

WORKSHEET- DUAL NATURE OF MATTER AND RADIATION

A. PARTICLE NATURE OF LIGHT – THE PHOTON

(3 Marks Questions)

1. The energy flux of sunlight reaching the surface of the earth is $1.388 \times 10^3 \text{ W/m}^2$. How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm.

Sol. Energy of each photon, $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{550 \times 10^{-9}} = 3.62 \times 10^{-19} \text{ J}$

Number of photons incident on earth's surface per second per square metre
 $= \frac{\text{Total energy per square metre per second}}{\text{Energy of each photon}} = \frac{1.388 \times 10^3}{3.62 \times 10^{-19}} = 3.8 \times 10^{21}$

2. A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light? (b) At what rate are the photons delivered to the sphere?

Sol. Power, $P = 100 \text{ W}$, wavelength of sodium light, $\lambda = 5.89 \times 10^{-9} \text{ m}$

(a) Energy of each photon associated with sodium light, $E = hv = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{589 \times 10^{-9}} = 3.38 \times 10^{-19} \text{ J}$

(b) Number of photons delivered to sphere per second

As $P = nE$

$n = \frac{\text{Energy radiated per second}}{\text{Energy of each photon}} = \frac{100}{3.38 \times 10^{-19}} = 3 \times 10^{20} \text{ photon/s.}$

3. Estimating the following two numbers should be interesting. The first number will tell you why radio engineers do not need to worry much about photons! The second number tells you why our eye can never 'count photons', even in barely detectable light.

(a) The number of photons emitted per second by a Medium wave transmitter of 10 kW power, emitting radiowaves of wavelength 500 m.

(b) The number of photons entering the pupil of our eye per second corresponding to the minimum intensity of white light that we humans can perceive ($\sim 10^{-10} \text{ W m}^{-2}$). Take the area of the pupil to be about 0.4 cm^2 , and the average frequency of white light to be about $6 \times 10^{14} \text{ Hz}$.

Sol. a) Power of the medium wave transmitter, $P = 10 \text{ kW} = 10^4 \text{ W} = 10^4 \text{ J/s}$

Hence, energy emitted by the transmitter per second, $E = 10^4$

Wavelength of the radio wave, $\lambda = 500 \text{ m}$

The energy of the wave is given as: $E = \frac{hc}{\lambda}$

Where, h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s

$$\therefore E_1 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{500} = 3.96 \times 10^{-28} \text{ J}$$

Let n be the number of photons emitted by the transmitter.

$$\therefore nE_1 = E$$

$$\begin{aligned} n &= \frac{E}{E_1} \\ &= \frac{10^4}{3.96 \times 10^{-28}} = 2.525 \times 10^{31} \\ &\approx 3 \times 10^{31} \end{aligned}$$

The energy (E_1) of a radio photon is very less, but the number of photons (n) emitted per second in a radio wave is very large.

The existence of a minimum quantum of energy can be ignored and the total energy of a radio wave can be treated as being continuous.

(b) Intensity of light perceived by the human eye, $I = 10^{-10} \text{ W m}^{-2}$, Area of a pupil, $A = 0.4 \text{ cm}^2 = 0.4 \times 10^{-4} \text{ m}^2$, Frequency of white light, $\nu = 6 \times 10^{14} \text{ Hz}$,

The energy emitted by a photon is given as: $E = h/\nu$ Where, h = Planck's constant = 6.6×10^{-34} Js

$$E = 6.6 \times 10^{-34} \times 6 \times 10^{14} = 3.96 \times 10^{-19} \text{ J}$$

Let n be the total number of photons falling per second, per unit area of the pupil. The total energy per unit for n falling photons is given as: $E = n \times 3.96 \times 10^{-19} \text{ Js}^{-1} \text{ m}^{-2}$

The energy per unit area per second is the intensity of light, $E = I$

$$n \times 3.96 \times 10^{-19} = 10^{-10}$$

$$\begin{aligned} n &= \frac{10^{-10}}{3.96 \times 10^{-19}} \\ &= 2.52 \times 10^8 \text{ m}^2 \text{ s}^{-1} \end{aligned}$$

The total number of photons entering the pupil per second is given as:

$$nA = n \times A = 2.52 \times 10^8 \times 0.4 \times 10^{-4} = 1.008 \times 10^4 \text{ s}^{-1}$$

This number is not as large as the one found in problem (a), but it is large enough for the human eye to never see the individual photons.

(5 Marks Questions)

- Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

- (a) Find the energy and momentum of each photon in the light beam,
 (b) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section which is less than the target area), and
 (c) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

Sol. Here $\lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$, $P = 9.42 \text{ mW} = 9.42 \times 10^{-3} \text{ W}$

(a) Energy of each photon, $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}} = 3.14 \times 10^{-19} \text{ J}$.

Momentum of each photon, $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{632.8 \times 10^{-9}} = 1.05 \times 10^{-27} \text{ kg ms}^{-1}$.

(b) Number of photons arriving per second at the target,

$$N = \frac{P}{E} = \frac{9.72 \times 10^{-3}}{3.14 \times 10^{-19}} = 3 \times 10^{16} \text{ photons per second}$$

(c) Momentum of a hydrogen atom = Momentum of a photon or $mv = p$

Therefore velocity, $v = \frac{p}{m} = \frac{1.05 \times 10^{-27} \text{ kg ms}^{-1}}{1.67 \times 10^{-27} \text{ kg}} = 0.63 \text{ ms}^{-1}$.

B. PHOTOELECTRIC EFFECT

(1 Mark Question)

1. The frequency (ν) of incident radiation is greater than threshold frequency (ν_0) in a photocell. How will the stopping potential vary if frequency (ν) is increased, keeping other factors constant?

Sol. Given, the frequency of the incident radiation is greater than the threshold frequency. Therefore, the value of stopping potential (V_0) increases with increase in frequency (ν) of the incident radiation and K.E. increases.

2. If the intensity of the incident radiation in a photocell is increased, how does the stopping potential vary?

Sol. The stopping potential does not depend on the intensity of incident radiation, so stopping potential will remain unchanged.

3. Two metals A and B have work functions 2eV and 4eV respectively. Which of the two metals has a smaller threshold wavelength?

Sol. As $\lambda = hc/w$.

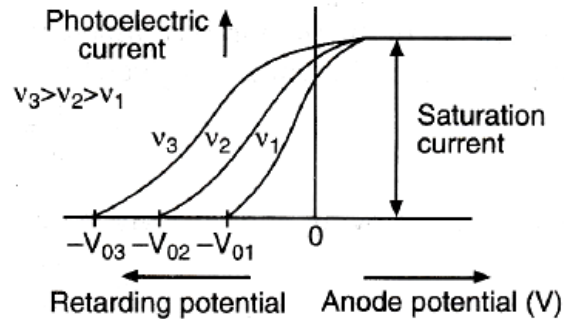
So, 4eV. Thus metal B has lower threshold wavelength.

4. Does the 'stopping potential' in photoelectric emission depend upon (i) the intensity of the incident radiation in a photocell? (ii) the frequency of the incident radiation?

Sol. No, the stopping potential does not depend upon the intensity of incident radiation but depends on the nature of photosensitive surface and frequency of the incident radiation.

5. Show graphically how the stopping potential for a given photosensitive surface varies with the frequency of the incident radiation.

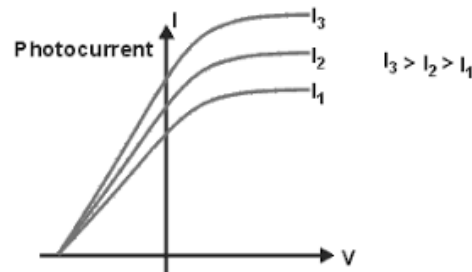
Sol.



6. State Einstein photoelectric equation.

Sol. Einstein's photoelectric equation states that $E_k = h\nu - \phi$.

7. In an experiment on photoelectric effect, the following graphs were obtained between the photoelectric current (I) and the anode potential (V). Name the characteristic of the incident radiation that was kept constant in this experiment.



Sol. As the value of stopping potential is same for all the curves, so the frequency of incident radiations is kept constant but their intensity is different.

