CLASS – 12

WORKSHEET- RAY OPTICS AND OPTICAL INSTRUMENTS

A. REFLECTION

(1 Mark Question)

1. When an object is placed between f and 2f of a concave mirror, would the image formed be, (i) real or virtual or (b) diminished or magnified?

Sol. When an object is placed between f and 2f of a concave mirror, the image formed is real and magnified.

(2 Marks Questions)

2. Use the mirror equation to show that an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

Sol. From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Now for a concave mirror, f < 0 and for an object on f the left of mirror, u < 0.

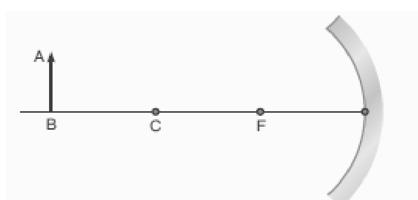
∴ 2f < u < f or $\frac{1}{2f} > \frac{1}{u} > \frac{1}{f}$ or $-\frac{1}{2f} < -\frac{1}{u} < -\frac{1}{f}$

Or $\frac{1}{f} - \frac{1}{2f} < \frac{1}{f} - \frac{1}{u} < \frac{1}{f} - \frac{1}{f}$ or $\frac{1}{2f} < \frac{1}{v} < 0$

This implies that v < 0 so that image is formed on left. Also the above inequality implies 2f > v or |2f|<|v| [since 2f and v are negative]

i.e. the real image is formed beyond 2f.

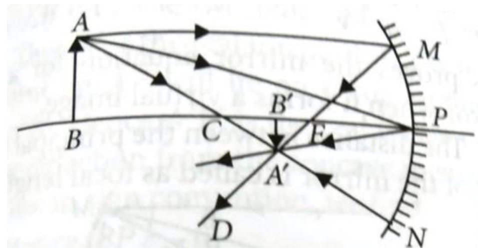
3. An object AB is kept in front of a concave mirror as shown in the figure.



(i) Complete the ray diagram showing the image formation of the object.

(ii) How will the position and intensity of the image be affected if the lower half of the mirror's reflecting surface is painted black?

Sol. (i)



(ii) Position of image will remain same/unchanged and intensity of image will decrease.

4. (a) Draw a ray diagram for a convex mirror showing the image formation of an object placed anywhere in front of the mirror.
(b) Use this ray diagram to obtain the expression for its linear magnification.

Sol. (a)

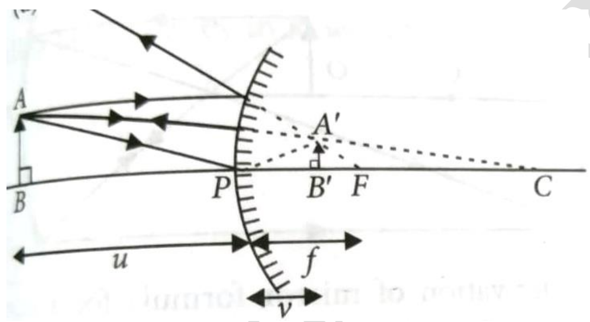


Figure shows the formation of image A'B' of a finite object AB by a convex mirror.

(b) Now, ΔABP – ΔA'B'P

$\therefore \frac{A'B'}{AB} = \frac{PB'}{PB}$

Applying the new Cartesian sign convention, A'B' = h_2, AB = h_1, PB' = v, PB = - u

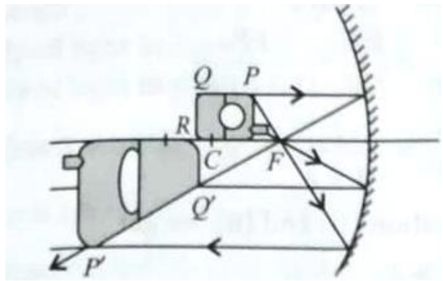
$\therefore \frac{h_2}{h_1} = \frac{v}{-u}$

Linear magnification, m = $\frac{h_2}{h_1} = -\frac{v}{u}$

(3 Marks Questions)

5. A mobile phone lies along the principal axis of a concave mirror. Show, with the help of a suitable diagram, the formation of its image. Explain why magnification is not uniform.

Sol. The formation of the image of the cell phone is shown in figure. The aprt which is at R will be damaged at R and will be of the same size i.e., Q'R = QR. The other end P of the mobile phone is highly magnified by the concave mirror.



Thus the different parts of the mobile phone are magnified in different proportions because of other different locations form the concave mirror.

6. A small candle, 2.5 cm in size is placed at 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how would the screen have to be moved?
By how much the screen has to be moved if the candle is moved towards the mirror?

Sol. Here h_1 = 2.5cm, u -= - 27cm, R = - 36cm, f = R/2 = - 18cm [Since R is –ve for a concave mirror)

By mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-18} + \frac{1}{27} = \frac{-3+2}{54} = -\frac{1}{54}$ or v = - 54cm

Thus the screen should be placed at 54cm from the mirror on the same side as the object.

Magnification, m = $\frac{h_2}{h_1} = -\frac{v}{u} = -\frac{-54}{-27} = -2$

Therefore, size of image, h_2= - 2 × 2.5 = - 5 cm

Negative sign allows that the same is real and inverted.

If the candle is moved closer to the mirror, the image moves away from mirror, so the screen would have to be moved further and farther from the mirror. Closer than 18cm from the mirror (when the focal length point is crossed), the image becomes virtual and cannot be taken on screen.

7. A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror.

Sol. Here h_1 = 4.5cm, u = - 12cm, f = +15cm [since f is +ve for a convex mirror]

By mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15} + \frac{1}{12} = \frac{4+5}{60} = \frac{9}{60} = \frac{3}{20}$ or v = + 20/3 = + 6.67cm

As v is +ve image is virtual and erect and is formed at 6.67cm behind the mirror.

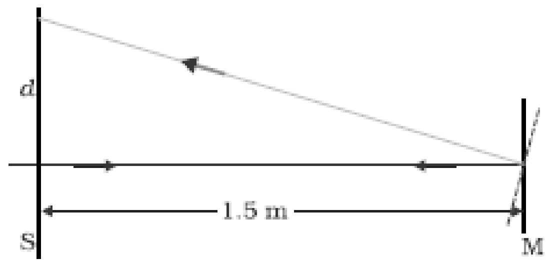
Magnification, m = $\frac{h_2}{h_1} = -\frac{v}{u} = -\frac{20}{3\times(-12)} = \frac{5}{9}$

Size of image $h_2 = \frac{5}{9} \times h_1 = \frac{5}{9} \times 4.5$ = 2.5 cm.

As the needle is moved farther from the mirror, the image shifts towards the focus (but never beyond F) and goes on decreasing in size.

8. Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in Fig. A current in the coil produces a deflection of 3.5° of the

mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away?



Sol. When the mirror is turned through angle θ, from position M to M', the reflected rays turns through angle 2θ, so that the reflected spot moves on the screen from position P and Q and ∠POQ = 2θ × 2 × 3.5° = 7°

Now tan 2θ = d/1.5.

∴ Displacement of reflected spot on the screen is d = 1.5 tan 2θ

= 1.5 × tan 7° = 1.5 × 0.1228m = 0.1842m = 18.4cm.

9. A 5cm long needle is placed 10cm from a convex mirror of focal length 40cm. Find the position, nature and size of the image of the needle. What happens to the size of the image when the needle is moved further away from the mirror?

Sol. Here h_1 = +5cm, u = - 10cm, f = +40cm

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{40} + \frac{1}{10} = \frac{5}{40} = \frac{1}{8}$

Or v = +8cm

As v is +ve, the image is virtual and erect and is formed at 8cm behind the mirror.

Magnification , m = $\frac{h_2}{h_1} = -\frac{v}{u} = -\frac{+8}{-10} = +0.8.$

Size of the image, h_2 = 0.8 × h_1 = 0.8 × 5 = 4cm

As the needle is moved farther away from the mirror, the image shifts towards the focus and its size goes on decreasing. When the needle is far off, it appears almost as a point image at the focus.

10. An object is placed at a distance of 40cm on the principal axis of a concave mirror of radius of curvature 30cm. By how much does the image move if the object is shifted towards the mirror through 15cm?

Sol. In the first case: u = - 40cm, R = -30cm or f = -15cm

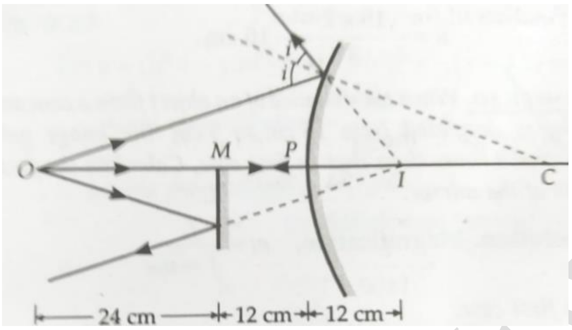
From the mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{15} + \frac{1}{40} = -\frac{1}{24}$ or v = - 24 cm

In the second case: The object is shifted towards the mirror by 15cm, so u ' = - (40 – 15) = - 25 cm

From mirror formula, $\frac{1}{v'} = \frac{1}{f'} - \frac{1}{u'} = -\frac{1}{15} + \frac{1}{25} = -\frac{2}{75}$

Or v' = 37.5 cm
Distance through which the image shifts = v' – v = -37.5 + 24 = - 13.5cm.

11. An object is placed exactly midway between a concave mirror of radius of curvature 40cm and a convex mirror of radius of curvature 30cm. The mirrors face each other and are 50cm apart. Determine the nature and position of the image formed by successive reflections first at the concave mirror and then at the convex mirror.



Sol. The image I of the object O formed at plane mirro should be at 24cm behind the mirror as 12cm behind the convex mirror. For no parallax between the images formed by the two mirrors, the image formed by the convex morror should also lie at I. Therefore, for convex mirror u = OP = - 36 cm; v = PI = +12cm

Therefore, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v} = -\frac{1}{36} + \frac{1}{12} = \frac{-1+3}{36} = \frac{1}{18}$ or f = 18cm.

Radius of curvature of convex mirror = 36cm.

12. When the distance of an object from a concave mirror is decreased from 15cm to 9cm, the image gets magnified 3 times than that in first case. Calculate the focal length of the mirror.

Sol. Magnification, m = $\frac{f}{f-u}$

In the first case, u = -15cm, ∴ m = $\frac{f}{f+15}$

In the second case, u = - 9cm, ∴ m = $\frac{f}{f+9}$

But m'= 3m

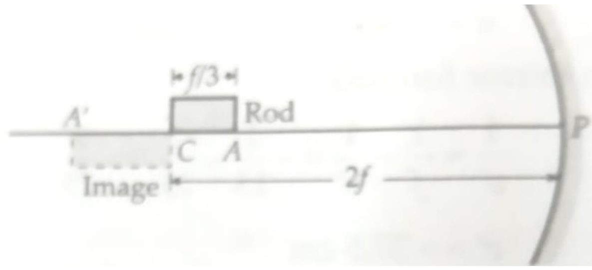
Or $\frac{f}{f+9} = \frac{3\times f}{f+15}$

Or f +15 = 3f + 27

f = - 6 cm.

13. A thin rod of length f/3 is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. What will be the magnification?

Sol. The image of the rod placed along the optical axis will touch the rod only when one end of the rod AC is at the centre of curvature of the concave mirror (PC = 2f, AC = f/3). Then the image of the end C of the rod will be formed at the same point C.



For the end A of the rod, we have u = PA = PC – AC = 2f - $\frac{f}{3} = \frac{5f}{3}$

From the mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{f} - \frac{3}{5f} = \frac{2}{5f}$

Thus the image of A is formed at A' at a distance 5f/2 from the pole P(PA' = 5f/2)

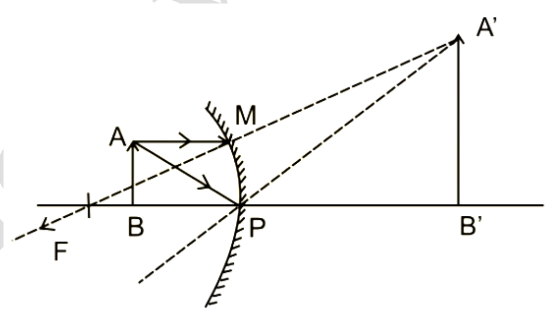
Length of the image = A'C = PA' - PC = 5f/2 – 2f = f/2.

Therefore magnification = $\frac{CA'}{CA} = \frac{f/2}{f/3}$ = 1.5.

(5 Marks Questions)

14. An object is placed in front of a concave mirror. It is observed that a virtual image is formed. Draw the ray diagram to show the image formation and hence derive the mirror equation: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$.

Sol. Consider an object AB placed on the principal axis of a concave mirror between its pole P and focus F. As shown in figure, a virtual and erect image A'B' is formed behind the mirror after reflection from the concave mirror.



Using the Cartesian sign convention, we find that Object distance, BP = - u, Image distance, PB' = v, Focal length, FP = - f, Radius of curvature, CP = - R = - 2f

Now ΔABC – ΔA'B'C

$\therefore \frac{AB}{A'B'} = \frac{CB}{CB'} = \frac{CP-BP}{CP+PB'} = \frac{-2f+u}{-2f+v}$ …(i)

Also, ΔMPF = ΔA'B'F

$\therefore \frac{MP}{A'B'} = \frac{FP}{FB'} = \frac{FP}{FP+PB'}$ or $\frac{AB}{A'B'} = \frac{-f}{-f+v}$ …(ii)

From equations (i) and (ii) we get

$\frac{-2f+u}{-2f+v} = \frac{-f}{-f+v}$

Or $2f^2 - fu - 2fv + uv = 2f^2 - fv$

Or – fv – fu + uv = 0

Or uv = fv + fu

Dividing both sides by uvf, we get

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

This proves the mirror equation for a concave mirror when it forms a virtual image.

