kg m^{-1}. Let ρ' be the density of water at the given depth. \If V and V' are volumes of certain mass M of ocean water at surface at a given depth, then V = M/ρ and V' = M/ρ'.

Therefore change in volume, $\Delta V = V' = M(1/\rho - 1/\rho')$

Therefore volumetric strain, $\frac{\Delta V}{V} = M\left(\frac{1}{\rho} - \frac{1}{\rho'}\right) \times \frac{\rho}{M} = 1 - \frac{\rho}{\rho'}$

Or $\frac{\Delta V}{V} = 1 - \frac{1.03\times10^3}{\rho'}$ …(i)

As bulk modulus B = $\frac{PV}{\Delta V}$ or $\frac{\Delta V}{V} = \frac{P}{B}$

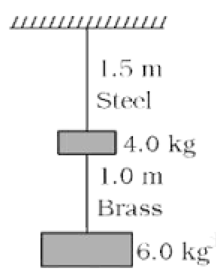
Therefore $\frac{\Delta V}{V} = (80.0\times1.013\times10^5)\times45.8\times10^{-11} = 3.712\times10^{-3}$

Putting this value in (i) we get $1 - \frac{1.03\times10^3}{\rho'} = 3.712 \times10^{-3}$

Or $\rho' = \frac{1.03\times10^3}{1*3.712\times10^{-3}} = 1.034\times10^3$ kg m^{-1}

(5 marks questions)

35. Two wires of diameter 0.25cm, one made of steel and the other of brass are loaded as shown in figure. The unloaded length of steel wire is 1.5m and that of brass wire is 1.0m. Compute the elongations of the steel and the brass wire.



Sol. Y for steel, $Y_{1 = 2\times10^{11}}$ Nm^{-2}, Y for aluminium, $Y_2 = 7\times10^{10}$ Nm^{-2}, length of each wire, l = 2m, diameter, d = 2.0mm.

Therefore radius, r = 1mm = 10^{-3}m

Area of cross section. $A = \pi r^2 = \pi(20^{-3})^2\ m^2 = n \times 10^{-6}m^2$

Total extension $(\Delta l_1 - \Delta l_2) = 0.90mm = 9\times10^{-4}$, We know $Y = \frac{FL}{A\Delta l}$ or $\Delta l = \frac{FL}{YA}$

Therefore $(\Delta l_1 + \Delta l_2) = \frac{FL}{A}\left(\frac{1}{Y_1} + \frac{1}{Y_2}\right)$

Or $9\times10^{-4} = \frac{F\times2}{\pi\times10^{-6}} \times \left(\frac{1}{2\times10^{11}} + \frac{1}{7\times10^{10}}\right)$

Or $F = \frac{9\times10^{-4}\times3.14\times10^{-6}\times1.4\times10^{11}}{2\times2.7} = 73.3N$

36. (a) Explain why should the beams used in the construction of bridges have large depth and small breadth.

(b) Why are girders given I shape?

Sol. (a) A large depth and small breadth" gives a beam a large second moment of area which in turn gives it a greater bending stiffness to loads in the vertical direction, which are the primary loads expected on most bridges.

(b) The I shape girders provide large load bearing surface and enough depth to prevent bending. This shape also reduces the weight of the beam without effecting its strength.

F. ASSERTION REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false **(d) If both assertion and reason are false**

(e) If assertion is false but reason is true

1. Assertion: Then stretching of a coil is determined by its shear modulus.

Reason: Shear modulus change only shape of a body keeping its dimensions unchanged.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion. Because the stretching of coil simply changes its shape without any change in the length of the wire used in coil. Due to which shear modulus of elasticity is involved.

2. Assertion: Steel is more elastic than rubber.

Reason: Under given deforming force, steel is deformed less than rubber.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion. Elasticity is a measure of tendency of the body to regain its original configuration. As steel is deformed less than rubber therefore steel is more elastic than rubber.

3. Assertion: Strain is a unitless quantity.

Reason: Strain is equivalent to force.

Ans. (c) Assertion is true but reason is false

Strain is the ratio of change in dimensions of the body to the original dimensions. Because this is a ratio, therefore, it is a dimensionless quantity.

4. Assertion: It is easier to spray water in which some soap is dissolved.

Reason: Soap is easier to spread.

Ans. (c) Assertion is true but reason is false

On adding soap, surface tension of water decreases, the spraying of water becomes easy.

5. Assertion: It is better to wash the clothes in cold soap solution.

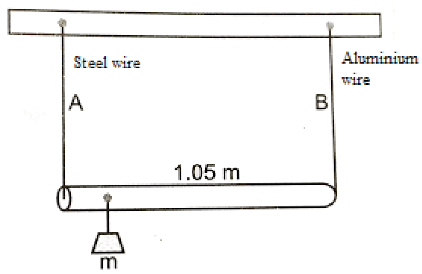
Reason: The surface tension of cold solution is more than the surface tension of hot solution.