8. How does maximum kinetic energy of electrons emitted vary with work function of the metal?

Sol. The maximum kinetic energy of emitted electrons, $K_{\max} = \frac{1}{2} m v_{\max}^2 = h\nu - W_0$. Clearly, the larger the work function of the metal, lesser is the maximum K.E. of the photoelectrons.

9. If the maximum kinetic energy of electrons emitted in a photocell is 5eV, what is the stopping potential?

Sol. The stopping potential $V_0 = 5$ volt (negative)

10. Define the term work function for photoelectric effect.

Sol. The photoelectric work function is the minimum photon energy required to liberate an electron from a substance, in the photoelectric effect. If the photon's energy is greater than the substance's work function, photoelectric emission occurs and the electron is liberated from the surface.

11. Electrons emitted from a photosensitive surface when it is illuminated by green light but electrons does not take place by yellow light. Will the electrons be emitted when the surface is illuminated by (i) red light and (ii) blue light?

Sol. (i) No, electrons are not emitted by yellow light, because $\nu_{\text{red}} < \nu_{\text{yellow}}$
 (ii) Yes, electrons are emitted by blue light, because $\nu_{\text{blue}} > \nu_{\text{green}}$

12. Ultraviolet radiations of different frequencies ν_1 and ν_2 are incident on two photosensitive materials having work functions W_1 and W_2 ($W_1 > W_2$) respectively. The kinetic energy of the emitted electrons is same in both the cases. Which one of the two radiations will be of higher frequency?

Sol. $h\nu_1 = K + W_1$
 $h\nu_2 = K + W_2$
 As $W_1 > W_2$
 so $\nu_1 > \nu_2$

13. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then

- (a) no electrons will be emitted as only photons can emit electrons.
 (b) electrons can be emitted but all with an energy, E_0 .
 (c) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function).
 (d) electrons can be emitted with any energy, with a maximum of E_0 .

Sol. (d)
 When a beam of electrons of energy E_0 is incident on a metal surface kept in vacuum of evacuated chamber so electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0 when part of incident energy of electron is used in liberating the electrons from the surface of metal. So, maximum energy of emitted electrons can be E_0 .

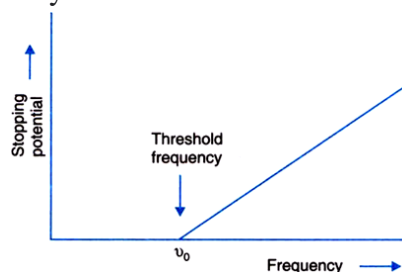
(2 Marks Questions)

14. Obtain the expression for the maximum kinetic energy of the electron emitted from a metal surface in terms of the frequency of the incident radiation and the threshold frequency.

Sol. The maximum kinetic energy of a photoelectron is given by $E = hf - W$, where h is the Planck constant, f is the frequency of the incident photon, and W is the work function of the metal surface.

15. Define the term 'threshold frequency' for photoelectric effect. Show graphically how stopping potential for a given metal varies with frequency of incident radiation. What does the slope of this graph represent?

Sol. Threshold frequency: It is the minimum frequency of the incident light or radiation that will produce a photoelectric effect i.e. ejection of photoelectrons from a metal surface is known as threshold frequency for the metal.



The slope represents threshold frequency.

16. Explain laws of photoelectric emission on the basis of Einstein's photoelectric equation.

Sol. The emission of electrons stops below a certain minimum frequency known as threshold frequency. According to Einstein's equation of photoelectric effect, $E_i = h\nu_i = h\nu_0 + KE$ where E_i is energy of incident photon and ν_0 is threshold frequency.

17. How does the value of work function influence the kinetic energy of electrons liberated during photoelectric emission?

Sol. Work function of metal:-

The amount of energy required to remove an electron from an atom is specified as the work function of a certain metal. The energy of the electron spinning around the nucleus is the same as this. We beam a light of a specific frequency on a metal in the photoelectric effect. The atom receives energy from light. The work function of a metal is defined as the minimal amount of energy required to simply remove an electron from an atom.

The law of conservation of energy states:

A photon's energy is equal to the sum of its threshold energy and its kinetic energy.

Energy of a photon = Threshold energy + Kinetic Energy

The element's work function equals the threshold energy.

As a result, work function = photon energy - metal kinetic energy.

In equation form, we can write it as: $K = h\nu - q$, $K = h\nu - q$,

Where, K is the kinetic energy, ν is the frequency, and q is the metal's work function.

The kinetic energy of the released photo electron falls as the work function of the metal increases.

18. When a monochromatic yellow coloured light beam is incident on a given photosensitive surface, photoelectrons are not ejected, while the same surface gives photoelectrons when exposed to green coloured monochromatic beam. What will happen if the same

photosensitive surface is exposed to (i) violet and (ii) red coloured, monochromatic beam of light? Justify your answer.

Sol. (i) Violet light has higher frequency than green light, so it can eject electrons from the photosensitive surface. (ii) Red light has lower frequency than yellow light, it cannot eject electrons from the photo-sensitive surface.

19. A source of light is placed at a distance of 50cm from a photocell and the cut off potential is found to be V_0 . If the distance between the light source and photocell is made 25cm, what will be the new cut off potential? Justify your answer

Sol. By changing the position of source of light from photocell, there will be a change in the intensity of light falling on photocell. As stopping potential is independent of the intensity of the incident light, hence stopping potential remains same i.e., V_0 .

20. Using Einstein's photoelectric equation to explain (i) independence of maximum energy of emitted photoelectrons from intensity of incident light, (ii) existence of threshold frequency for emission of photoelectrons.

21. For a photosensitive surface, threshold wavelength is λ_0 . Does photo emission occur if the wavelength (λ) of the incident radiation is (i) more than λ_0 (ii) less than λ_0 ? Justify your answer

Sol. Einstein's photoelectric equation gives us $h\nu = \nu_0 + \frac{1}{2}mv_{\max}^2$

$$h \frac{c}{\lambda} = h \frac{c}{\lambda_0} + \frac{1}{2}mv_{\max}^2$$

(i) When $\lambda > \lambda_0$, $\frac{1}{2}mv_{\max}^2$ is negative. Therefore, photoemission will not occur.

(ii) When $\lambda < \lambda_0$, $\frac{1}{2}mv_{\max}^2$ is positive. So, photoemission will occur

Here λ_0 is threshold frequency.

22. The work function of lithium is 2.3eV. What does it mean? What is the relation between the work function 'W' and threshold wavelength ' λ ' of a metal?

Sol. Work function of lithium = 2.3eV

This means that to remove outermost electron from the ground shell of the lithium atom, an energy of 2.3eV is required.

The relation between work function (W) and wavelength (λ) is given as $W = h\nu$

$$\text{i.e., } W = \frac{hc}{\lambda}$$

23. The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted?

Sol. Photoelectric cut off voltage, $V_0 = 1.5\text{V}$

The maximum kinetic energy of the emitted photoelectrons is given by $K_e = eV_0$

$$K_e = 1.6 \times 10^{-19} \times 1.5\text{J} = 2.4 \times 10^{-19}\text{J}.$$

Therefore the maximum kinetic energy of photoelectrons emitted in the given experiment is $2.4 \times 10^{-19}\text{J}$.

24. The threshold frequency for a certain metal is $3.3 \times 10^{14}\text{Hz}$. If light of frequency $8.2 \times 10^{14}\text{Hz}$ is incident on the metal, predict the cutoff voltage for the photoelectric emission.

Sol. Threshold frequency of the metal, $\nu_0 = 3.3 \times 10^{14}\text{Hz}$, frequency of light incident on the metal, $\nu = 8.2 \times 10^{14}\text{Hz}$, Charge on one electron, $e = 1.6 \times 10^{-19}\text{C}$, Planck's constant, $h = 6.626 \times 10^{-34}\text{Js}$

Cut off voltage for photoelectric emission from the metal V_0

The equation for the cut off energy is given by $eV_0 = h(\nu - \nu_0)$

$$V_0 = \frac{h(\nu - \nu_0)}{e} = \frac{6.626 \times 10^{-34} \times (8.2 \times 10^{14} - 3.3 \times 10^{14})}{1.6 \times 10^{-19}} = 2.03\text{V}$$

25. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm?

Sol. No.