15. Use the mirror equation to deduce that:

(i) an object placed between f and 2f of a concave mirror produces a real image beyond 2f.

(ii) a convex mirror always produces a virtual image independent of the location of the object.

(iii) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(iv) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image.

Sol. (i) Same as sol. Q 2.

(ii) For convex mirror, $f > 0$ and for an object on left, $u < 0$. From mirror formula,

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

This implies that $\frac{1}{v} > 0$ or $v > 0$

This shows that whatever be the value of v, a convex mirror forms a virtual image on the right.

(iii) For convex mirror, $f > 0$ and for an object on the left $u < 0$ so mirror formula,

$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

Implies that $\frac{1}{v} > \frac{1}{f} \left[\because\ -\frac{1}{u} \text{ is + ve quantity}\right]$ or $v < f$

This show that the image is located between the pole and the focus of the mirror.

(iv) From mirror formula, $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$

For a concave mirror, $f < 0$ and for an object located between the pole and focus of a concave mirror, $f < u < 0$.

$\therefore \frac{1}{f} > \frac{1}{u}$ or $\frac{1}{f} - \frac{1}{v} = 0$ or $\frac{1}{v} > 0$

i.e., a virtual image is formed on the right.

Also, $\frac{1}{v} < \frac{1}{|u|}$ or $v > |u| \therefore |m| = \frac{v}{|u|} > 1$

i.e. image is enlarged.

B. REFRACTION THROUGH PLANE SURFACE

(1 Mark Question)

1. For the same value of angle of incidence, the angles of refraction in three media A, B and C are 15°, 25° and 35° respectively. In which media would the velocity of light be minimum?

Sol. Refractive index, $\mu = c/v = \sin i/\sin r$
As $\sin 15° < \sin 25° < \sin 35°$
So, $v_A < v_B < v_C$
Hence in medium A, velocity of light is minimum.

2. State the criteria for the phenomenon of total internal reflection of light to take place.

Sol. Essential conditions for total internal reflection:
(i) Light should travel from a denser medium to a rarer medium.
(ii) Angle of incidence in denser medium should be greater than the critical angle for the pair of media in contact.

3. Define refractive index of a media.

Sol. It is the ratio of speed of light in vacuum to the speed of light in media.

4. Why does sun appear red at sunrise and sunset?

Sol. During sunrise or sunset, the sun is near the horizon. Sunlight has to travel a greater distance so shorter waves of blue region are scattered away by the atmosphere, red waves of longer wavelength are least scattered and reach the observer. So the sun appears ed.

5. Why does bluish colour predominate in a clear sky?

Sol. The amount of scattering as per Rayleigh's law depends upon wavelength.
Scattering $\propto \frac{1}{\lambda^4}$
As $\lambda_B < \lambda_R$
Hence blue colour scatters more and also blue colour is most sensitive to oru eyes than any colour like violet and indigo. Thus the part of atmosphere which we observe as sky has scattering of blue colour mostly, thus sky appears to be bluish.

6. Do the frequency and wavelength change when light passes from a rarer to a denser medium?

Sol. When light passes from a rarer to a denser medium, wavelength fo light changes byt frequency remains unchanged.

7. Does critical angle depend on colour of light? Explain.

Sol. Yes, As $\sin i_c = 1/\mu$ i.e. i_c depends on μ, but μ depends on wavelength λ. Hence i_c depends on colour of light.

8. During summer noon, why do the trees and houses on the other side of an open ground appear to be shaking?

Sol. Open ground becomes very hot during a summer noon. It heats up the air in contact. Convection currents are set up in air. Light rays passing through this air changes their path due to refraction. This gives shaking appearance to the objects from which these light rays start.

9. How is the refractive index of a medium related to the wavelength of incident light?

Sol. The refractive index μ of a material is related to wavelength l by the Cauchy's relation

$\mu = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4}$ where a, b, c are constants which depend on the nature of the material.

10. Eye is more sensitive to yellow colour. Why do then we use traffic light stop signals of red colour?

Sol. According to Rayleigh's law of scattering, the intensity of light of wavelength λ present in the scattered light is inversely proportional to fourth power of wavelength λ: $I \propto 1/\lambda^4$. In the visible spectrum, red colour has the largest wavelength, it is least scattered and can be easily observed even in foggy and dust conditions.

11. Why is yellow medium light used for illumination in foggy conditions?

Sol Yellow light has longer wavelength than green, blue or violet components of white light. As scattered intensity $\propto 1/\lambda^4$, so yellow colour is least scattered and produces sufficient illumination.

12. In which direction relative to the normal does a ray of light bend, when it enters obliquely a medium in which its speed is increased?

Sol. The ray of light bends away from the normal.

(2 Marks Questions)

13. Calculate the speed of light in a medium whose critical angle is 30°.

Sol. Here $i_c = 30°$

As $\mu = \frac{c}{v} = \frac{1}{\sin i_c}$

$\therefore v = c \sin i_c = 3 \times 10^{98} \times \sin 30°$

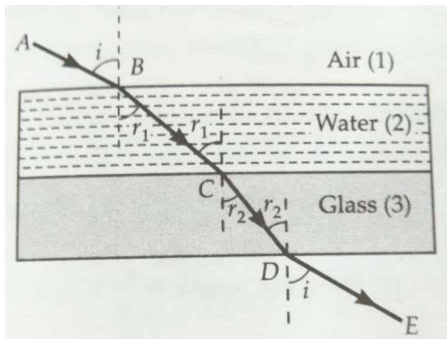
$= 3 \times 10^8 \times 0.5 = 1.5 \times 10^8 \text{ ms}^{-1}$.

14. Find the maximum angle of refraction when a ray of light is refracted from glass (μ = 1.5) to air.

Sol. When the ray of light is incident at the critical angle i_c, the angle of refraction is maximum and is equal to 90°.

15. For a ray of light suffering refraction through a combination of thee media, show that: ${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$.

Sol.



For the ray going from medium 1 to medium 2, ${}^1\mu_2 = \sin i/\sin r_1$

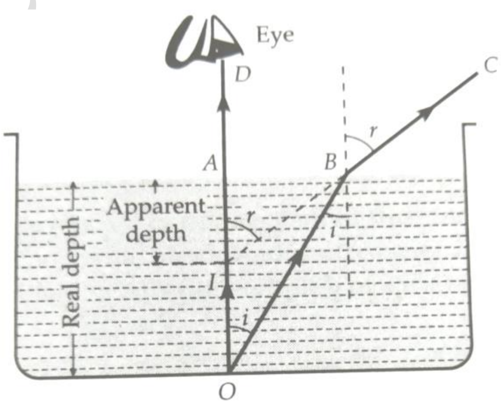
For the ray going from medium 2 to medium 3, ${}^2\mu_3 = \sin r_1/\sin r_2$

For the ray going from medium 3 to medium 1, ${}^3\mu_1 = \sin r_2/\sin i$

Multiplying the above three equations, we get ${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$

16. Deduce the relation: $\mu = \dfrac{\text{Re al Depth}}{\text{Apparent depth}}$

Sol. Figure shows a point O placed at the bottom of a beaker filed with water. The rays OA and OB strating from O are refracted along AD and BC, respectively. These rays appear to diverge from point I.



So I is the virtual image of O. Clearly, the apparent depth Al is smaller than the real depth AO. This is why a water tank appears shallower.or an object placed at the bottom appears to be raised.

From Snell's law we have ${}^{\omega}\mu_a = \frac{\sin i}{\sin r} = \frac{\sin\angle AOB}{\sin\angle AIB} = \frac{AB/BO}{AB/BI} = \frac{BI}{BO}$

As size of the pupil is small, the ray BC will enter the ye only if B is close to A. Then BI = AI and BO= AO.

$\therefore {}^{a}\mu_{\omega} = \frac{1}{{}^{\omega}\mu_{a}} = \frac{AO}{AI}$

Or refractive index = $\frac{\text{real Depth}}{\text{Apparent depth}}$

(3 Marks Questions)

17. Refractive index of glass is 1.5. Calculate the velocity of light in glass if velocity of light in vacuum is 3×10^{6} ms^{-1}. Also calculate the critical angle for glass-air interface.

Sol. $v = \frac{c}{\mu} = \frac{3\times10^{8}}{1.5} = 2\times10^{8}$ ms^{-1}.

Also, $\sin i_c = \frac{1}{\mu} = \frac{1}{1.6} = \frac{2}{3} = 0.6667$

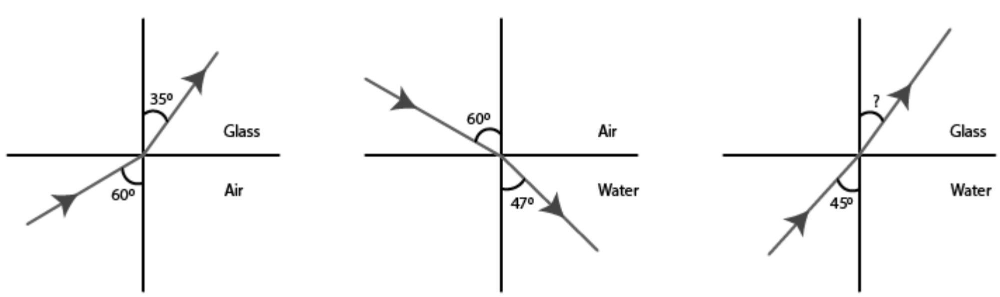
$\therefore i_c = 41°49'$.

18. State the reason for the following observations recorded from the surface of moon: (i) sky appears dark, (ii) rainbow is never formed.

Sol. (i) Moon has no atmosphere. So there is nothing to scatter sunlight towards the moon. No skylight reaches moon surface. Sky appears black in the day time as it does at night.

(iii) No water vapours are present at moon's surface. No clouds are formed. There are no rains on the moon. So rainbow is never observed.

19. The figures show the refraction of a ray in air incident at 60° with the normal to a glass-air and water-air interface, respectively. Predict the angle of refraction in glass when the angle of incidence in water is 45° with the normal to a water-glass interface.



Sol. From figure a, ${}^{a}\mu_g = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 35°} = \frac{0.8660}{0.5736} = 1.51$

From figure b, ${}^{a}\mu_w = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 41°} = \frac{0.8660}{0.6561} = 1.32$

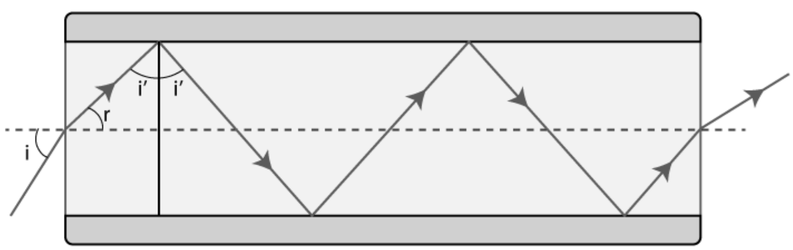
From figure c, ${}^{w}\mu_g = \frac{{}^{a}\mu_g}{{}^{a}\mu_w} = \frac{\sin i}{\sin r}$

Or $\frac{1.51}{1.32} = \frac{sin45°}{sin\, r} = \frac{0.7071}{sin\, r}$
Or sin r = $\frac{1.32\times0.7071}{1.51}$= 0.6181
r = 38.2°.

20. A small pin fixed on a tabletop is viewed from above from a distance of 50 cm. By what distance would the pin appear to be raised if it is viewed from the same point through a 15 cm thick glass slab held parallel to the table? Refractive index of glass = 1.5. Does the answer depend on the location of the slab?

Sol. The distance through which the pin appears to be raised is d = Real thickness of slab – Apparent thickness of slab.
= Real thickness of slab – $\frac{\text{Real thickness of slab}}{\mu} = t - \frac{t}{\mu} = t\left(1 - \frac{1}{\mu}\right)$
Here t= 15cm, μ = 1.5
Therefore d = $15\left(1 - \frac{1}{1.5}\right) = 15\left(\frac{1.5-1}{1.5}\right)$ = 5cm.

21. (i) Figure below shows a cross-section of a 'light pipe' made of a glass fiber of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of the angles of the incident rays with the axis of the pipe for which total reflections inside the pipe take place, as shown in the figure. (ii) What is the answer if there is no outer covering of the pipe?



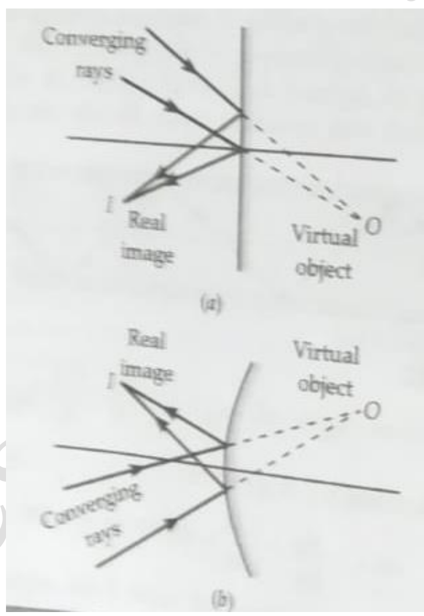
Sol. Given μ_2 = 1.68, μ_1 = 1.44, $\mu = \frac{\mu_1}{\mu_2} = \frac{1}{\sin i'_c}$
∴ Critical angle i'_c is given by $\sin i'_c = \frac{\mu_1}{\mu_2} = \frac{1.44}{1.68}$= 0.8571 so, i'_c = 59°
Total internal refelction will occur if the angle i' > i'_c, i.e. if i' > 59° or when r < r_{max}, where r_{max} = 90° - 59° = 31°.
Using Snell's law, $\frac{\sin i_{max}}{\sin r_{max}} = 1.68$
Or sin i_{max} = 1.68 × sin r_{max} = 1.68 × sin31° = 1.68 × 0.5150 = 0.8662
Therefore, i_{max} = 60°
Thus incident ray which make angles in the range 0 < i < 60° with the axis of the pipe will suffer total internal reflections in the pipe.