Ans. (d) Both assertion and reason are false

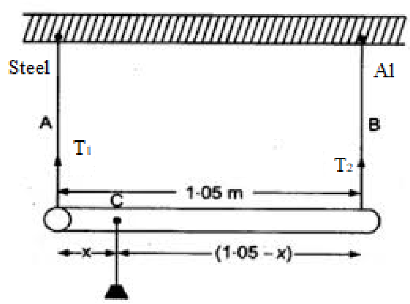
The soap solution has less surface tension as compared to ordinary water and its surface tension decreases further on heating. The hot soap solution can, therefore spread over large surface area and also it has more wetting power. It is on account of this property that hot soap solution can penetrate and clean the clothes better than the ordinary water.

G. CHALLENGING PROBLEMS

1. A rod of length 1.05m having negligible mass is supported at its ends by two wires of steel (wire A) and aluminium (wire B) of equal lengths as shown in figure. The cross section areas of wires A and B are 1.0 mm^2 and 2.0 mm^2 respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stresses and (b) equal strains in both steel and aluminium wires?



Sol.



Suppose the mass m is suspended at distance x from wire A. let T_1 and T_2 be the tensions in the steel and aluminium wires respectively

(a) Stress in steel wire= T_1/A_1 and stress in aluminium wire = T_2/A_2

As both the stresses are equal so, $\frac{T_1}{A_1} = \frac{T_2}{A_2}$ or $\frac{T_1}{T_2} = \frac{A_1}{A_2} = \frac{1.0\text{mm}^2}{2.0\text{mm}^2} = \frac{1}{2}$

Now the moments about C are equal because the system is in equilibrium.

Therefore, $T_1x = T_2(1.05 - x)$

Or $\frac{T_1}{T_2} = \frac{1.05-x}{x}$ or $\frac{1}{2} = \frac{1.05-x}{x}$ or x = 2.10 – 2x 0r x = 0.7m (from steel wire)

(b) Strain = $\frac{\text{Stress}}{\text{Young's modulus}}$

Therefore strain in steel wire = $\frac{T_1/A_1}{Y_1} = \frac{T_1}{A_1Y_1}$

Strain in aluminium wire = $\frac{T_2}{A_2Y_2}$

For two strains to be equal $\frac{T_1}{A_1Y_1} = \frac{T_2}{A_2Y_2}$ or $\frac{T_1}{T_2} = \frac{A_1Y_1}{A_2Y_2}$

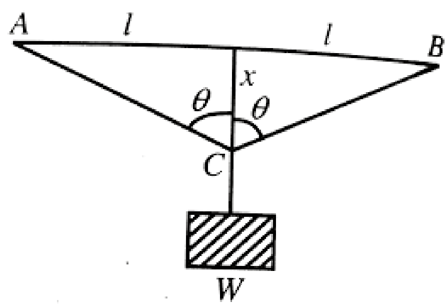
= $\frac{1.0mm^2\times200\times10^9Pa}{2.0mm^2\times70\times10^9Pa} = \frac{10}{7}$

Again $T_z = T_2(1.05 - x)$

Or 10x = 73.35 – 7x =r x = $\frac{7.35}{17}$ = 0.43m (from steel wire)

2. A wire of cross sectional area A is stretched horizontally between two clamps located at a distance 2l metres from each other. A weight W kg is suspended from the midpoint of the wire. If the vertical distance through which the midpoint of the wire moves down be x < l, then find (i) the strain produced in the wire (ii) the stress on the wire, (iii) If Y is the Young's modulus of the wire, then find the value of x

Sol.



(i) Here change in length is $\Delta l = [\,AC + BC\,] - 2l = 2(l^2 + x^2) - 2l$

$= 2l(1 + x^2/l^2) - 2l = x^2/l$

$\therefore$ strain = $\Delta l/2l = x^2/2l^2$

(ii) Stress = T/A = Wl/2Ax

(iii) $Y = \frac{Stress}{strain} = \frac{Wl}{2Ax} \times \frac{2l^2}{x^2} = \frac{Wl^3}{Ax^3}$

Or $x = l\left[\frac{W}{YA}\right]^{1/3} = \frac{1.0}{2}\left[\frac{0.100\times9.8}{2\times10^{11}\times0.5\times10^{-6}}\right]^{1/3}$

$= 0.5(9.8\times10^{-8})^{1/3} = 0.5\times2.14\times10^{-2} = 1.07\times10^{-2}$m = 1.07cm.