Work function of the metal, $\phi_0 = 4.2\text{eV}$

Charge on an electron, $e = 1.6 \times 10^{-19}\text{C}$

Planck's constant, $h = 6.626 \times 10^{-34}\text{Js}$

Wavelength of the incident radiation, $\lambda = 330\text{nm} = 330 \times 10^{-9}\text{m}$

Speed of light, $c = 3 \times 10^8\text{m/s}$

The energy of the incident photon is given as:

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}} = 6.0 \times 10^{-19}\text{J}$$

$$= \frac{6.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.76\text{eV}$$

It can be observed that the energy of the incident radiation is less than the work function of the metal. Hence, no photoelectric emission will take place.

26. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.

Sol. Given, for the first condition,

Wavelength of light, $\lambda = 600$

Wavelength of light, $\lambda' = 400$ nm

Also, maximum kinetic energy for the second condition is equal to the twice of the kinetic energy in first condition, then,

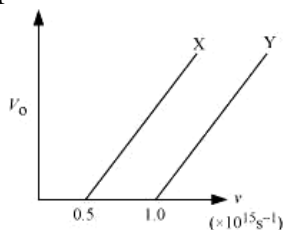
$$\begin{aligned}
 K'_{\max} &= 2K_{\max} \\
 \text{Here, } K'_{\max} &= \frac{hc}{\lambda} - \phi \\
 \Rightarrow 2K_{\max} &= \frac{hc}{\lambda'} - \phi_0 \\
 \Rightarrow 2\left(\frac{1240}{600} - \phi\right) &= \left(\frac{1240}{400} - \phi\right) \quad [:: hc \approx 1240 \text{ eV-nm}] \\
 \Rightarrow \phi &= \frac{1240}{1200} = 1.03 \text{ eV}
 \end{aligned}$$

(3 Marks Questions)

27. Radiation of frequency 10^{15} Hz are incident on two photosensitive surfaces A and B. Following observations are recorded: Surface A: No photoemission takes place. Surface B: Photoemission takes place but photoelectrons on the basis of Einstein's photoelectric equation. How will the observation with surface B change when the wavelength of incident radiations is decreased?

Sol. From the observations made (parts A and B) on the basis of Einstein's photoelectric equation, we can draw following conclusions: For surface A, the threshold frequency is more than 10^{15} HZ, hence no photoemission is possible. For surface B the threshold frequency is equal to the frequency of given radiation.

28. The following graph shows the variation of stopping potential V_0 with the frequency ν of the incident radiation for two photosensitive metals X and Y:



- Which of the metals has larger threshold wavelength? Give reasons.
- Explain, giving reason, which metal gives out electrons, having larger kinetic energy, for the same wavelength of the incident radiation.

(iii) If the distance between the light source and metal X is halved, how will the kinetic energy of electrons emitted from it change? Give reasons.

Sol. (i) Let ν_i = frequency of incident radiation on metal Y, ν_i' = frequency of incident radiation on metal X

Since $\nu_i > \nu_i'$ therefore $\lambda_i < \lambda_i'$ [since $\nu_i = C/\lambda_i$ and $\nu_i' = C/\lambda_i'$]

Therefore metal X has larger threshold wavelength.

(ii) Since the kinetic energy of the emitted electrons is directly proportional to the frequency of incident radiation, metal Y having larger incident frequency will have larger kinetic energy. So, E_{av_i} , therefore metal Y has larger kinetic energy.

(iii) Kinetic energy of the emitted photoelectrons is independent of the intensity of the incident light. Hence kinetic energy of the emitted photoelectrons remains unchanged if the distance between the light source and metal X is halved.

29. In the figure above,

(i) Explain which metal has a smaller threshold wavelength.

(ii) Explain, giving reason, which metal emits photoelectrons having smaller kinetic energy.

(iii) If the distance between the light source and metal P is doubled, how will the stopping potential change?

Sol. Same as 28

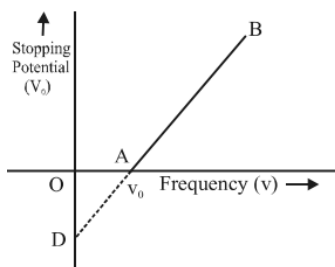
(i) metal Y has smaller threshold wavelength

(ii) metal X emits smaller kinetic energy.

(iii) On doubling the distance between the light source and the cathode of the cell, the intensity of light incident on the photocell becomes one-fourth. As stopping potential does not depend on intensity, the stopping potential remains unchanged.

30. Sketch a graph between the frequency of incident radiations and stopping potential for a given photosensitive material. What information can be obtained from the value of the intercept on the potential axis?

Sol.



From Einstein's photoelectric equation $E_k = h\nu - W$

$$eV_0 = h\nu - W$$

$$V_0 = h/e \cdot \nu - W/e$$

Comparing with $y = mx + c$ The intercept W/e will help to obtain the work function of the substance.

(i) Photoelectric Current : As the distance of the light source from the cathode is reduced, the intensity of light is increased. Thus, photoelectric current is increased because more photo electrons will get emitted.

(ii) Stopping Potential : The stopping potential remains unaffected by reducing the distance of the light source from the cathode, as frequency is not changed on reducing the distance between source of light and cathode.

31. Define the terms threshold frequency and stopping potential in relation to the phenomenon of photoelectric effect. How is the photoelectric current affected on increasing the (i) frequency (ii) intensity of the incident radiations and why?

Sol. Threshold frequency is the minimum frequency of the incident light which can cause the ejection of electrons without giving them additional energy. The amount of potential that is required to stop the electron having the maximum kinetic energy from moving is known as stopping potential.

Stopping potential : The minimum negative potential given to the anode of a photocell for which the photoelectric current becomes zero is called stopping potential.

(i) The increase of frequency of incident radiation has no effect on the photoelectric current.

(ii) The photoelectric current increases proportionally with the increase in intensity of incident radiation.

32. If the frequency of incident radiation on a photo cell is doubled for the same intensity, what changes will you observe in (i) the kinetic energy of photoelectrons emitted (ii) Photoelectric current and (iii) Stopping potential? Justify your answer in each case.

Sol. (i) The K.E. of the photoelectron becomes more than double of its original energy. As the work function of the metal is fixed, so incident photon of higher energy will impart more energy to the photoelectron. (ii) The increase in frequency of incident radiation has no effect on photoelectric current.

33. When light of wavelength 400nm is incident on the cathode of a photocell, the stopping material recorded is 6V. If the wavelength of the incident light is increased to 600nm, calculate the new stopping potential.

Sol. Let W be the work function of the cathode surface, and ν the frequency of light falling on the surface. Then, according to Einstein's photoelectric equation, the maximum kinetic energy E_k of the emitted electrons is given by

$$E_k = h\nu - W$$

$$= \frac{hc}{\lambda} - W \quad \text{where } \lambda \text{ is the wavelength of the incident light.}$$

If the cut-off potential is V_0 , then the maximum kinetic energy of the

electron, $E_k = eV_0$. Thus $eV_0 = \frac{hc}{\lambda} - W$ or $V_0 = \frac{hc}{e\lambda} - \frac{W}{e}$

If the wavelength of the incident light is increased from λ_1 to λ_2 , then the change (decrease) in the stopping potential will be

$$\begin{aligned} V_0 &= (V_0)_2 - (V_0)_1 = \left(\frac{hc}{e\lambda_2} - \frac{W}{e} \right) - \left(\frac{hc}{e\lambda_1} - \frac{W}{e} \right) \\ &= \frac{hc}{e} \left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right) = \frac{hc}{e} \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} \right) \end{aligned}$$

Here,

$$\lambda_1 = 400 \text{ nm} = 400 \times 10^{-9} \text{ m}$$

$$\lambda_2 = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$$

$$\begin{aligned} \therefore \Delta V_0 &= \frac{(6.6 \times 10^{-34} \text{ Js}) \times (3.0 \times 10^8 \text{ ms}^{-1})}{(1.6 \times 10^{-19} \text{ C})} \left[\frac{(400 - 600) \times 10^{-9} \text{ m}}{(400 \times 10^{-9} \text{ m})(600 \times 10^{-9} \text{ m})} \right] \\ &= -1.03 \text{ J/C} = -1.03 \text{ V} \end{aligned}$$

$$\begin{aligned} \text{Now, } (V_0)_2 &= (V_0)_1 + \Delta V_0 = 6.0 \text{ V} + (-1.03 \text{ V}) \\ &= 4.97 \text{ V} \end{aligned}$$

34. Ultraviolet light of wavelength 2271 \AA from 100 W mercury source radiates a photo cell made of molybdenum metal. If the stopping potential is 1.3 V , estimate the work function of the metal. How would the photocell respond to high intensity (10^5 Wm^{-2}) red light of wavelength 6328 \AA produced by a He-Ne laser?