(5 Marks Questions)

22. Answer the following questions:

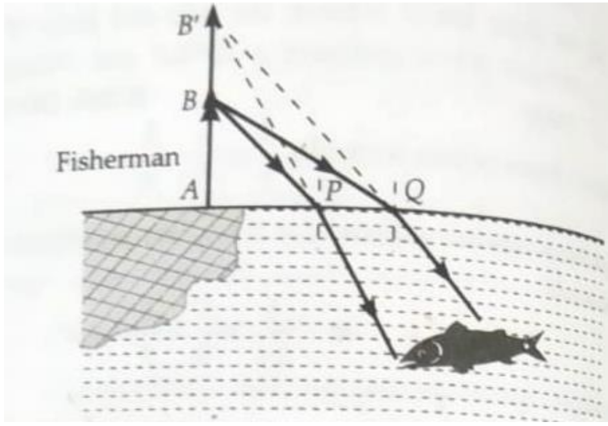
(i) You have learnt that plane and convex mirrors produce virtual images of objects. Can they produce real images under some circumstances? Explain.
(ii) A virtual image, we always say, cannot be caught on a screen. Yet when we 'see' a virtual image, we are obviously bringing it on to the 'screen' (i.e., the retina) of our eye. Is there a contradiction?
(iii) A diver underwater, looks obliquely at a fisherman standing on the bank of a lake. Would the fisherman look taller or shorter to the diver than what he actually is?
(iv) Does the apparent depth of a tank of water change if viewed obliquely? If so, does the apparent depth increase or decrease?
(v) The refractive index of diamond is much greater than that of ordinary glass. Is this fact of some use to a diamond cutter?

Sol. (i) Yes, a plane or convex mirror can produce a real image if the object is virtual. As Shown in figure a and b if a plane or a convex mirror is placed in the path of rays converging to a point, the rays get reflected to a point in front of the mirror. Real image can be obtained on a screen.

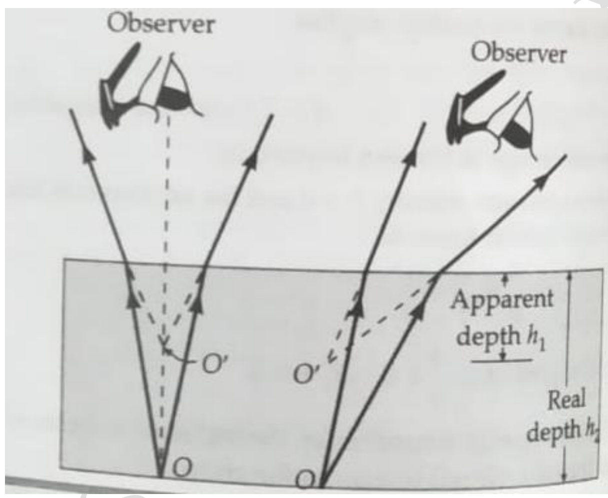


(ii) When the reflected and refracted rays are divergent, the image is virtual. These rays are converged by the eyelens to form a real image on the retina. The virtual image serves as a virtual object. Also the screen is not located at the position of virtual image. So there is no contradiction.

(iii) The man looks taller to a diver under water. As the fisherman is in air, light rays travel from rater to denser medium. They bend towards the normal and hence appear to come from a larger distance as shown in figure, it may be noted that the points P and Q are in fact so close that the rays through these points can enter the small aperture of the eye of the fish. Here AB = Real height of the man, AB' = Apparent height of the man.



(iv) Yes, the apparent depth decreases for oblique viewing from its value of normal viewing. This is obvious from the ray diagrams shown.

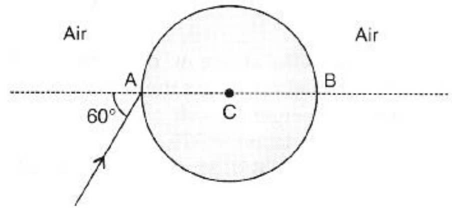


(v) Yes, refractive index of diamond is high (μ = 2.42) so its critical angle is small i_e = 24°). A diamond cutter makes use of this large range of angles of incidence (24° to 90°) to ensure that light entering diamond suffers total internal reflection, several times. When light emerges out, it produces sparkling effect.

C. REFRACTION THROUGH CURVED SURFACE

(1 Mark Question)

1. A ray of light falls on a transparent sphere with centre C as shown in the figure. The ray emerges from the sphere parallel to the line AB. Find the angle of refraction at A if refractive index of the material of the sphere is √3.



Sol. From Snell's laws, we have $\frac{\sin(i)}{\sin(r)} = \mu$

At A, I = 60°, $\mu = \sqrt{3}$

Now $\sin(r) = \frac{\sin(i)}{\mu}$

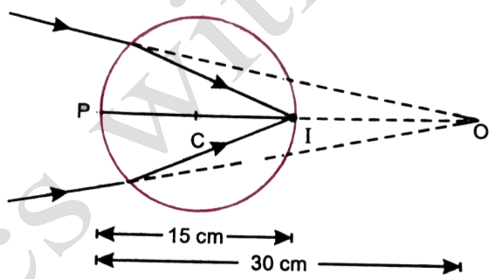
$\Rightarrow \sin(r) = \frac{\sin(60°)}{\sqrt{3}} = \frac{1}{2} \Rightarrow r = \sin^{-1}\left(\frac{1}{2}\right)$

$\therefore$ r = 30°.

(3 Marks Questions)

2. The diameter of a glass sphere is 15cm. A beam of light strikes the sphere, which converges at point 30cm behind the pole of the spherical surface. Find the position of the image if $\mu = 1.5$.

Sol. In the absence of glass sphere, the light rays will converge at point O. So, O acts as virtual object for the image I for the second surface.



Therefore, u = OI = OP = OP – IP = 30 – 15 = 15cm, $\mu_1 = 1$, $\mu_2 = 1.5$, R = + 15/2 = + 7.5cm.

As the light passes from rarer to denser medium, so

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Or $\frac{1.5}{v} - \frac{1}{30} = \frac{1.5-1}{7.5} = \frac{0.5}{7.5} = \frac{1}{15}$

Or $\frac{1.5}{v} = \frac{1}{15} + \frac{1}{30} = \frac{1}{10}$

Or v = +10 × 1.5 = + 15cm.

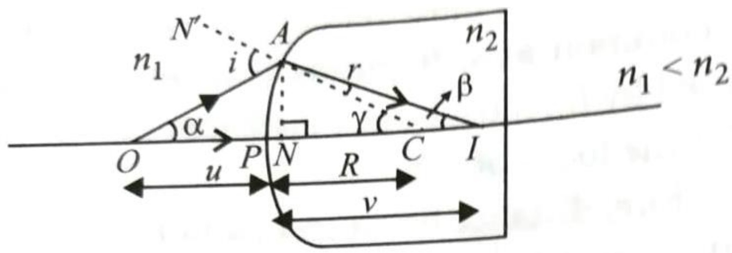
Thus the image is formed at the other end (l) of the diameter.

(5 Marks Questions)

3. Obtain lens makers formula using the expression: $\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}$

Here the ray of light propagating from a rarer medium of refractive index (n_1) to a denser medium of refractive index (n_2) is incident on the convex side of spherical refracting surface of radius of curvature R.

Sol. Refraction at convex spherical surface: When object is in rarer medium and image formed is real.



In ΔOAC, $i = \alpha + \gamma$ and in ΔAIC, $\gamma = r + \beta$ or $r = \gamma \square \beta$

∴ By Snells' law ${}^1n_2 = \frac{\sin i}{\sin r} = \frac{i}{r} = \frac{\alpha+\gamma}{\gamma-\beta}$ or $\frac{n_2}{n_1} = \frac{\alpha+\gamma}{\gamma-\beta}$ or $n_2\gamma - n_2\beta = n_1\alpha + n_1\gamma$ or $(n_2 - n_1)\gamma = n_1\alpha + n_2\beta$ …(i)

As α, β and γ are small and P and BN lie close to each other.

So, $\alpha = \tan\alpha = \frac{AN}{NO} = \frac{AN}{PO}$

$\beta = \tan\beta = \frac{AN}{NI} = \frac{AN}{PI}$

$\gamma = \tan\gamma = \frac{AN}{NC} = \frac{AN}{PC}$

On using them in equation (i) we get

$(n_2 - n_1)\frac{AN}{PC} = n_1\frac{AN}{PO} + n_2\frac{AN}{PI}$ or $\frac{n_2-n_1}{PC} = \frac{n_1}{PO} + \frac{n_2}{PI}$ …(ii)

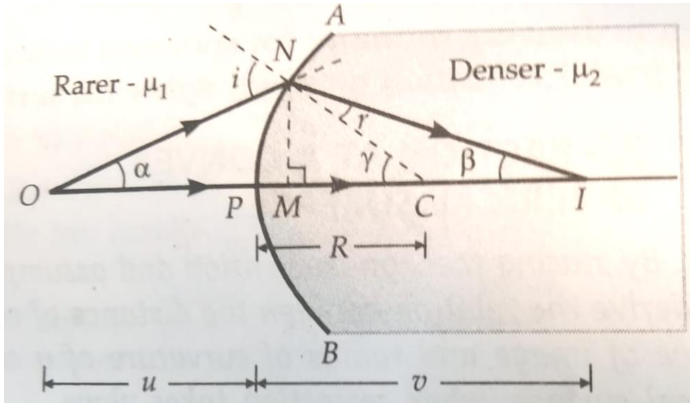
Where PC = +R, radius of curvature, PO = - u, object distance, PI = +v, image distance

So, $\frac{n_2-n_1}{R} = \frac{n_1}{-u} + \frac{n_2}{v}$ or $\frac{n_2-n_1}{R} = \frac{n_2}{v} - \frac{n_1}{u}$

This give formula for refraction at spherical surface when object is in rarer medium.

4. Derive the expression for convex surface: $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$ where symbols have their usual meanings.

Sol.



Suppose all the rays are paraxial. Then the angles i, r, α, β and γ will be small.
Therefore, $\alpha = \tan\alpha = NM/OM = NM/OP$ [since P is close to M]
$\beta = \tan\beta = NM/MI = NM/PI$ and $\gamma = \tan\gamma = NM/MC = NM/PC$
From Snell's law of refraction, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$
As i and r are small, therefore $i/r = \mu_2/\mu_1$
Or $\mu_1 i = \mu_2 r$
Or $\mu_1[\alpha + \gamma] = \mu_2[\gamma - \beta]$
Or $\mu_1\left[\frac{NM}{OP} + \frac{NM}{PC}\right] = \mu_2\left[\frac{NM}{PC} - \frac{NM}{PI}\right]$
Or $\mu_1\left[\frac{1}{OP} + \frac{1}{PC}\right] = \mu_2\left[\frac{1}{PC} - \frac{1}{PI}\right]$
Or $\frac{\mu_1}{OP} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$
Using new Cartesian sign convention, we find object distance OP = - u, Image distance, PI = +v, Radius of curvature, PC = +R.
$\therefore \frac{\mu_1}{-u} + \frac{\mu_1}{v} = \frac{\mu_2 - \mu_1}{R}$ or $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

D. LENS

(1 Mark Question)

1. A concave lens of refractive index 1.5 is immersed in medium of refractive index 1.65. What is the nature of the lens?

Sol. Focal length of a concave lens is negative. Using lens maker's formula,
$\frac{1}{f} = \left(\frac{\mu_l}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
Here $\mu_l = 1.5$, $\mu_m = 1.65$
Also $\frac{\mu_l}{\mu_m} < 1$, so $\left(\frac{\mu_l}{\mu_m} - 1\right)$ is negative and focal length of the given lens becomes positive.
Hence, it behaves as a convex lens.

2. When does a convex lens behave as a concave lens?

Sol. When a lens is placed inside a transparent medium of refractive index greater than that iof its own material, it behaves as a concave lens.

3. What happens to a focal length of a convex lens, when it is immersed in water?

Sol. Focal length f of a convex lens is related to the refractive index as $f \propto \frac{1}{\mu - 1}$

As ${}^{w}\mu_g < {}^{a}\mu_g$ so focal length of the convex lens wil increase when it is immersed in water.

4. A lens of glass is immersed in water. What will be its effect on the power of the lens?

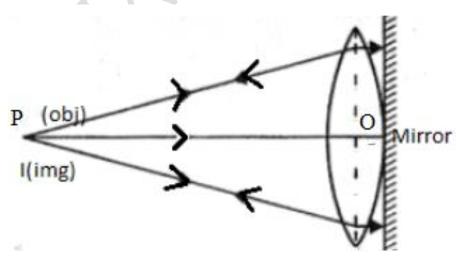
Sol. Power of a lens $P \propto (\mu - 1)$. As ${}^{w}\mu_g < {}^{a}\mu_g$ so power of the glass lens will decrease when it is immersed in water.

5. How does the focal length of a convex lens change if monochromatic red light is used instead of violet light?

Sol. Focal length, $f \propto 1/\mu - 1$. As $\mu_R < \mu_V$, so the focal length fo the combination decreases.

6. A convex lens is placed in contact with a plane mirror. A point object at a distance of 20cm on the axis of this combination has its image coinciding with itself. What is the focal length of the lens?

Sol.



From figure, focal length of lens = OP = 20cm.

(2 Marks Questions)

7. The focal length of an equiconcave lens is ¾ times of radius of curvature of its surfaces. Find the refractive index of the material of the lens. Under what condition will this lens behave as a converging lens?