Plot a graph showing the variation of photoelectric current with anode potential for two light beams of same wavelength but different intensity.

- Sol. (a) From Einstein's equation, $h\nu = \phi_0 + K = \phi_0 + eV_s$

Or $\phi_0 = h\nu - eV_s = \frac{hc}{\lambda} - eV_s$ (Equation is independent of the power of the source)

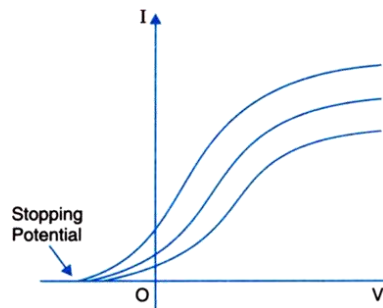
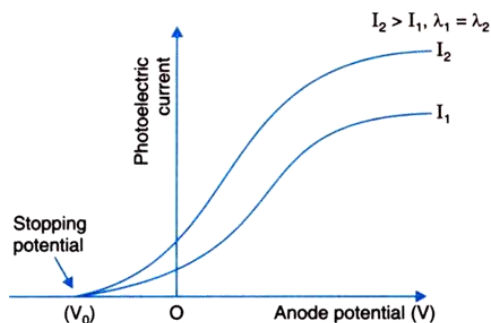
$$\phi_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2271 \times 10^{-10}} - 1.3 \text{ eV} = \left(\frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2271 \times 10^{-10} \times 1.6 \times 10^{-19}} - 1.3 \right) \text{ eV} = 5.5 \text{ eV} - 1.3 \text{ eV} = 4.2 \text{ eV}$$

(b) Threshold frequency, $\nu_0 = \frac{\phi_0}{h} = \frac{4.2 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.0 \times 10^{15} \text{ Hz}$ and the frequency of red light from the source is 10^5 W/m^2 .

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{6.325 \times 10^{-10}} = 4.7 \times 10^{14} \text{ Hz}$$

Since frequency of red light is less than threshold frequency so photocell will not respond to red light, however high (10^5 W/m^2) be the intensity of light.

(c)



35. Find the
 (a) maximum frequency, and
 (b) minimum wavelength of X-rays produced by 30 kV electrons.

Sol. (a). Potential of the electrons, $V=30\text{kV}=3\times 10^4\text{V}$

Hence, energy of the electrons, $E=3\times 10^4\text{eV}$

Where,

$e =$ Charge on an electron $= 1.6\times 10^{-19}\text{C}$

Maximum frequency produced by the X-rays is ν

The energy of the electrons is given by the relation is $E=h\nu$

Where,

$h =$ Planck's constant $= 6.626\times 10^{-34}\text{Js}$

$\nu=E/h=7.24\times 10^{18}\text{Hz}$

(b). Energy of a electron, $E=30\times 10^3\text{eV}$

Let Maximum frequency produced by the X-rays be ν

$\nu=E/h=1.6\times 10^{-19}\times 3\times 10^4/6.626\times 10^{-34}=7.24\times 10\text{-Hz}$ where h is the Planck's constant.

The minimum wavelength produced by the X-rays is given as-

$\lambda=c/\nu=0.0414\text{nm}$

36. The work function of caesium metal is 2.14 eV. When light of frequency $6\times 10^{14}\text{Hz}$ is incident on the metal surface, photoemission of electrons occurs. What is the
 (a) maximum kinetic energy of the emitted electrons,
 (b) Stopping potential, and
 (c) maximum speed of the emitted photoelectrons?

Sol. Given: The work function of caesium metal is 2.14 eV and the frequency of light is $6\times 10^{14}\text{Hz}$.

(a) The maximum kinetic energy (eV) in photoelectric effect is given as,

$\text{K.E.} = h\nu - \phi$

Where, the Planck's constant is h , the charge on electron is e , the work function of caesium metal is ϕ and the frequency of light is ν .

By substituting the given values in above equation, we get

$$\text{K.E.} = 6.626 \times 10^{-34} \times 6 \times 10^{14} = 1.6 \times 10^{-19} - 2.14 = 2.485 - 2.14 = 0.34 \text{ eV}$$

Thus, the maximum kinetic energy of emitted electron is 0.34 eV.

(b) The stopping potential is given as,

$$V_0 = \text{K.E.} / e$$

By substitute the values in above equation, we get

$$V_0 = 0.345 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19} = 0.34 \text{ V}$$

Thus, the stopping potential of the material is 0.34 V.

(c) The maximum speed of the photoelectrons is given as,

$$\text{K.E.} = \frac{1}{2} m v^2 \text{ or } v = \sqrt{2 \times \text{K.E.} / m}$$

Where, the maximum speed of electron is v and the mass of electron is m .

By substitute the given values in above equation, we get

$$v = \sqrt{2 \times 0.345 \times 1.6 \times 10^{-19}} = 9.1 \times 10^{-10} = 3.483 \times 10^5 \text{ m/s} = 348.3 \text{ km/s}$$

Thus, the maximum speed of emitted photoelectron is 348.3 km/s.

37. In an experiment on photoelectric effect, the slope of the cut-off voltage versus frequency of incident light is found to be $4.12 \times 10^{-15} \text{ V s}$. Calculate the value of Planck's constant.

Sol. Here $\frac{\Delta V}{\Delta \nu} = 4.12 \times 10^{-15} \text{ V s}$, $e = 1.6 \times 10^{-19} \text{ C}$

$$\text{Planck's constant, } h = \frac{\Delta V}{\Delta \nu} \cdot e = 4.12 \times 10^{-15} \times 1.6 \times 10^{-19} = 6.592 \times 10^{-34} \text{ Js.}$$

38. Light of frequency $7.21 \times 10^{14} \text{ Hz}$ is incident on a metal surface. Electrons with a maximum speed of $6.0 \times 10^5 \text{ m/s}$ are ejected from the surface. What is the threshold frequency for photoemission of electrons?

Sol. Here $\nu = 7.21 \times 10^{14} \text{ Hz}$, $v_{\text{max}} = 6.0 \times 10^5 \text{ ms}^{-1}$

$$\text{From Einstein's photoelectric equation, } K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = h\nu - W_0 = h(\nu - \nu_0)$$

$$\text{Therefore } \nu - \nu_0 = \frac{\frac{1}{2} m v_{\text{max}}^2}{h} = \frac{1 \times 9.1 \times 10^{-31} \times (6.0 \times 10^5)^2}{2 \times 6.63 \times 10^{-34}} = 2.47 \times 10^{14} \text{ Hz}$$

$$\text{Or } \nu_0 = \nu - 2.47 \times 10^{14} = 7.21 \times 10^{14} - 2.47 \times 10^{14} = 4.74 \times 10^{14} \text{ Hz.}$$

39. Light of wavelength 488 nm is produced by an argon laser which is used in the photoelectric effect. When light from this spectral line is incident on the emitter, the stopping (cut-off) potential of photoelectrons is 0.38 V. Find the work function of the material from which the emitter is made.

Sol. Here $\lambda = 488 \text{ nm} = 488 \times 10^{-9} \text{ m}$, $V_0 = 0.38 \text{ V}$

$$\text{From Einstein's photoelectric equation, } K_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = h\nu - W_0$$

$$\text{Or } eV_0 = \frac{hc}{\lambda} - W_0$$

$$\therefore W_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{488 \times 10^{-9}} - 1.6 \times 10^{-19} \times 0.38 = 4.076 \times 10^{-19} - 0.608 \times 10^{-19}$$

$$= 3.468 \times 10^{-19} \text{J} \text{ or } W_0 = 3.47 \times 10^{-19} \text{J} = 2.17 \text{eV}.$$

40. (a) An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at 0.45 \AA . What is the maximum energy of a photon in the radiation?