Sol. Given $f = \frac{-3R}{4}$. From lens maker's formula,

$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \frac{1}{\frac{-3R}{4}} = (\mu - 1)\left[\frac{1}{-R} - \frac{1}{R}\right]$

$\frac{4}{3R} = \frac{2(\mu-1)}{R} \Rightarrow \mu = \frac{5}{3}$

It will behave as a converging lens if $(\mu - 1) < 0$ or $\mu < 1$.

8. The radius of curvature of each surface of a convex lens of refractive index 1.5 is 40cm. Calculate its power.

Sol. Here $\mu = 1.5$, $R_1 = +40cm = 0.40m$, $R_2 = -40cm = -0.40$ m.

$P = \frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$= (1.5 - 1)\left[\frac{1}{40} + \frac{1}{40}\right] = 2.5D$

9. Two thin lenses of focal lengths +10cm and – 5cm are kept in contact. What is the (i) focal length and (ii) power of the combination?

Sol. Here $f_1 = +10cm = 0.10m$, $f_2 = -5cm = -0.05m$.

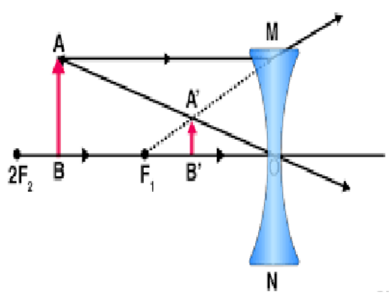
(i) $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.10} + \frac{1}{-0.05} = -10$

$f = -\frac{1}{10}m = -10cm$

(ii) $P = P_1 + P_2 = \frac{1}{f_1} + \frac{1}{f_2} = -10D$

10. Derive the lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, for a concave lens, using necessary ray diagram.

Sol.



As $\Delta A'B'O \sim \Delta ABO$

So, $\frac{A'B'}{AB} = \frac{B'O}{BO}$ …(1)

Also, $\Delta A'B'F \sim \Delta MOF$

So, $\frac{A'B}{MO} = \frac{FB'}{FO}$

But MO = AB, therefore $\frac{A'B'}{AB} = \frac{FB'}{FO}$ …(2)

From (1) and (2) we get

$\frac{B'O}{BO} = \frac{FB'}{FO} = \frac{FO - B'O}{FO}$

Using new Cartesian sign convention, we get BO = - u, B'O= - v, FO = -f

$\therefore \frac{-v}{-u} = \frac{-f+v}{-f}$

Or vf = uf – uv or uv = uf – vf

Dividing both sides by uvf, we get $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

11. Double-convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required if the focal length is to be 20cm? [Ans. 22.0 cm]

____

____

____

____

12. The image of a small electric bulb fixed on the wall of a room is to be obtained on the opposite wall 3 m away by means of a large convex lens. What is the maximum possible focal length of the lens required for the purpose? [Ans. 0.75m]

____

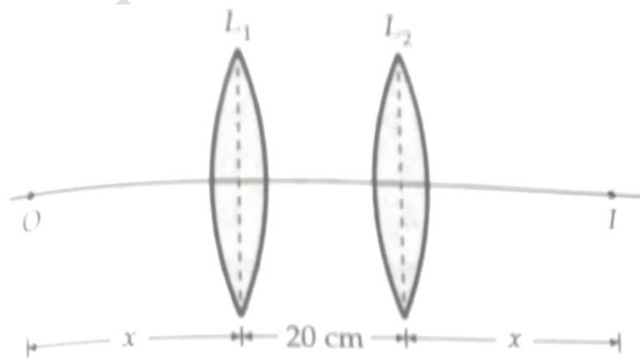
____

____

____

13. A screen is placed 90 cm from an object. The image of the object on the screen is formed by a convex lens at two different locations separated by 20 cm. Determine the focal length of the lens.

Sol. As shown in figure, let O and I be the positions of object and image respectively and L_1 and L_2 be the two conjugate positions of the lens.



Obviously x + 20 + x = 90cm or x = 35cm

When the lens is in position, L_1, we have u = - x = - 35cm, v = 20 + x = 20 +35 = 55cm

$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{55} + \frac{1}{35} = \frac{7+11}{385} = \frac{18}{385}$ or f = $\frac{385}{18}$ = 21.4cm

(3 Marks Questions)

14. A convex lens of focal length 0.2m and made of glass (μ = 1.50) is immersed in water (μ = 1.33). Find the change in focal length of the lens.

Sol. For glass lens in air, $^a\mu_s = 1.5$, $f_a = 0.2$m

As $\frac{1}{f_a} = (^a\mu_g - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$\therefore \frac{1}{0.2} = (1.5 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Or $\frac{1}{R_1} - \frac{1}{R_2} = 10$

For same lens in water, $^a\mu_w = 1.33$

$\frac{1}{f_w} = (^a\mu_w - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

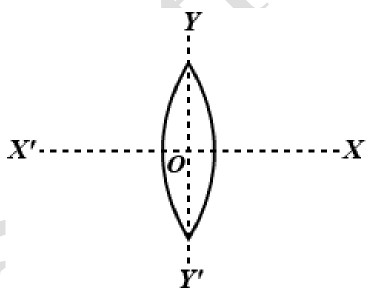
$= \left(\frac{^a\mu_g}{^a\mu_w}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$= \left(\frac{1.5}{1.33} - 1\right) \times 10 = \frac{0.17 \times 10}{1.33}$

Or $f_w = \frac{133}{170} = 0.78$ m

$\therefore$ Change in focal length = $f_w - f_a$ = 0.78 – 0.20 = 0.58m.

15. An equiconvex lens of focal length 15cm is cut into two equal halves as shown in fig. What is the focal length of each half?



Sol. For the equiconvex lens, let $R_1 = +R$, $R_2 = -R$.

Then from the lens maker's formula, $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} + \frac{1}{R}\right] = \frac{2(\mu-1)}{R}$ …(i)

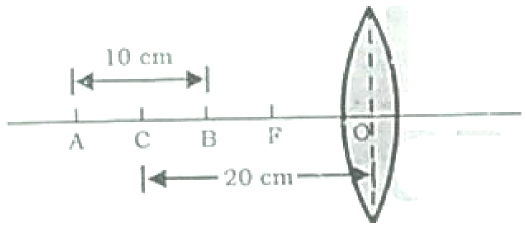
For each half lens, $R_1 = R$, $R_2 = -\infty$

$\therefore \frac{1}{f'} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{-\infty}\right] = \frac{\mu-1}{R}$ …(ii)

Dividing (i) by (ii) we get f'/f = 2

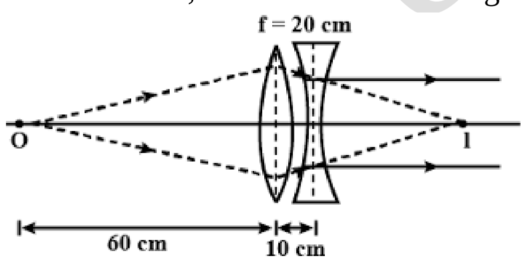
Or f' = 2f = 2 × 15 = 30cm.

16. A needle 10cm long is placed along the axis of a convex lens of focal length 10cm such that the middle point of the needle is at a distance of 20cm from the lens. Find the length of the image of the needle.



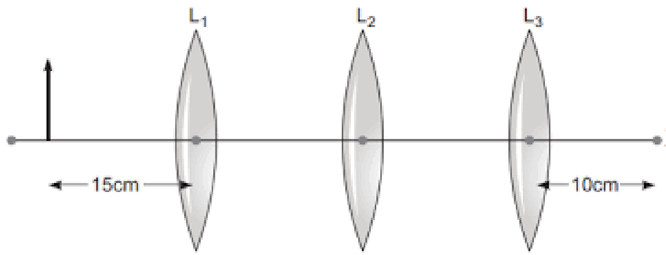
Sol. Figure shows a needle AB of length 10cm placed on the axis of convex lens.
Here CO = 20cm, AO = 20+5 = 25cm, BO = 20 – 5 = 15cm.
For the image of end A of the needle, u_1 = AO = - 25cm, f= + 10cm
Using thin lens formula, $\frac{1}{v_1} = \frac{1}{f} + \frac{1}{u_1} = \frac{1}{+10} + \frac{1}{-25} = \frac{3}{50}$
Or v_1 = 50/3 = 16.67cm
For the image of end B of the needle, u_2 = BO = - 15cm, f = +10cm
$\therefore \frac{1}{v_2} = \frac{1}{f} + \frac{1}{u_2} = \frac{1}{+10} + \frac{1}{-15} = \frac{1}{30}$ or v_2 = 30cm
Hence the length of the image of needle Ab = $v_2 - v_1$ = 30 – 16.67 = 13.33 cm.

17. From the ray diagram shown below, calculate the focal length of the concave lens.



Sol. For the convex lens, f = +20cm, u = -60cm, v = ?
From the lens formula, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{20} - \frac{1}{60} = +\frac{20}{60} = +\frac{1}{30}$
Or v = +30cm
The image I' formed by the convex lens serves as an object for the concave lens. But the rays converging on the concave lens become parallel after refraction through it and form image at infinity.
∴ For the concave lens: u = + (30 – 10) = +20cm, v = ∞, f = ?
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{\infty} - \frac{1}{20} = -\frac{1}{20}$
Or f = - 20cm.

18. You are given three lenses L_1, L_2 and L_3 each of focal length 10cm. An object is kept at 15cm in front of L_3 as shown. The final real image is formed at the focus I of L_3. Find the separations between L_1, L_2 and L_3.



Sol. For lens L_1, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{v}-\frac{1}{-15}=\frac{1}{10}$

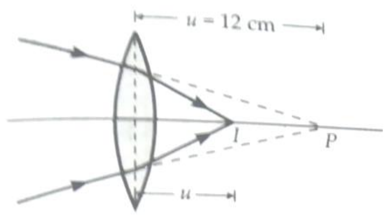
or $\frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$ or v = 30cm.

The image formed by L_1 must lie on the focus of L_2. Only then the rays form L_2 will become parallel to the principal axis and the final image formed by L_3 will lie at its focus i.e., at a distance of 10cm from L_3.

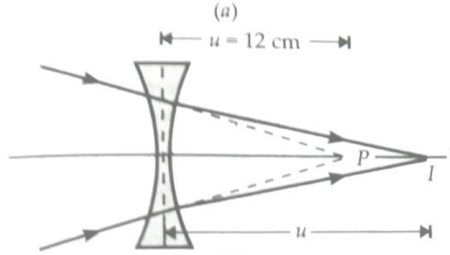
∴ $L_1L_2 = 30 + 10 = 40$cm, L_2L_3 can be any finite distance.

19. A beam of light converges at a point P. Now a lens is placed in the path of the convergent beam 12cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20cm, and (b) a concave lens of focal length 16cm?

Sol. Here the point P on the right of the lens acts as a virtual object but the image I is real, as shown in figure a and b.



(a)



(b)

(a) For convex lens, u = +12cm, f = +20cm

Now, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ ∴ $\frac{1}{v}-\frac{1}{12}=\frac{1}{20}$

Or $\frac{1}{v}=\frac{1}{20}+\frac{1}{12}=\frac{3+5}{60}=\frac{8}{60}$

Or v = 15/2 = 7.5cm

Thus the beam converges at a point 7.5cm to the right of the lens.

(b) For concave lens, u = +12cm, f = - 16cm

Now, $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{-16}+\frac{1}{12}=\frac{-3+4}{48}=\frac{1}{48}$

Therefore v = 48cm

Thus the beam converges at point 48cm to the right of the lens.

20. An object of size 3.0cm is placed 14cm in front of a concave lens of focal length 21cm. Describe the image produced by the lens. What happens if the object is moved further away from the lens?

Sol. Here h_1 = 3cm, u = -14cm, f = -21cm, v = ?

For a lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-21} + \frac{1}{-14} = \frac{-2-3}{42} = \frac{-5}{42}$

Or v = - 8.4 cm

Negative v indicates that the image is virtual, erect and is formed at 8.4cm from the lens on the same side as the object.

As m = $\frac{h_2}{h_1} = \frac{v}{u}$

Therefore size of image, $h_2 = \frac{v}{u} \times h_1 = \frac{-8.4}{-14} \times 3\text{cm} = 1.8\text{cm}$

i.e., the image is diminished in size.

21. What is the focal length of a convex lens of focal length 30cm in contact with a concave lens of focal length 20cm? Is the system a converging or a diverging lens? Ignore the thickness of the lenses, Ignore the thickness of the lenses.

Sol. Here f_1 = +30cm (convex lens) , f_2 = - 20 cm (concave lens)

Focal length of the combination is given by $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{-20} = -\frac{1}{60}$

Or f = -60cm.

The negative value of f indicates that the combination behaves as a diverging lens.

22. (a) Determine the 'effective focal length' of the combination of the two lenses in previous Question, if they are placed 8.0 cm apart with their principal axes coincident. Does the answer depend on which side of the combination a beam of parallel light is incident? Is the notion of effective focal length of this system useful at all?

(b) An object 1.5 cm in size is placed on the side of the convex lens in the arrangement (a) above. The distance between the object and the convex lens is 40 cm. Determine the magnification produced by the two-lens system, and the size of the image.

Sol. (a) (i) Let the parallel beam of light be incident from the left on the convex lens first. Then f_1 = 30cm, $u_1 = -\infty$

$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

$\therefore \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{30} - \frac{1}{\infty} = \frac{1}{30}$

Or v_1 = +30cm

Thus the image becomes a virtual object for the second lens so that f_2 = - 20cm, u_2 = + (30 – 8) = 22cm

Now, $\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{20} + \frac{1}{22} = \frac{-11+10}{220} = \frac{-1}{220}$

Or v_2 = - 220cm

The parallel beam appears to diverge from a point 220 – 4 = 216 cm from the centre of the two lens system.

(ii) Le the parallel beam be incident from the left on the concave lens first. Then, f_1 = -20cm, u_1 = - ∞

As $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

$\therefore \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{20} + \frac{1}{-\infty} = -\frac{1}{20}$ or v_1 = - 20cm.