(b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Sol. (a) Here $\lambda_{\min} = 0.45 \text{ \AA} = 0.45 \times 10^{-10} \text{m}$

$$\begin{aligned} \text{The maximum energy of an X ray photon is } E_{\max} &= h\nu_{\max} = \frac{hc}{\lambda_{\min}} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.45 \times 10^{-10}} \text{J} = \frac{6.63 \times 3 \times 10^{-16}}{0.45 \times 1.6 \times 10^{-19}} \text{eV} = 27.6 \times 10^3 \text{eV} = 27.6 \text{keV} \end{aligned}$$

(b) To get photons of energy 27.6keV , electrons of at least 27.6keV must strike the target of the X ray tube. Hence the accelerating potential should be greater than 27.6eV or it should be of the order of 30keV .

41. In an accelerator experiment on high-energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2BeV into two γ -rays of equal energy. What is the wavelength associated with each γ -ray? ($1 \text{BeV} = 10^9 \text{eV}$)

Sol. Energy of two γ -rays = energy of electron positron pair = $10.2 \text{BeV} = 10.2 \times 10^9 \text{eV}$

$$\begin{aligned} \text{Therefore energy of each } \gamma\text{-ray photon is } E &= 5.1 \times 10^9 \text{eV} = 5.1 \times 10^9 \times 1.6 \times 10^{-19} \text{J} \\ &= 5.1 \times 1.6 \times 10^{-10} \text{J} \end{aligned}$$

$$\text{But } E = h\nu = hc/\lambda$$

Hence wavelength associated with γ -ray is

$$\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5.1 \times 1.6 \times 10^{-10}} \text{m} = 2.436 \times 10^{-16} \text{m}.$$

42. Ultraviolet light of wavelength 2271 \AA from a 100W mercury source irradiates a photo-cell made of molybdenum metal. If the stopping potential is -1.3V , estimate the work function of the metal. How would the photo-cell respond to a high intensity ($\sim 10^5 \text{W m}^{-2}$) red light of wavelength 6328 \AA produced by a He-Ne laser?

Sol. Here $\lambda = 2271 \text{ \AA} = 2271 \times 10^{-10} \text{m}$, $V_0 = 1.3 \text{V}$, $h = 6.63 \times 10^{-34} \text{Js}$, $e = 1.6 \times 10^{-19} \text{C}$, $W_0 = ?$

Einstein's photoelectric equation is Maximum KE of emitted photon = $eV_0 = h\nu - W_0$

$$\text{Therefore } W_0 = h\nu - eV_0 = \frac{hc}{\lambda} - eV_0$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2270 \times 10^{-10}} = 1.6 \times 10^{-19} \times 1.3 = [8.72 \times 10^{-19} \text{ } 2.08 \times 10^{-19} \text{J}]$$

$$= \frac{6.64 \times 10^{-19}}{1.6 \times 10^{-19}} \text{eV} = 4.5 \text{eV} \text{ [since } 1 \text{eV} = 1.6 \times 10^{-19} \text{J}]$$

$$\text{Threshold frequency is } \nu_0 = \frac{W_0}{h} = \frac{6.64 \times 10^{-19}}{6.63 \times 10^{-34}} \text{Hz} = 1.0 \times 10^{15} \text{Hz}$$

$$\text{For red light, } \lambda = 6328 \text{ \AA} = 6328 \times 10^{-10} \text{m}$$

Therefore corresponding frequency for red light will be $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{6328 \times 10^{-10}} = 4.74 \times 10^{10} \text{ Hz}$

As $\nu < \nu_0$ so there will be no photoelectric emission with this red light, howsoever high its intensity may be.

43. Monochromatic radiation of wavelength 640.2 nm ($1 \text{ nm} = 10^{-9} \text{ m}$) from a neon lamp irradiates photosensitive material made of cesium on tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photo-cell. Predict the new stopping voltage.

Sol. Here $\lambda_1 = 640.2 \times 10^{-9} \text{ m}$, $V_{01} = 0.54 \text{ V}$, $\lambda_2 = 427.2 \times 10^{-9} \text{ m}$, $V_{02} = ?$

From Einstein's photoelectric equation, $eV_0 = \frac{hc}{\lambda} - W_0$

From neon lamp, $eV_{01} = \frac{hc}{\lambda_1} - W_0$

For iron source, $eV_{02} = \frac{hc}{\lambda_2} - W_0$

Therefore, $V_{02} - V_{01} = \frac{hc}{e} \left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right]$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}} \times \left[\frac{1}{427.2 \times 10^{-9}} - \frac{1}{640.2 \times 10^{-9}} \right] = \frac{6.63 \times 3 \times 10^2}{1.6} \times \frac{213.0}{427.2} = 0.97 \text{ V}$$

Therefore $V_{02} = V_{01} + 0.97 = 0.54 + 0.97 = 1.51 \text{ V}$.

44. The work function for the following metals is given:

Na: 2.75 eV; K: 2.30 eV; Mo: 4.17 eV; Ni: 5.15 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away?

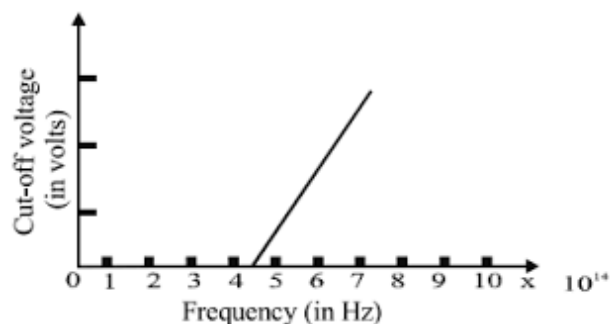
Sol. Wavelength of incident radiation is $\lambda = 3300 \text{ Å} = 3300 \times 10^{-10} \text{ m}$

$$\text{Energy of an incident photon, } E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{3300 \times 10^{-10}} \text{ J} = \frac{6.63 \times 3 \times 10^{-18}}{33 \times 1.6 \times 10^{-19}} \text{ eV} = 3.75 \text{ eV}$$

As the energy of incident photon is less than the work functions of Mo and Ni so the metals Mo and Ni will not give photoelectric emission.

If the laser is brought closer, intensity of radiation increases. This does not affect the result regarding Mo and Ni, but the photoelectric current will increase for Na and K with the increase in intensity.

45. Figure shows the plot of cut off voltage vs frequency of radiation incident on a metal.



Calculate:

(a) the threshold frequency (b) Planck's constant

Sol. (a) From the given graph, the threshold frequency is $\nu_0 = 4.5 \times 10^{14} \text{ Hz}$

(b) Using Einstein's photoelectric equation, $eV = h\nu = W_0$

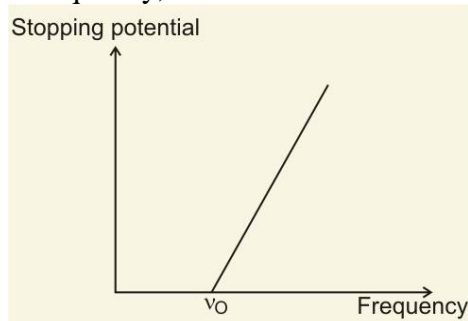
On differentiation, we get $e\Delta V = h\Delta\nu$

$$h = \frac{\Delta V}{\Delta\nu} \cdot e = \frac{2.5-2}{(10-9) \times 10^{14}} \times 1.6 \times 10^{-19} = 8 \times 10^{-34} \text{ Js.}$$

(5 Marks Questions)

46. Draw a graph showing the variation of stopping potential with frequency of incident radiations in relation to photoelectric effect. Deduce an expression for the slope of this graph using Einstein's photoelectric equation.

Sol. a) Due to the increase in frequency, there is an increase in kinetic energy of the electron.



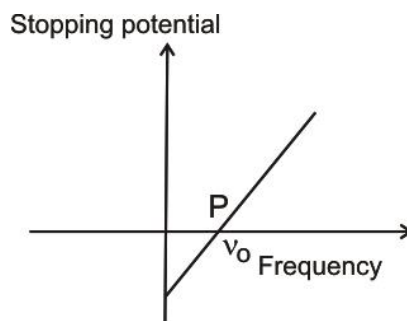
So, the stopping potential increases with increase in the frequency of incident radiation in relation to the photoelectric effect.