This image becomes a real object for the second lens so that f_2 = -30 cm, u_2 = - (20+8) = -28cm,

Now, $\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{30} - \frac{1}{28} = \frac{14-15}{420} = \frac{-1}{420}$ or v_2 = - 420cm.

Thus the parallel incident beam appears to diverge from a point 420 – 4 = 416cm on the left of the centre of the two lens system.

Clearly the answer depends on which side of the lens system the parallel beam is incident. The rotation of effective focal length, therefore, does not seem to be meaningful for this system.

(b) Here u_1 = - 40cm, f_1 = 30cm

As, $\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$

$\therefore \frac{1}{v_1} + \frac{1}{40} = \frac{1}{30}$ or $\frac{1}{v_1} = \frac{1}{30} - \frac{1}{40} = \frac{4-3}{120} = \frac{1}{120}$ or v_1 = 120cm

Magnitude of magnification due to the first (convex) lens is $m_1 = \frac{v}{|u|} = \frac{120}{40} = 3$

Thus image becomes a virtual object for the second lens so that u_2 = + (120 – 8) = +112cm, f_2 = - 20cm.

Now, $\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = -\frac{1}{20} + \frac{1}{112} = \frac{-112+20}{112\times 20} = \frac{-92}{112\times 20}$

Or $v_2 = -\frac{112\times 20}{92}$cm = - 24.9cm

Magnitude of magnification due to second (concave) lens is $m_2 = \frac{|v_2|}{u_2} = \frac{112\times 20}{92\times 112} = \frac{20}{92}$

Net magnitude of magnification due to the two lens system is, $m = m_1 \times m_2$

$= \frac{3\times 20}{92}$ = 0.652

Size of image, $h_2 = mh_1$ = 0.652 × 1.5 = 0.98cm.

23. A large card divided into squares each of size 1 mm^2 is being viewed from a distance of 9 cm through a magnifying glass (converging lens has a focal length of 9 cm) held close to the eye. Determine:

(a) the magnification produced by the lens? How much is the area of each square in the virtual image?

(b) the angular magnification (magnifying power) of the lens?

(c) Is the magnification in (a) equal to the magnifying power in (b)? Explain.

Sol. (a) Here area of each square (or object) = 1 mm^2, u = - 9cm, f = +10cm

As $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{9} = \frac{9-10}{90} = -\frac{1}{90}$ or v = - 90cm

Magnitude of magnification is $m = \frac{v}{|u|} = \frac{90}{9} = 10$

Area of each square in the virtual image = $(10)^2$ × 1 = 100 mm^2 = 1cm^2

(b) Magnifying power. $M = \frac{D}{|u|} = \frac{25}{9}$ = 2.8.

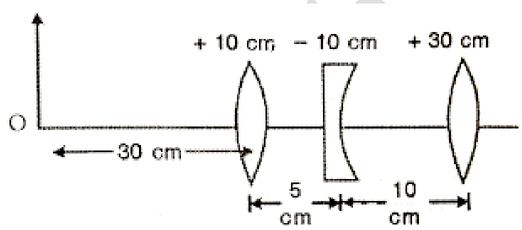
(c) No. magnification of an image by a lens and angular magnification (or magnifying power) of an optical instrument are two separate things. The latter is the ratio of the angular size of the object (which is equal to the angular size of the iamge even if the imae is magnified) to the angular zie of the object if placed at near point (25cm). Thus magnification magnitude is |v/u| and magnifying power is 25/|u|.
Only when the image is located at the near point |v| = 25cm, the two quantities are equal.

24. Derive the formula of effective focal length of two lenses having focal lengths f_1 and f_2 placed in contact with each other.

____
____
____
____
____
____

(5 Marks Questions)

25. Three lenses of focal lengths +10cm, - 10cm and +30cm are arranged coaxially as in the figure given below. Find the position of the final image formed by the combination.



Sol For lens L_1: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ where f = +10cm

$$\frac{1}{v_1} = \frac{1}{10} - \frac{1}{30}$$

$$\frac{1}{v_1} = \frac{3-1}{30} = \frac{2}{30} \Rightarrow v_1 = 15\text{cm}$$

For lens L_2: v_1 = 15cm, u = 10cm, f = - 10cm

Position of final image,

$$\frac{1}{v_2} = \frac{1}{f} + \frac{1}{u} = \frac{1}{10} - \frac{1}{10} \Rightarrow v_2 = \infty$$

∴ For third lens L_3 object is at infinity, hence final image is formed at focus of L_3 at a distance of 30cm.

E. PRISM

(2 Marks Questions)

1. Calculate the refractive index of the material of an equilateral prism for which the angle of minimum deviation is 60°.

Sol. For an equilateral prism, A = 60°, Also δ_m = 60°.

∴ Refractive index of the prism material is

$$\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{60°+60°}{2}}{\sin\frac{60°}{2}}$$

$$= \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3}$$

2. Calculate the angle of minimum deviation for an equilateral prism of refractive index √3.

Sol. Here A = 60°, $\mu = \sqrt{3}$, δ_m = ?

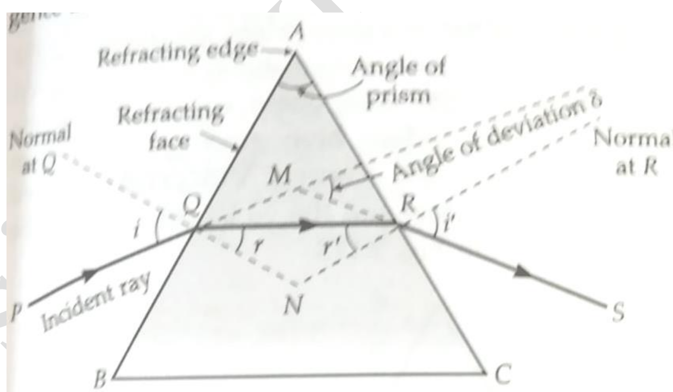
As, $\mu = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$

∴ $\frac{\sin\frac{60°+\delta_m}{2}}{\sin 30°}$ or $\sin\frac{60°+\delta_m}{2} = \sqrt{3} \times \frac{1}{2} = \sin 60°$ or $\sin\frac{\delta_m+60°}{2} = 60°$

Or δ_m = 120 – 60 = 60°.

3. Show that in case of a prism: A + δ = i + i' where the symbols have their usual meanings.

Sol.



From the quadrilateral AQNR, A + ∠QNR = 180°

From the triangle QNR, r + r' = ∠QNR = 180°

∴ A = r + r'

Now from the triangle MQR, the deviation produced by the prism is

δ = ∠MQR + ∠MRQ – (I - r) + (I' - r')

Or δ = deviation at the first face + deviation at the second face

= (I + I') – (r + r')

Or δ = I + I' = A

Or I + I' = A + δ

i.e., Angle of incidence + Angle of emergence = Angle of prism + Angle of deviation.

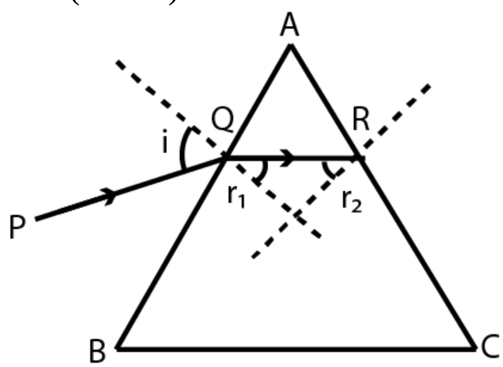
4. At what angle should a ray of light be incident on the face of a prism of refracting angle 60° so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524.

Sol. The refracted ray QR will just suffer total internal reflection if it is incident at the critical angle i_e.

Thus $r_2 = i_c$

Now $\sin i_c = 1/\mu = 1/1.524 = 0.6542$

Therefore, $i_c = som^{-1}(0.6542) = 41°$



But $r_1 + r_2 = A$

Therefore, $r_1 = A - r_2 = A - i_c = 60° - 41° = 19°$.

From Snell's law, $\mu = \sin / \sin r_1$

Therefore, $\sin i_1 = \mu \sin r_1 = 1.524 \times \sin 19°$

$= 1.524 \times 0.3256 = 0.4962$

Hence, $i_1 = \sin^{-1}(0.4962) = 30°$.

5. You are given prisms made of crown glass and flint glass with a wide variety of angles. Suggest a combination of prisms which will

(a) deviate a pencil of white light without much dispersion,

(b) disperse (and displace) a pencil of white light without much deviation.

Sol. Two identical prisms made of the same material placed with their base on opposite sides (or the incident white light) and faces touching (or parallel) will neither deviate nor disperse, but will merely produce a parallel displacement of the beam.

Now angular dispersion produced by crown glass prism is, $\delta_b - \delta_r = (\mu_b - \mu_r) A$

Mean deviation produced by flint glass prism is $\delta_y = (\mu_y - 1)A$

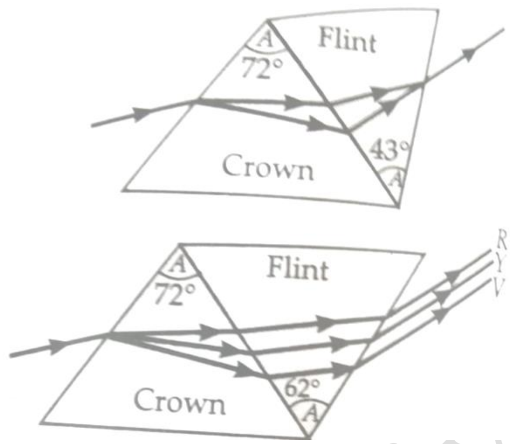
Angular dispersion produced by flint glass prism is $\delta_b' = \delta_r' = (\mu_y' - 1)A'$

$\delta_y' = (\mu_y' - 1)A$

(a) To deviate a beam without dispersion the net angular dispersion produced by the combination must be zero, i.e., $(\mu_b - \mu_r)A + (\mu_b' - \mu_r')A' = 0$

Or $A' = \frac{(\mu_b - \mu_r)}{(\mu_b' - \mu_r')} A$

Negative sign show that the two prisms must be placed with their bases n opposite sides. As (μ_b' – μ_r') for flint glass is more than (μ_b – μ_r) for crown glass prism so that the dispersion due to the first is mollified by the second.



(b) To produce dispersion without deviation, the net mean deviation should be zero, i.e$(\mu_y - 1)A + (\mu_y' - 1)A' = 0$

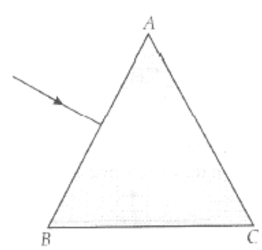
Or $A' = \frac{\mu_y - 1}{\mu_y' - 1} A$

We take a crown glass prism of certain angle and to on increasing the angel of flint glass prism till the deviations due to the two prisms are equal and opposite. However, the flint glass prism angle will still be smaller than tat of crown glass because flint glass ahs higher refractive index that of crown glass.

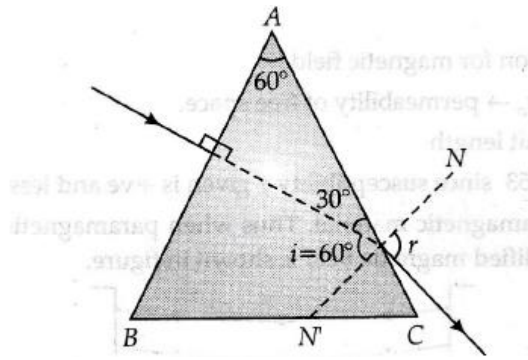
Due to adjustments involved for many colours, the above combinations are not very accurate arrangements for the purpose required.

(3 Marks Questions)

6. The figure shows a light falling normally on the face AB of an equilateral glass prism having refractive index 3/2, placed in water of refractive index 4/3. Will this ray suffer total internal reflection on striking the face AC? Justify your answer.



Sol.



Critical angle for the given pair of media

$\theta c = \sin^{-1}\left(\frac{\mu_w}{\mu_g}\right)$

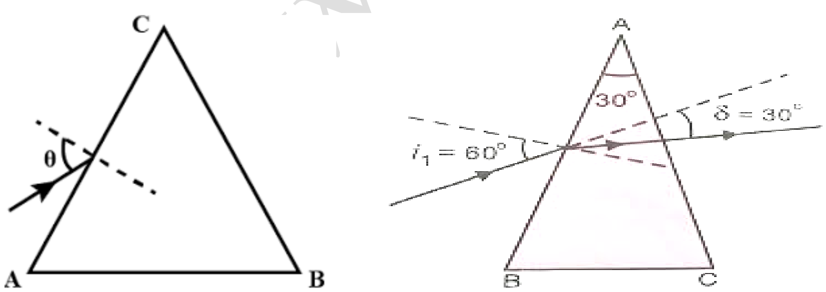
$= \sin^{-1}\left(\frac{4/3}{3/2}\right) = \sin^{-1}\left(\frac{8}{9}\right)$

Sin θ_c = 8/9= 0.89

Now, sin60° = $\sqrt{3}/2$ = 0.86

On face AC, angle of incidence is less than that of critical angle, so there wil be no total internal reflection.

7. A ray of light PQ is incident at an angle of 60° on the face AB of a prism of angle 30°, as shown in figure. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face BC through which it emerges. Also calculate the refractive index of the prism material



Sol. Here i = 60°, A = 30°.

As the emergent ray makes an angle of 30° with the incident ray, so angle of deviation, δ = 30°

Now i + i' = A + δ

Therefore Angle of emergence, i' = A + δ – i = 30° + 30° - 60° = 0°

Thus the emergent is perpendicular to the face BC through which it merges, as shown in figure b,

When i' = 0°, r' = 0°

∴ r = A – r' = 30° - 0° = 30°

$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

8. A prism is made of glass of unknown refractive index. A parallel beam of light is incident on the face of the prism. The angle of minimum deviation is measured to be 40°. What is the refractive index of the material of the prism? The refracting angle of the prism is 60°. If the prism is placed in water (refractive index 1.33), predict the new angle of minimum deviation of a parallel beam of light.

Sol. When the prism is paled in air, δ_m= 40°, A = 60°

$${}^a\mu_g = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}} = \frac{\sin\frac{60°+40°}{2}}{\sin\frac{40°}{2}} = \frac{\sin 50°}{\sin 20°} = \frac{0.7660}{0.5000} = 1.532$$

When the prism is placed in water,

$${}^w\mu_g = \frac{\sin\frac{A+\delta'_m}{2}}{\sin\frac{A}{2}}$$

or $\frac{{}^a\mu_g}{{}^a\mu_w} = \frac{\sin\frac{60°+\delta'_m}{2}}{\sin\frac{60°}{2}}$

or $\frac{1.532}{1.33} = \frac{\sin\frac{60°+\delta'_m}{2}}{\sin 30°}$

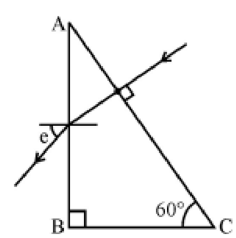
or $\sin\frac{60°+\delta'_m}{2} = \frac{1.532}{1.33} \times 0.5 = 0.5759$

$\therefore 30° + \frac{\delta'_m}{2} = \sin^{-1}(0.5759) = 35°10'$

$\delta'_m = 10°20'$

(5 Marks Questions)

9. Calculate the angle of emergence (e) of the ray of light incident normally on the face AC of a glass prism ABC of refractive index √3. How will the angle of emergence change qualitatively, if the ray of light emerges from the prism into a liquid of refractive index 1.3 instead of air?



Sol. From Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ or μ sin i = μ sinr, $\mu_p \sin 30° = \mu_a \sin e$

$\sqrt{3}\frac{1}{2}$ = sine [∵ m_{air} = 1] or e = 60°

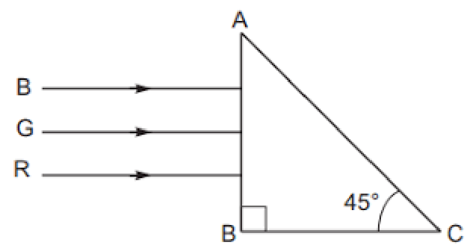
When the ray of light emerges into a liquid of m_L = 1.3,

⇒ $\mu_p \sin 30° = \mu_L \sin e$

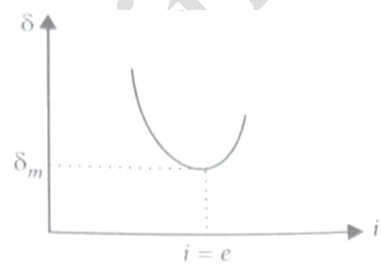
$\sqrt{3}\frac{1}{2}$ = sine ⇒ $\sin^{-1}\left(\frac{\sqrt{3}}{2\times 1.3}\right)$ = 41.78°

Thus the angle of emergence decreases.

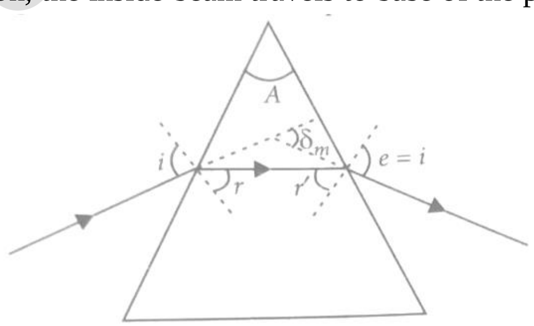
10. (i) A ray of monochromatic light is incident on one of the faces of an equilateral triangular prism of refracting angle A. Trace the path of ray passing through the prism. Hence, derive an expression for the refractive index of the material of the prism in terms of the angle of minimum deviation and its refracting angle. (ii) Three light rays red (R), green (G) and blue (B) are incident on the right angled prism abc. The refractive indices of the material of the prism for red, green and blue wavelengths are respectively 1.39, 1.44 and 1.47. Trace the paths of these rays reasoning out the difference in their behavior.



Sol. (i) If graph is plotted between angle of incidence I and angle of deviation δ, it is found that the angle of deviation δ first decreases with increase in angel of incidence I and then becomes minimum 'δ_m' when i = e and then increase with increase in angle of incidence i. Figure show the path of a ray of light suffering refraction through a prism of refracting angle 'A'.



At minimum deviation, the inside beam travels to base of the prism.



i = e; r = r'

δ_m = (I + e) – (r + r')

δ_m = 2i – 2r …(i)

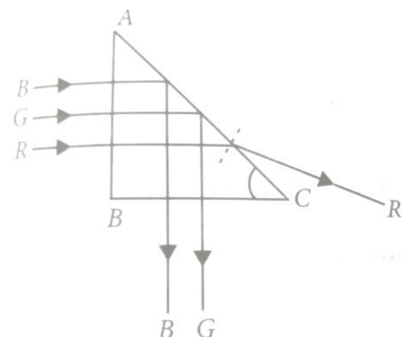
Also r + r' = A = 2r …(ii)

So, angle of incidence using equation (i)

$i = \frac{A+\delta_m}{2}$, angle of refraction, r = A/2

Now refractive index of the material of prism ${}^a\mu_g = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$ where A is the "refracting angle" of the prism and A = 60° for an equiangular prism.

(ii) Critical angle for



(a) Red light is sin c_r = 1/1.39 = 0.7197 or c_r = 46°

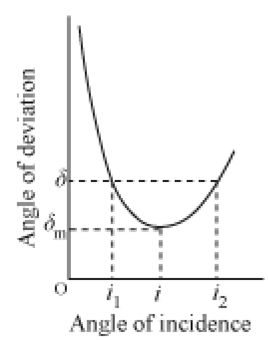
(b) Green light is sin c_g = 1/1.44 = 0.6944 or c_4 = 44°

(c) Blue light is sin c_b = 1/1.47 = 0.6802 or c_b = 43°

As angle of incidence I = 45° of red light ray on face AC is less than its critical angle of 46°, so erd light wil merge out of face AC.

11. Draw a graph to show the variation of the angle of deviation 'δ' with that the angle of incidence 'i' for a monochromatic ray of light passing through a glass prism of refracting angle 'A'. Hence deduce the relation: $\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

Sol.



When a prism in sin the position of minimum deviation, a ray of light passes symmetrically (parallel to the base) through the prism so that i = i' , r = r', $\delta = \delta_m$

As A + δ = i + i'

So, $A + \delta_m$ = i + i'or $i = \frac{A+\delta_m}{2}$

Also, A = r + r' o r = r + r = 2r

Therefore, r = A/2

From Snell's law, the refractive index of the material of the prism will be

$$\mu = \frac{\sin i}{\sin r} \text{ or } \mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

F. OPTICAL INSTRUMENTS

(1 Mark Question)

1. A compound microscope is used because a realistic simple microscope does not have ________ magnification.

Sol. A compound microscope is used because a realistic simple microscope does not have large magnification.

(2 Marks Questions)

2. For a normal eye, the far point is at infinity and the near point of distinct vision is about 25cm in front of the eye. The cornea of the eye provides a converging power of about 40 dioptres, and the least converging power of the eye-lens behind the cornea is about 20 dioptres. From this rough data estimate the range of accommodation (i.e., the range of converging power of the eye-lens) of a normal eye.

Sol. To see objects at infinity, the eye uses its least converging power = 40+ 20 = 60 dioptres

∴ Approximate distance between the retina and the cornea eyelens = focal length of the eyelens = $\frac{100}{P} = \frac{100}{60} = \frac{5}{3}$cm

To focus an object at the near point on the retina, we have u = - 25cm, v = $\frac{5}{3}$ cm

∴ Focal length f should be given by $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{3}{5} + \frac{1}{25} = \frac{15+1}{25} = \frac{16}{25}$

f = $\frac{25}{16}$ cm

∴ Corresponding converging power = 64 dioptres

Power of eyelens = 64 – 40 = 24 dioptres

Thus the range of accommodation of the eyelens is roughly 20 to 24 dioptres.

3. Does the human eye partially lose its ability of accommodation when it undergoes short-sightedness (myopia) or long-sightedness (hypermetropia)? If not, what might cause these defects of vision?

____

____

____

____

4. Spectacles of power −1.0 dioptre is being used by a person suffering from myopia for distant vision. He also needs to use separate reading glass of power + 2.0 dioptres when he turns old. Explain what may have happened.

____
____
____
____

5. A person looking at a cloth with a pattern consisting of vertical and horizontal lines is able to see the vertical lines more distinctly than the horizontal ones. What is this defect due to? How is such a defect of vision corrected?

____
____
____
____

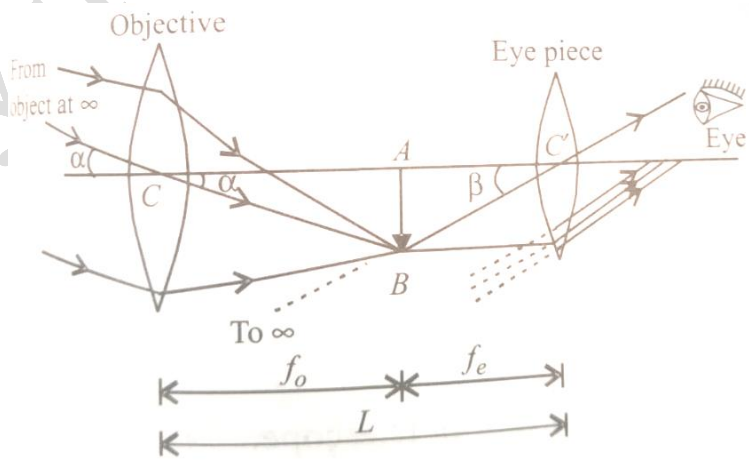
(3 Marks Questions)

6. (a) Draw a ray diagram depicting the formation of the image by an astronomical telescope in normal adjustment.

(b) You are given the following three lenses. Which two lenses will you use as an eyepiece and as an object to construct an astronomical telescope? Give reason.

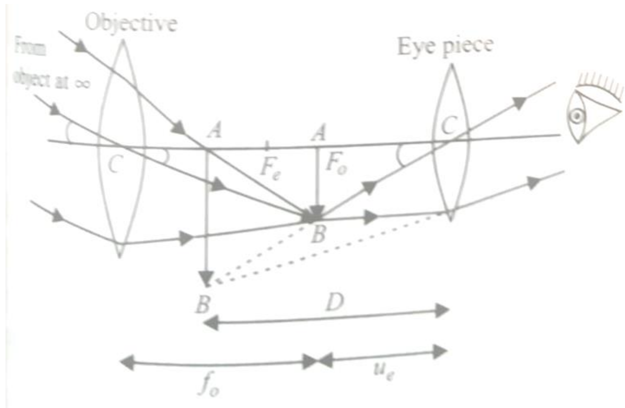
Lenses	Power (D)	Aperture (cm)
L_1	3	8
L_2	6	1
L_3	10	1

Sol. When final image is formed at infinity:



Magnification, $M = -\frac{f_0}{f_e}$

When the final image is formed at least distance of distinct vision:



Magnification, M = $-\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$

Resolving power of telescope, R = $\frac{a}{1.22\lambda}$

R ∝ a and R ∝ $\frac{1}{\lambda}$

7. Which two of the following lenses L_1, L_2 and L_3 will you select as objective and eyepiece for constructing best possible (i) telescope (ii) microscope? Give reason to support your answer.

Lenses	Power (P)	Aperture (A)
L_1	6D	1cm
L_2	3D	8cm
L_3	10D	1cm

Sol. An astronomical telescope should have an objective of larger aperture and longer focal length while an eyepiece of small aperture and small focal length. Therefore, we will use L_2 as an objective and L_3 as an eyepiece.

For constructing microscope, L_3 should be used as objective and L_3 as eyepiece because both the lenses of microscope should have short focal lengths of the focal length of objective should be smaller than the eyepiece.

8. A person wears glasses of power – 2.5D, Is the person far sighted or near sighted? What is the far point of the person without glasses?

Sol. Here P = - 2.5D

Negative power show that the lens is concave, so the person is near sighted.

f = $\frac{1}{P}=\frac{1}{-25}\text{m}=-\frac{2}{5}\text{m}$ = - 40cm

The object placed at infinity from the corrective lens must produce the virtual image at the far point. Therefore, u = - ∞, v = ?

From the lens formula $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{-40}+\frac{1}{-\infty}=\frac{1}{-40}-0=-\frac{1}{40}$

Or v = - 40cm.

Thus the far point of the eye is at 40cm from the eye.

9. A simple microscope is rated 5X for a normal relaxed eye. What will be its magnifying power for a relaxed farsighted eye whose near point is 40cm?

Sol. For normal eye: D = 25cm, m = 5

As $m = \frac{D}{f} \therefore 5 = \frac{25}{f}$ or f = 5cm

For relaxed farsighted eyes: D' = 40cm, f = 5cm

$\therefore m = \frac{D'}{f'} = \frac{40}{5} = 8$

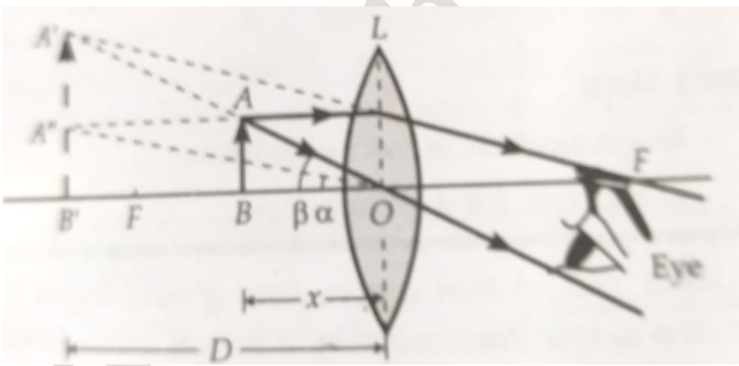
Thus the magnifying power of the simple microscope is 8X in the second case.