(b)

$$eV_0 = h\nu - h\nu_0$$

$$V_0 = \frac{h}{e}(\nu - \nu_0)$$

Point p on the graph shows the threshold frequency.



$$e\nu_o = h\nu - \phi_o$$

$$\nu_o = \frac{h}{e}\nu - \frac{\phi_o}{e}$$

Hence, the threshold frequency ν_o is the intercept along ν (frequency) axis.

Now, the slope of the given graphs gives $\frac{h}{e}$

$$\text{slope} = \frac{h}{e}$$

Hence, plank's constant $h = e \times \text{slope}$ slope of the graph

47. Define the terms (i) threshold frequency and (ii) stopping potential, with reference to photoelectric effect. Calculate the maximum kinetic energy of electrons emitted from a photosensitive surface of work function 3.2eV, for the incident radiation of wavelength 300nm.

Sol. Same as 31

Energy of incident radiation is given by:

$$E = \lambda/hc = 6.64 \times 10^{-34} \times 3 \times 10^8 / 300 \times 10^{-9}$$

$$= 6.64 \times 10^{-19} = 4.05 \text{eV}$$

The work function of the metal is equal to 2.54eV

Hence the maximum kinetic energy of an ejected photoelectron would be $4.05 - 2.54 = 1.59 \text{eV}$

And the stopping potential needed to stop this would be 1.59V.

48. A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photo-cell, the following lines from a mercury source were used:

$$\lambda_1 = 3650 \text{ \AA}, \lambda_2 = 4047 \text{ \AA}, \lambda_3 = 4358 \text{ \AA}, \lambda_4 = 5461 \text{ \AA}, \lambda_5 = 6907 \text{ \AA},$$

The stopping voltages, respectively, were measured to be:

$$V_{01} = 1.28 \text{ V}, V_{02} = 0.95 \text{ V}, V_{03} = 0.74 \text{ V}, V_{04} = 0.16 \text{ V}, V_{05} = 0 \text{ V}$$

Determine the value of Planck's constant h , the threshold frequency and work function for the material.

Sol. (i) Using $v = c/\lambda$, we first determine frequency in each case and then plot a graph between stopping potential V_0 and frequency v .

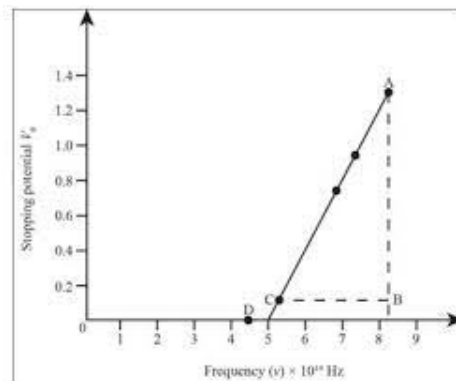
$$v_1 = \frac{c}{\lambda_1} = \frac{3 \times 10^8}{3650 \times 10^{-10}} = 8.219 \times 10^{14} \text{ Hz}$$

$$v_2 = \frac{c}{\lambda_2} = \frac{3 \times 10^8}{4047 \times 10^{-10}} = 7.412 \times 10^{14} \text{ Hz}$$

$$v_3 = \frac{c}{\lambda_3} = \frac{3 \times 10^8}{4358 \times 10^{-10}} = 6.884 \times 10^{14} \text{ Hz}$$

$$v_4 = \frac{c}{\lambda_4} = \frac{3 \times 10^8}{5461 \times 10^{-10}} = 5.493 \times 10^{14} \text{ Hz}$$

$$v_5 = \frac{c}{\lambda_5} = \frac{3 \times 10^8}{6907 \times 10^{-10}} = 4.343 \times 10^{14} \text{ Hz}$$



The first four points lie nearly on a straight line which intercepts the v axis of threshold frequency $v_0 = 5.0 \times 10^{14} \text{ Hz}$.

The fifth point ($v = 4.3 \times 10^{14} \text{ Hz}$) corresponds to $v < v_0$ so there is no photoelectric emission and no stopping voltage is required to stop the current. The slope of V_0 versus v graph is

$$\frac{\Delta V}{\Delta v} = \frac{(1.28 - 0) \text{ V}}{(8.2 - 5.0) \times 10^{14} \text{ s}^{-1}} = 4.0 \times 10^{-15} \text{ Vs}$$

From Einstein's photoelectric equation, $KE = eV = hv - W_0$

So, $e\Delta V = h\Delta v$ [W_0 is constant]

$$\text{Or } \frac{\Delta V}{\Delta v} = \frac{h}{e}$$

$$\text{Hence } \frac{h}{e} = 4.0 \times 10^{-15} \text{ Vs}$$

$$\text{Planck's constant, } h = e \times 4.0 \times 10^{-15} \text{ Js} = 1.6 \times 10^{-19} \times 4.0 \times 10^{-15} \text{ Js} = 6.4 \times 10^{-34} \text{ Js}$$

(ii) Threshold frequency, $v_0 = 5.0 \times 10^{14} \text{ Hz}$

$$\text{Therefore work function, } W_0 = hv_0 = 6.4 \times 10^{-34} \times 5.0 \times 10^{14} \text{ J} = \frac{6.4 \times 5.0 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV} = 200 \text{ eV}$$

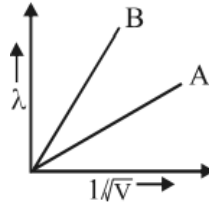
C. WAVE NATURE OF MATTER

(1 Mark Question)

1. Why are de Broglie waves associated with a moving football not visible?

Sol. de-Broglie waves associated with a moving football are not visible because in accordance with formula $\lambda = h/mv$, the value of wavelength is extremely small (of the order of 10^{-34} m).

2. Two lines A and B, in the plot given below show the variation of de Broglie wave length, λ versus $1/\sqrt{V}$, where V is the accelerating potential difference, for two particles carrying the same charge. Which one of the two represents a particle of smaller mass?



Sol. de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mqV}} = \frac{h}{\sqrt{2mq}} \cdot \frac{1}{\sqrt{V}}$

Therefore slope of λ versus graph $\frac{1}{\sqrt{V}} = \frac{h}{\sqrt{2mq}}$

For the particle of the same charge q, slope $\propto \frac{1}{\sqrt{m}}$

As the slope of line A is smaller than that of line B, line B has smaller mass.

3. de Broglie wavelength associated with an electron associated through a potential difference V is λ . What will be its wavelength when the accelerating potential is increased to 4V?

Sol. When potential difference is increased 4 times, wavelength becomes half its value.

4. An electron, an alpha particle and a proton have the same kinetic energy. Which one of these particles has the largest de Broglie wavelength?

Sol. As mass of electron is least, so electron has largest de-Broglie wavelength.

5. An electron and alpha particle have the same de Broglie wavelength associated with them. How are their kinetic energies related to each other?

Sol. Given $\lambda_{\text{electron}} = \lambda_{\alpha}$

de Broglie wavelength associated with a particle of mass m and energy E is

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\therefore \frac{h}{\sqrt{2m_e E_e}} = \frac{h}{\sqrt{2m_{\alpha} E_{\alpha}}}$$

That is kinetic energy of electron and α -particle are in inverse ratio of these masses.

6. An electron and alpha particle have the same kinetic energy. How are the de Broglie wavelength associated with them related?

Sol. Same Kinetic energy means electron has highest velocity than a proton and an alpha particle. An electron has least momentum. Hence, it has largest de Broglie wavelength and alpha particle has least wavelength.

7. An electron and a proton have the same de Broglie wavelength associated with them. How are their kinetic energies related to each other?

Sol. de Broglie wavelength,

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$

Given $\lambda_e = \lambda_p$

$$\therefore \frac{h}{\sqrt{2m_e E_e}} = \frac{h}{\sqrt{2m_p E_p}}$$

$$\Rightarrow \frac{E_e}{E_p} = \frac{m_p}{m_e} \approx 1840$$

i. e., K.E. of electron = 1840 x (K.E. of proton)

8. With what purpose was famous Davisson-Germer experiment with electrons performed?

Sol. Davisson-Germer experiment was performed to verify wave nature of electrons.