10. A reflecting type telescope has a concave reflector of radius of curvature 120cm. Calculate focal length of eyepiece to secure a magnification of 20.

Sol. $f_e = \frac{f_0}{m} = \frac{R_0/2}{m} = \frac{120/2}{20} = 3$cm.

11. Draw a ray diagram of simple microscope. Deduce the formula for its angular magnification when the image is formed at the least distance of distinct vision.

Sol.



The magnifying power of a simple microscope is defined as the ratio of the angle subtended by the image and the object at the eye, when both are at the least distance of distinct vision from the eye. Thus

Magnifying power= $\frac{\text{Angle subtended by the imae at theleast distance of distinct vision}}{\text{Angle subtended by the object at the least distance of distinct vision}}$

As the eye is held close to the lens, the angles subtended by the lens may be taken to the angles subtended at the eye. The image A'B is formed at the least distance vision 'D'. Let ∠A'OB' = β. Imagine the object AB to be displaced to position A'"B' at distance D from the lens. Let ∠A"OB' = α. Then magnifying power

$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$ [since α, β are small angles]

$= \frac{AB/OB}{A''B'/OB'} = \frac{AB/OB}{AB/OB'}$ [Since A"B' = AB]

$= \frac{OB'}{OB} = \frac{-D}{-x}$ or m = D/x

Let f be the focal length of the lens. As the image is formed at the least distance of distinct vision form the lens, so v = - D

Using thin lens formula, $\frac{1}{v} = \frac{1}{u} - \frac{1}{f}$

We get. $\frac{1}{-D} = \frac{1}{-x} = \frac{1}{f}$

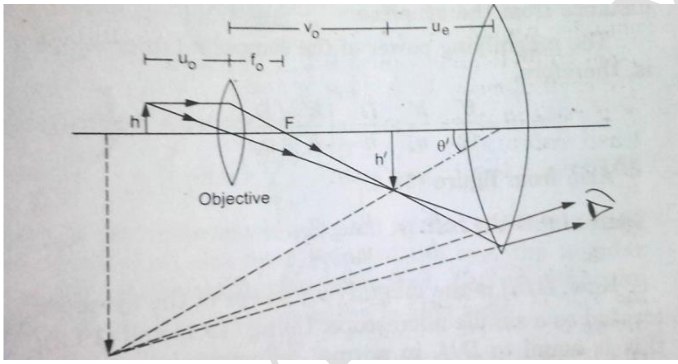
Or $\frac{1}{x} = \frac{1}{D} + \frac{1}{f}$

Or $\frac{D}{x} = 1 + \frac{D}{f}$

Therefore, m = 1+ $\frac{D}{f}$

12. Draw a labeled diagram to show the formation of an image by a compound microscope. Write the expression for its magnifying power.

Sol.



The magnifying power of a compound microscope is defined as the ratio of the angle subtended at the eye by the final virtual image to the angle subtended at the eye by the object, when both are at the least distance of distinct vision, form the eye.

$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{h'/u_e}{h/D} = \frac{h\prime}{h}.\frac{D}{u_e} = m_0 m_e$

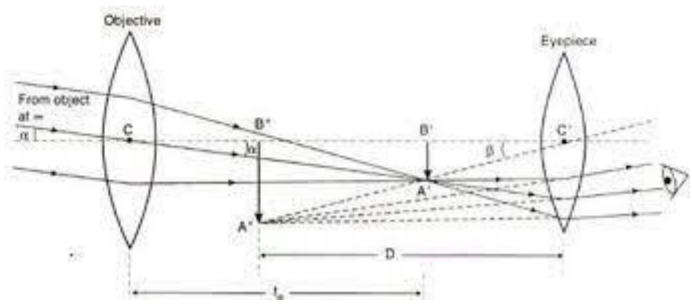
Here $m_0 = h/h' = v_0/u_0$

As the eyepiece acts as a simple microscope, so

$m_e = \frac{D}{u_e} = 1 + \frac{D}{f_e} \quad \therefore m = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

13. Draw a labeled ray diagram to show the image formation in a refracting type astronomical telescope. Why should the diameter of the objective of a telescope be large?

Sol.



It is a convex lens of large focal length and a much larger aperture. It faces the distant object. In order to form bright image of the distant objects, the aperture of the objective is taken large so that it can gather sufficient light from the distant objects.

14. A compound microscope consists of an objective lens of focal length 2.0cm and an eyepiece of focal length 6.25cm separated by a distance of 15 cm. How far from the objective should an object be placed in order to obtain the final image at (a) the least distance of distinct vision (25 cm), and (b) at infinity? What is the magnifying power of the microscope in each case?

Sol. Here f_0 = 2.0cm, f_e= 6.25cm, μ_0 = ?

(a) When the final image is obtained at least distance of distinct vision:

$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$\therefore \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-0.25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25} = -\frac{1}{5}$

Or u_e = -5cm

Now distance between object and eyepiece = 15cm

∴ distance of the image from object is, $v_{0=}$ = 15 – 15 = 10cm

$\therefore \frac{1}{u_0} - \frac{1}{v_0} = \frac{1}{f_0} = \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} = -\frac{2}{5}$

Or $u_0 = -\frac{5}{2}$ = - 2.5cm

Therefore distance of object from object = - 2.5cm

Magnifying power, m = $m_0 \times m_e = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

$= \frac{10}{25}\left(1 + \frac{25}{6.25}\right)$m = 20.

(b) When the final image is formed at infinity

Here $v_e = \infty$, f_e = 6.25cm

As $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \therefore \frac{1}{\infty} - \frac{1}{u_e} = \frac{1}{f_e}$

Or $u_e = - f_e$ = - 6.25cm

Distance between object and eyepiece = 15cm

∴ distance of the object from the image formed by itself. v0 = 15 – 6.5 = 8.75cm.

Also, f_0 = +2.cm

$\therefore \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{8.75} - \frac{1}{2} = \frac{2-8.75}{17.5} = \frac{-6.75}{17.5}$

Or $u_0 = -\frac{17.5}{6.75}$ = - 2.59cm

∴ The distance of the object from objective = - 2.59cm.

Magnifying power, m = $m_0 \times m_e = \frac{v_0}{u_0} \times \frac{25}{6.25} = \frac{27}{8} \times 4 = 13.46$ = 13.5.

15. A person with a normal near point (25 cm) using a compound microscope with objective of focal length 8.0 mm and an eyepiece of focal length 2.5cm can bring an object placed at 9.0 mm from the objective in sharp focus. What is the separation between the two lenses? Calculate the magnifying power of the microscope.

Her f_0 = 0.8cm, u_0 = - 0.9cm, v_0 = ?

As $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

$\therefore \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{0.8} - \frac{1}{0.9} = \frac{0.9-0.8}{0.9\times0.8} = \frac{0.1}{0.8\times0.9}$

Or $v_0 = \frac{0.8x\times0.9}{0.1}$ = 7.2cm

Now for the eyepiece, we have, f_e = 2.5cm, v_e = - D = - 25 cm, u_e = ?

$\therefore \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{25} = \frac{-1-10}{25} = \frac{-11}{25}$

Or $u_e = -\frac{25}{11}$ = - 2.27cm

Hence the separation between the two lenses = $u_0 + |u_e|$ = 7.2 + 2.27 = 9.47cm

Magnifying power, m = $m_0 \times m_e = \frac{v_0}{|u_0|}\left(1 + \frac{D}{f_e}\right) = \frac{7.2}{0.9}\left(1 + \frac{25}{2.5}\right)$ = 88.

16. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece?

Sol. Here f_0 = 144cm, f_e = 6cm

For a small telescope set in normal adjustment the magnifying power is m = $\frac{f_0}{f_e} = \frac{144}{6} =$ 24

Separation between the objective and th eyepiece = $f_0 + f_e$ = 144 + 6 = 150cm.

17. (i) A giant refracting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece of focal length 1.0 cm is used, what is the angular magnification of the telescope?

(ii)If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m and the radius of lunar orbit is 3.8×10^8m.

Sol. Here f_0 = 15m, f_e = 1.0cm = 0.01m.

(i) Angular magnification, m = f_0/f_e = 15/0.01 = 1500.

(ii) Let d be the diameter of the image in metres. Then angle subtended by the moon will be, $\alpha = \frac{\text{Diameter of moon}}{\text{Radius of lunar orbit}} = \frac{3.48\times10^6}{3.8\times10^8}$

Angle subtended by the image formed by the objective will also be equal to α and is given by, $\alpha = \frac{\text{Diameter of image of moon}}{f_0} = \frac{d}{15}$

$\therefore \frac{d}{15} = \frac{3.48\times10^6}{3.8\times10^8}$

Diameter of image of moon, d = $\frac{3.48\times10^{6}\times15}{3.8\times10^{8}} = \frac{3.48\times15\times10^{-2}}{3.8}$ = 13.73cm.

18. The virtual image of each square in the figure is to have an area of 6.25 mm^2. Find out, what should be the distance between the object in Exercise 86 and the magnifying glass? If the eyes are too close to the magnifier, would you be able to see the squares distinctly?

[Ans. – 15cm]

19. An angular magnification (magnifying power) of 30X is desired using an objective of focal length 1.25 cm and an eyepiece of focal length 5 cm. How will you set up the compound microscope?

Sol. We assume the microscope is common usage, i.e., the final image is formed at the least distance of distinct vision, D = 25cm, f_e = 5cm

∴ Angular magnification of the eye piece is $m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$

As total magnification, $m = m_e \times m_0$

∴ Angular magnification of the objective is $m_0 = \frac{m}{m_e} = \frac{30}{6} = 5$

As real image is formed by the objective, therefore, $m_0 = \frac{v_0}{u_0} = -5$ or $v_0 = -5u_0$, f_0 = 1.25cm

Now, $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$

Or $\frac{1}{-5u_0} - \frac{1}{u_0} = \frac{1}{1.25}$

Or $\frac{-6}{5u_0} = \frac{1}{1.25}$ or $u_0 = -\frac{6\times1.25}{5}$ = - 1.5cm

Thus the object should be held at 1.5cm in front of the objective lens.

Also, $v_0 = -5u_0 = -5 \times (-1.5) = 7.5$cm

As $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

∴ $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{5}$ [$v_e = -D = -25$cm] or

$= \frac{-1-5}{25} = -\frac{6}{25}$ or $v_e = \frac{-25}{6}$ = - 4.17 cm

∴ Separation between the objective and the eyepiece = $|u_e| + |v_0|$ = 4.17 + 7.5 = 11.67cm.

20. A small telescope has an objective lens of focal length 140 cm and an eyepiece of focal length 5.0 cm. What is the magnifying power of the telescope for viewing distant objects when

(a) the telescope is in normal adjustment (i.e., when the final image is at infinity)?

(b) the final image is formed at the least distance of distinct vision (25 cm)?

Sol. Here f_0 = 140cm, f_e = 5.0cm

(a) Its normal adjustment: magnifying power, $m = f_0/f_e = 140/5 = 28$

(b) When the final image is formed at the least distance of distinct vision (25cm):

$M = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right) = \frac{140}{5}\left(1+\frac{5}{25}\right) = 28 \times 1.2 = 33.6$

21. (a)For a telescope, what is the separation between the objective lens and the eyepiece?
(b) If this telescope is used to view a 100 m tall tower 3 km away, what is the height of the image of the tower formed by the objective lens?
(c)What is the height of the final image of the tower if it is formed at 25 cm?

Sol. (a) In normal adjustment, the separation between objective and eyelens = $f_0 + f_e$ = 140 + 5 = 145cm.

(b) Angle subtended by the 100m tall tower at 3knm away is $\alpha = \tan\alpha = \frac{100}{3\times10^3} = \frac{1}{30}$rad

Let h be the height of the image of tower formed by the objective. Then angle subtended by the image produced by the objective will also be equal to α and is given by

$\alpha = \frac{h}{f_0} = \frac{h}{140}$

$\therefore \frac{h}{140} = \frac{1}{30}$

Or $h = \frac{140}{30} = \frac{14}{3}$ = 4.67cm

(c) Magnification produced by the eyepiece is $m_e = 1+\frac{D}{f_e} = 1 + \frac{25}{5} = 6$

∴ Height of the final image = $h \times m_e = \frac{14}{3} \times 6$ = 28cm.

G. CASE STUDY QUESTIONS

Read the blow case and answer the questions that follow:

1. **OPTICAL FIBRE**: Optical fibre works on the principle of total internal reflection. Light rays can be used to transmit a huge amount of data, but there is a problem here – the light rays travel in straight lines. So, unless we have a long straight wire without any bends at all, harnessing this advantage will be tedious. Instead, the optical cables are designed such that they bend all the light rays inwards (using TIR). Light rays travel continuously bouncing off the optical fibre walls and transmitting end to end data. It is usually made of plastic of glass.

Modes of transmission: Single mode fibre is used for long distance transmission, while multi mode fibre is used for shorter distances. The outer cladding of these fibres needs better protection than metal wires. Although light signals do degrade over progressing distances due to absorption and scattering. Then, optical regenerator system is necessary to boost the signal.