9. A particle is dropped from a height H. The de Broglie wavelength of the particle as a function of height is proportional to

- (a) H (b) $H^{1/2}$ (c) H^0 (d) $H^{-1/2}$

Ans. (d)

Velocity of a body freely falling a height H is $v = \sqrt{2gH}$

so, $\lambda = h/mv = h/m\sqrt{2gH} = h/m\sqrt{2g}\sqrt{H}$

(h, m and g are constant)

here, $h/m\sqrt{2g}$ is also constant

So, $h \propto 1/\sqrt{H}$ or $\lambda \propto H^{-1/2}$

10. A proton, a neutron, an electron and an α -particle have same energy. Then their de Broglie wavelengths compare as

- (a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (b) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$ (c) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (d) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

Ans. (b)

The relation between λ and K is given by $\lambda = \frac{h}{\sqrt{2mk}}$

So, for the given value of kinetic energy K $\frac{h}{\sqrt{2k}}$ is constant

Thus, $\lambda \propto \frac{1}{\sqrt{m}}$

therefore $\Rightarrow \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha$

$$\Rightarrow \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_\alpha}}$$

If ($m_p = m_n$), then $\lambda_p = \lambda_n$

if ($m_\alpha > m_p$), then $\lambda_\alpha < \lambda_p$

if ($m_e < m_n$), then $\lambda_e > \lambda_n$
Hence $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$

11. An electron is moving with an initial velocity $v = v_0 \hat{i}$ and is in a magnetic field $B = B_0 \hat{j}$. Then it's de Broglie wavelength
(a) remains constant. (b) increases with time.
(c) decreases with time. (d) increases and decreases periodically.

Ans. (a)

A given that $v = v_0 \hat{i}$ and $B = B_0 \hat{j}$

Force on moving electron due to perpendicular magnetic field B, is $F = -e(\mathbf{v} \times \mathbf{B})$

$$F = -e[V_0 \hat{i} \times B_0 \hat{j}] = -ev_0 B_0 (\hat{i} \times \hat{j})$$

$$\Rightarrow -v_0 B_0 \hat{k} \text{ (since } \hat{k} = \hat{i} \times \hat{j}\text{)}$$

So, the force is perpendicular to v and B , both as the force is \perp to the velocity so the magnitude of v will not change, so momentum is ($-mv$) will remain same of constant in magnitude. Hence, de-Broglie $\lambda = h/mv$ remains constant.

12. An electron (mass m) with an initial velocity $v = v_0 \hat{i}$ ($v_0 > 0$) is in an electric field $E = -E_0 \hat{i}$ ($E_0 = \text{const}$ $t > 0$). It's de Broglie wavelength at time t is given by

(a) $\frac{\lambda_0}{\left(1 + \frac{eE_0 t}{m v_0}\right)}$ (b) $\lambda_0 \left(1 + \frac{eE_0 t}{m v_0}\right)$ (c) λ_0 (d) $\lambda_0 t$.

Ans. (a)

13. An electron (mass m) with an initial velocity $v = v_0 \hat{i}$ is in an electric field $E = E_0 \hat{j}$. If $\lambda_0 = h/mv_0$, it's de Broglie wavelength at time t is given by

(a) λ_0 (b) $\lambda_0 \sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}$ (c) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}}}$ (d) $\frac{\lambda_0}{\left(1 + \frac{e^2 E_0^2 t^2}{m^2 v_0^2}\right)}$

Ans. (c)

14. A proton and an α -particle are accelerated, using the same potential difference. How are the de-Broglie wavelengths λ_p and λ_α related to each other?

Sol. It is given that the proton and α -particle are accelerated at the same potential difference so their kinetic energies will be equal.

The de-Broglie wavelengths λ_p and λ_α related to each other as

$$\lambda_p / \lambda_\alpha = p_\alpha / p_p = \frac{\sqrt{2m_\alpha E_\alpha}}{\sqrt{2m_p E_p}} = \sqrt{8} : 1$$

15. Do all the electrons that absorb a photon come out as photoelectrons?

Sol. No, most electrons get scattered into the metal. Only a few come out of the surface of the metal.

(2 Marks Questions)

16. Derive the expression for the de Broglie wavelength of an electron moving under a potential difference of V volt.

Sol. When a charged particle is accelerated by potential V , the kinetic energy is equal to electrostatic potential energy.

$$\text{Thus } \frac{1}{2} mv^2 = qV \text{ or } v = \sqrt{2qV/m}$$

$$\text{Hence the de Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{mV} = \frac{h}{\sqrt{2qV}}$$

17. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having velocities v_1 and v_2 respectively. Find the ratio of de Broglie wavelengths of two particles.

Sol. We know that, according to de-Broglie's hypothesis, the momentum of the particle is,

$$p = hc/\lambda$$

Here, h is Planck's constant, c is the speed of light and λ is the de Broglie wavelength.

According to the law of conservation of momentum, the momentum of a system remains conserved.

Therefore, we can write,

$$Mv = m_1v_1 + m_2v_2$$

Here, v is the velocity of parent particle, v_1 is the velocity of m_1 and v_2 is the velocity of m_2

Since the parent particle is at rest, the initial velocity v is zero. Therefore, the above equation becomes,

$$0 = m_1v_1 + m_2v_2$$

$$\Rightarrow m_1v_1 = -m_2v_2$$

Therefore, from the above equation, the momentum of the particle of mass m_1 and the momentum of the particle of mass m_2 is equal.

So, we can write, $p_1 = p_2$

$$\Rightarrow hc/\lambda_1 = hc/\lambda_2$$

Planck's constant h and speed of light c is constant for both particles. Therefore, the wavelength of these particles is the same. Therefore, we can write,

$$\therefore \lambda_1\lambda_2 = 1$$

18. The wavelength λ , of a photon and the de Broglie wavelength of an electron have the same value. Show that the energy of the photon is $2\lambda mc/h$ times the kinetic energy of the electron, where m , c , and h have their usual meanings.

(3 Marks Questions)

19. Show that the de Broglie wavelength λ of electrons of energy E is given by the relation $\lambda = h/\sqrt{2mE}$.

Sol. If one electron which is in rest is accelerated with potential difference V and due to this acceleration it acquires the velocity v then.

$$\frac{1}{2} m_e v^2 = eV \quad \text{where } m_e = \text{mass of electron}$$

$$e = \text{charge of electron of, } meV^2 \text{ } 2eV$$

If p is the momentum of the electron

20. Red light, however bright it is, cannot produce the emission of electrons from a clean zinc surface. But even weak ultraviolet radiation can do so. Why?

X-rays of wavelength ' λ ' falls on a photosensitive surface emitting electrons. Assuming that the work function of the surface can be neglected, prove that the de Broglie wavelength of electrons emitted will be $(h \lambda / 2mc)^{1/2}$.

Sol. The frequency of red light is less than the threshold frequency of zinc surface. Hence, it cannot cause photoelectric emission from zinc surface, whatever may be its intensity. The frequency of ultraviolet radiation is greater than the threshold frequency of zinc surface.

21. An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photoelectrons emitted from this surface have the same de

Broglie wavelength λ_1 , prove that $\lambda = \left(\frac{2mc}{h}\right) \lambda_1^2$

Sol. An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photoelectrons emitted from this surface have the de

Broglie wavelength λ' , then, A. $\lambda = \left(\frac{2mc}{h}\right) \lambda_1^2$.

22. Mention the significance of Davisson-Germer experiment. An α -particle and a proton are accelerated from rest through the same potential difference V . Find the ratio of de-Broglie wavelengths associated with them.

Sol. An α -particle and a proton are accelerated from rest through the same potential difference V . Find the ratio of de-Broglie wavelengths associated with them. The Davisson and Germer's experiment established famous de-Broglie hypothesis of wave-particle duality by confirming the wave nature of moving particle.

23. Calculate the **(a)** momentum, and **(b)** de Broglie wavelength of the electrons accelerated through a potential difference of 56 V.