Types of Optical Fibres: The types of optical fibres depend on the refractive index, materials used, and made of propagation of light. The classification based on the refractive index is as follows:

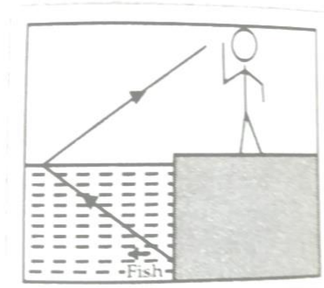
e Step Index Fibres: It consists of a core surrounded by the cladding, which has a single uniform index or refraction.

e Graded Index Fibres: The refractive index of the optical fiber decreases as the radial distance fron the fibre axis increases.

(i) Optical fibre works on the principle of
(a) scattering of light (b) diffraction of light
(c) total internal reflection of light (d) dispersion of light
Ans. (c)
The optical fibre works on the principle of total internal reflection.
(ii) For long distance transmission
(a) single mode fibre is used (b) multi-mode fibre is used
(c) both single and multi-mode are used
(d) any one of single mode or multi-mode may be used
Ans. (a)
Single mode fibre is used for long distance transmission, while multi mode fibre is used for shorter distances.
(iii) Optical fibre is made of
(a) copper (b) semiconductor (c) plastic or glass (d) superconductor
Ans. (c)
Optical fibre is usually made of plasic or glass, so that light rays can travel continuously, bouncing off the optical fibre walls and can be transmitting and to end data.
(iv) In graded index optical fibre
(a) the refractive index of the optical fibre increases as the radial distance from the fibre axis increases
(b) the refractive index of the optical fibre decrease as the radial distance from the fibre axis increases
(c) the refractive index of the optical fibre remains same throughout.
(d) inner side of cladding is mirrored to ensure reflection
Ans. (b)
In graded index fibres, the refractive index of the optical fibre decreases as the radial distance from the fibre axis increases.
(v) Light signal through optical fibre may degrade due to:
(a) refraction (b) refraction and reflection
(c) diffraction and scattering (d) scattering and absorption
Ans. (d)
Light signals do degrade over progressing distances due to absorption and scattering.

2. **Negative Refractive Index**: One of the most fundamental phenomena in optics is refraction. When a beam of light crosses the interface between two different materials, its path is altered depending on the difference in the refractive indices of the materials. The greater the difference the greater the refraction of the beam. For all known naturally occurring materials the refractive index assumes only positive values. But does this have to be the case?
In 1967, Soviet physicist Victor Veselago hypothesized that a material with a negative refractive index could exist without violating any of the laws of physics.
Veselago predicted that this remarkable material would exhibited a wide range of new optical phenomena. However, until recently no one had found such a material and Veselago's ideas had remained untested. Recently, meta material samples are being tested for negative refractive index. But the experiments show significant losses and this could be an intrinsic property of negative index materials.

Snell's law is satisfied for the materials having negative refractive index, but the direction of the refracted light ray is 'mirror imaged' about the normal to the surface.



There will be an interesting difference in image formation if a vessel is filled with "negative water" having refractive index – 1.33, instead of regular water having refractive index 1.33.

Say, there is fish in vessel filled with negative water. The position of the fish is such that the observer cannot see it due to normal refraction since the refracted rays does not reach to his eyes.

But due to negative refraction, he will be able to see it since the refracted ray now reaches his eyes.

(i) Who hypothesized a material may have negative refractive index?

(a) Joseph Von Fraunhofer (b) Augustin-Jean Fresnel

(c) Thomas Moore (d) Victor Veselago

Ans. (d)

In 1967, Soviet physicist Victor Veselago hypothesized that a material with a negative refractive index could exist without violating any of the laws of physics.

(ii) Is Snell's law applicable for negative refraction?

(a) Yes (b) No (c) Unpredictable (d) Yes, only for normal incidence

Ans. (a)

Snell's law is satisfied for the materials having negative refractive index, but the direction of the refracted light ray is 'mirror imaged' about the normal to the surface.

(iii) A ray is incident on normal glass and "negative glass" at an angle 60°. If the magnitude of angle of refraction in normal glass is 45°, then what will be the magnitude of angle of refraction in the "negative glass"?

(a) Less than 45↑8 (b) more than 45° (c) 45° (d) Unpredictable

Ans. (c)

The magnitude of angle of refraction in normal "negative glass" will also be 45°, but the direction of the refracted light ray is 'mirror-imaged' about the normal to the surface.

(iv) When the angle of incidence will be equal to angle of refraction for material having negative refraction index?

(a) When angle of incidence = 90° (b) When angle of incidence = 0°

(c) It will vary from material to material (d) It is never possible

Ans. (b)

Like normal refraction, for material having negative refraction index also when the angle of incidence is equal to 0°, then angle of refraction will be equal to angle of incidence, i.e., 0°.

(v) Which of the following is the intrinsic property of negative index materials?

(a) Significant gain of light energy due to refraction
(b) No loss of light energy due to refraction
(c) Significant loss of light energy due to refraction
(d) Loss of energy due to refraction in intermittent

Ans. (c)
Recently, meta material samples are being tested for negative refractive index. The experiments show significant losses and this could be an intrinsic property of negative index materials.

H. ASSERTION REASON TYEP QUESTION:

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true.

1. Assertion: Higher the refractive index of a medium or denser the medium, lesser is the velocity of light in that medium.
Reason: Refractive index is inversely proportional to velocity.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion
By Snell's law: $\frac{sin\, i}{sin\, r} = \frac{\mu_2}{\mu_1} = \frac{c/v_2}{c/v_1} = \frac{v_2}{v_1}$
Or $\mu_1 v_1 = \mu_2 v_2$
This shows that higher is the refractive index of a medium or denser the medium, lesser is the velocity of light in that medium.

2. Assertion: The illumination of earth's surface from Sun is more at noon than in the morning.
Reason: Luminance of a surface refers to brightness of the surface.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.
At noon, rays of sunlight falls normally on earth. Therefore $\theta = 0°$. According to Lambert's cosine law, $E \propto \cos\theta$ when $\theta = 0°$, $\cos\theta = \cos 0° = 1$ = max. Therefore E is maximum.

3. Assertion: The mirrors and in search lights are parabolic and not concave spherical.
Reason: In a concave spherical mirror the image formed is always virtual.

Ans. (c) Assertion is true but reason is false.
In search lights, we need an intense parallel beam of light. If a source is placed at the focus of a concave spherical mirror, only paraxial rays are rendered parallel. Due to large aperture of mirror, marginal rays give a divergent beam.
2. But in case of parabolic mirror, when source is at the focus, beam of light produced over the entire cross section of the mirror is a parallel beam.

4. Assertion: Critical angle of light passing from glass to air is minimum for violet colour.
Reason: The wavelength of blue light is greater than the light of other colours.

Ans. (c) Assertion is true but reason is false.
According to Snell's law the critical angle is given by $\sin C = 1/\mu$.
Where μ is refractive index of medium. Since μ decreases with increase in λ, hence C is minimum for the violet colour which has smallest wavelength.

5. Assertion: A ray incident along normal to the mirror retraces its path.
Reason: In reflection, angle of incidence is always equal to angle of refraction.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion
When light ray incident along normal to the mirrror, angle of incidence, $\angle i = 0°$. According to law of reflection, $\angle i = \angle r$, therefore angle of reflection, $\angle r = 0°$, i.e. th incident ray retraces its path.

6. Assertion: The diamond shines due to multiple total internal reflections.
Reason: The critical angle for diamond is 24.4°.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.
The brilliance of diamond is due to total internal reflection of light. μ for diamond is 2.42, so that critical angle for diamond air surface is C = 24.4° (from $\sin C = 1/\mu$). The diamond is cut suitable so that light entering the diamond from any face suffers multiple total internal reflections at the various faces and remains within the diamond. Hence the diamond sparkles.

7. Assertion: The sun looks reddish at the time of sunrise and sunset because of dispersion of light.
Reason: The wavelength of red colour is more than the wavelength of blue colour.

Ans. (e) Assertion is false but reason is true.
At the time of sunrise and se\unset, the rays from the sun have a travel a larger atmospheric distance. As $\lambda_b < \lambda_r$, and scattering intensity $\propto 1/\lambda^4$, therefore most of the blue and shorter wavelength is scattered away. Only red colour which is least scattered enters our eyes and appears to come from the sun. Hence the sun appears red.

8. Assertion: By increasing the diameter of the objective of telescope, we can increase its range.
Reason: The range of telescope tells us how far away a star of same standard brightness can be spotted by telescope.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.
The light gathering power (or brightness) of a telescope is directly proportional to the area of the objective lens i/e/. light gathering power $\propto \pi r^3 \propto \pi D^2/4$ where D is the diameter of the objective. Thus telescope will have large light gathering power if the diameter fo the objective lens is large. So by increasing the objective diameter even far off stars may produce images of optimum brightness.

9 Assertion: It is possible to eliminate dispersion by combining two prism of same refracting angles but of different materials.
Reason: The angular dispersion does not depends on refractive index of the material of the prism.

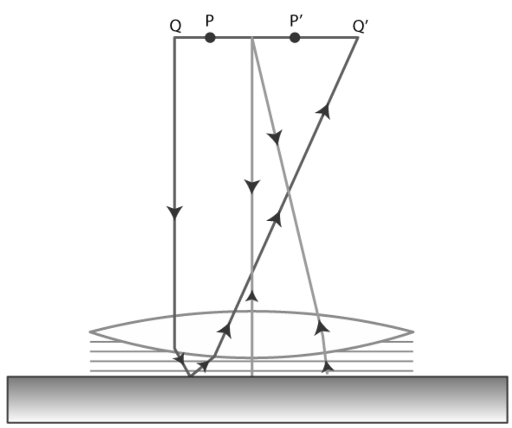
Ans. (d) Both assertion and reason are false
For a prism of small angle A, the angular dispersion produced = $(\mu_v - \mu_g)A$. This can be cancelled by a second prism of angle A’ made of different material such that $(\mu'_v - \mu'_g)A' = (\mu_v - \mu_g)A$. if only A = A’ then the dispersion produced by one prism cannot be cancelled by the dispersion produced by the other prism because, $(\mu'_v = \mu'_g)A' = (\mu_v - \mu_g)A$ for different materials. Therefore in order to eliminate dispersion by combining two prism they should have same refracting angle and made of same material.

10. Assertion: Optical fibres are used to transmit light without any loss in its intensity over distance of several kilometers.
Reason: Optical fibres are very thick and all the light is passed through it without any loss.

Ans. (c) Assertion is true but reason is false.
Optical fibre is extremely thin (radius of few microns) and long strand of very fine quality gals of quartz. When light is incident at a small angle at one end, it gets refracted into the strands (or fibres) and incident on the interface of the fibres and the coating.
The angle of incidence being greater than the critical angle the ray of light undergoes internal reflections. It suffers the internal reflection again and again, till the angle of incidence remains greater than the critical angle for fibre material with respect to coating. Due to successive total internal reflection there is no loss of intensity in optical fibres.

I. CHALLENGING PROBLEMS

1. Figure shows a biconvex lens (of refractive index 1.50) in contact with a liquid layer on top of a plane mirror. A small needle with its tip on the principal axis is moved along the axis until its inverted image is found at the position of the needle. The distance of the needle from the lens is measured to be 45.0 cm. The liquid is removed and the experiment is repeated. The new distance is measured to be 30.0 cm. What is the refractive index of the liquid?



Sol. Distance of the needle from the lens in the first case = Focal length F of the combination of the convex lens and planoconcave lens formed by the liquid, i.e., F = 45cm
Distance measured on second case,= Focal length of the convex lens, i.e. $f_1 = +30$cm
The focal length f_2 of the plano-concave lens is given by

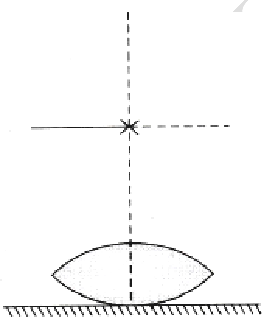
$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{F} \text{ or } \frac{1}{f_2}=\frac{1}{F}-\frac{1}{f_1}=\frac{1}{45}-\frac{1}{30}=\frac{2-3}{90}=-\frac{1}{90}$$

$\therefore f_2 = -90$cm
Now for the equiconvex lens, we have $R_1 = R$, $R_2 = -R$, $f = 30$cm, $\mu = 1.5$
Using Lens makers formula

$$\frac{1}{f}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

Or $\frac{1}{30} = (1.5-1)\left[\frac{1}{R}+\frac{1}{R}\right]$ = 0.5 ×$\frac{2}{R}$ or R = 0.5 × 2 × 30cm = 30cm.
For plano-convex lens, f = - 90cm, For concave surface, R_1 = - R = - 30cm, For plane surface, $R_2 = \infty$
As $\frac{1}{f} = (\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$
$\therefore \frac{1}{-90} = (\mu-1)\left[\frac{1}{-30}-\frac{1}{\infty}\right]$
Or $\mu - 1 = \frac{-30}{-90} = +\frac{1}{3}$
Or m = 1+ 1/3 = 1.33.

2. A symmetric biconvex lens of radius of curvature R and made of refractive index 1.5, is placed on a layer of liquid placed on top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x. On removing the liquid layer and repeating the experiment, the distance is found to be y. Obtain the expression for the refractive index of the liquid in terms of x and y.



Sol. Clearly, equivalent focal length of equiconvex lens and water lens, f = x
Focal length of equiconvex lens f_1 = y
Focal length f_2 of water lens is given by $\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$ or $f_2 = \frac{xy}{y-x}$
The water lens formed between the plane mirror and the equiconvex lens is a planoconcave lens. For this lens, R_1 = - R and $R_2 = \infty$
Using lens maker's formula, $\frac{1}{f_2} = (\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$
Or $\frac{y-x}{xy} = (\mu-1)\left[\frac{1}{-R}-\frac{1}{\infty}\right]$
Or $\mu - 1 = \frac{(x-y)R}{xy}$ or $\mu = 1+\frac{(x-y)R}{xy}$

SPACE FOR ROUGH WORK

SPACE FOR NOTES