Sol. (a) Kinetic energy of an electron, $K = 56\text{eV} = 56 \times 1.6 \times 10^{-19}\text{J}$

$$\text{Moment of an electron, } p = \sqrt{2mK} = \sqrt{2 \times 9.1 \times 10^{-31} \times 56 \times 1.6 \times 10^{-19}} \\ = \sqrt{1630.72 \times 10^{-50}} = 4.04 \times 10^{-24} \text{ kg ms}^{-1}$$

$$\text{(b) de Broglie wavelength, } \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{4.04} = 0.164 \times 10^{-9}\text{m} = 0.164\text{nm}.$$

24. What is the **(a)** momentum, **(b)** speed, and **(c)** de Broglie wavelength of an electron with kinetic energy of 120 eV.

Sol. Kinetic energy, $K = 120\text{eV} = 120 \times 1.6 \times 10^{-19}\text{J} = 1.92 \times 10^{-17}\text{J}$

$$\text{(a) Momentum of an electron, } p = \sqrt{2mK} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1.92 \times 10^{-17}} \\ = 5.91 \times 10^{-24} \text{ kg ms}^{-1}$$

$$\text{(b) Speed of an electron, } v = \frac{p}{m} = \frac{5.91 \times 10^{-24}}{9.1 \times 10^{-31}} = 6.5 \times 10^6 \text{ ms}^{-1}.$$

$$\text{(c) de-Broglie wavelength, } \lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{5.91 \times 10^{-24}} = 0.112 \times 10^{-9}\text{m} = 0.112 \text{ nm}$$

25. The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which

(a) an electron, and

(b) a neutron, would have the same de Broglie wavelength.

Sol. Here $589\text{nm} = 589 \times 10^{-9}\text{m}$

$$\text{But } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \text{ or } \lambda^2 = \frac{h^2}{2mK}$$

$$\text{Therefore } K = \frac{h^2}{2m\lambda^2}$$

$$\text{(a) Kinetic energy of an electron, } K = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (589 \times 10^{-9})^2} = 6.98 \times 10^{-25}\text{J} = 4.34 \mu\text{eV}.$$

$$\text{(b) Kinetic energy of neutron, } K = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.67 \times 10^{-27} \times (589 \times 10^{-9})^2} = 3.78 \times 10^{-28}\text{J} = 0.236 \text{ neV}.$$

26. An electron and a photon each have a wavelength of 1.00 nm. Find **(a)** their momentum, **(b)** the energy of the photon, and **(c)** the kinetic energy of electron.

Sol. Here $\lambda = 1.00\text{nm} = 1.00 \times 10^{-9}\text{m}$

(a) Both electrons and photon have same wavelength, so they have same momentum also.

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{1.00 \times 10^{-9}} = 6.63 \times 10^{-25} \text{ kg ms}^{-1}$$

$$\text{(b) Energy of a photon, } E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.00 \times 10^{-9}} = 19.89 \times 10^{-17}\text{J} \\ = \frac{19.89 \times 10^{-17}}{1.6 \times 10^{-19}} \text{eV} = 1.24 \times 10^3 \text{ eV} = 1.24 \text{ keV}$$

$$\text{(c) Kinetic energy of electron, } E = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} = 2.42 \times 10^{-19}\text{J}$$

$$= \frac{2.42 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.51 \text{ eV.}$$

27. (a) For what kinetic energy of a neutron will the associated de Broglie wavelength be $1.40 \times 10^{-10} \text{ m}$?

(b) Also find the de Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $(3/2) kT$ at 300 K.

Sol. Given $k = 1.38 \times 10^{-23} \text{ JK}^{-1}$

(a) de Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mk}}$

$$\text{Therefore Kinetic energy, } K = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.677 \times 10^{-27} \times (1.40 \times 10^{-10})^2} = 6.686 \times 10^{-21} \text{ J}$$

$$= \frac{6.686 \times 10^{-21}}{1.6 \times 10^{-19}} = 4.174 \times 10^{-2} \text{ eV}$$

(b) $E = 3/2 kT$

$$\text{Therefore } \lambda = \frac{h}{\sqrt{2mk}} = \frac{h}{\sqrt{3mkT}} = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.677 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m}$$

$$= \frac{6.63 \times 10^{-10}}{\sqrt{20.8}} = \frac{6.63 \times 10^{-10}}{4.56} \text{ m} = 1.45 \times 10^{-10} \text{ m} = 0.145 \text{ nm.}$$

28. Show that the wavelength of electromagnetic radiation is equal to the de Broglie wavelength of its quantum (photon).

Sol. For a photon, de Broglie wavelength, $\lambda = h/p$.

For an electromagnetic radiation of frequency ν and wavelength $\lambda' (= c/\nu)$.

$$\text{Momentum, } p = \frac{E}{c} = \frac{h\nu}{c} \text{ or } p = \frac{h}{c} \cdot \frac{c}{\lambda'} = \frac{h}{\lambda'}$$

$$\text{Then } \lambda' = \frac{h}{p} = \lambda$$

Thus the wavelength λ' of the electromagnetic radiation is the same as the de Broglie wavelength λ of the photon.

29. What is the de Broglie wavelength of a nitrogen molecule in air at 300 K? Assume that the molecule is moving with the root-mean square speed of molecules at this temperature. (Atomic mass of nitrogen = 14.0076 u, $k = 1.38 \times 10^{-23} \text{ Jk}^{-1}$)

Sol. Mass of N_2 molecule, $m = 2 \times 14.0076 \times 1.66 \times 10^{-27} \text{ kg} = 46.5 \times 10^{-27} \text{ kg}$

$T = 300 \text{ K}$

Average kinetic energy, $\frac{1}{2} mc^2 = 3/2 kT$

$$\text{Or } c = \sqrt{\frac{3kT}{m}}$$

$$\text{Therefore } \lambda = \frac{h}{mc} = \frac{h}{\sqrt{3mkT}} = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 46.5 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} \text{ m} = \frac{6.63 \times 10^{-34}}{\sqrt{577.59 \times 10^{-24}}}$$

$$= \frac{6.63 \times 10^{-10}}{24.03} \text{ m} = 0.0276 \times 10^{-9} \text{ m} = 0.028 \text{ nm.}$$

30. Crystal diffraction experiments can be performed using X-rays, or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 \AA , which is of the order of interatomic spacing in the lattice) ($m_e = 9.11 \times 10^{-31} \text{ kg}$).

Sol. For X ray photon: $\lambda = 1 \text{ \AA} = 10^{-10} \text{ m}$

$$\text{Energy of a photon, } E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{10^{-10}} \text{ J} = \frac{6.63 \times 3 \times 10^{-16}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 12.4 \times 10^3 \text{ eV} = 12.4 \text{ keV}$$

For electrons: $\lambda = 1 \text{ \AA} = 10^{-10} \text{ m}$, $m = 9.11 \times 10^{-31} \text{ kg}$

$$\text{Therefore momentum, } p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-10}} = 6.63 \times 10^{-24} \text{ kg ms}^{-1}$$

$$\text{Energy of an electron is } K = \frac{1}{2} mv^2 = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} \text{ J}$$

$$= \frac{6.63 \times 6.63 \times 10^{-17}}{2 \times 9.11 \times 1.6 \times 10^{-19}} \text{ eV} = 150.6 \text{ eV [since } p = mv]$$

Thus for the same wavelength, a photon has much greater energy than an electron.

31. (a) Obtain the de Broglie wavelength of a neutron of kinetic energy 150 eV. As you have seen in previous Qs, an electron beam of this energy is suitable for crystal diffraction experiments. Would a neutron beam of the same energy be equally suitable? Explain. ($m_n = 1.675 \times 10^{-27} \text{ kg}$)

(b) Obtain the de Broglie wavelength associated with thermal neutrons at room temperature (27°C). Hence explain why a fast neutron beam needs to be thermalised with the environment before it can be used for neutron diffraction experiments.

Sol. (a) Here $m_n = 1.675 \times 10^{-27} \text{ kg}$

$$K = 150 \text{ eV} = 150 \times 1.6 \times 10^{-19} \text{ J} = 2.4 \times 10^{-17} \text{ J}$$

$$\text{As } K = \frac{1}{2} mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m} \text{ or } p = \sqrt{2mK}$$

$$\text{Therefore de Broglie wavelength of neutrons is } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.675 \times 10^{-27} \times 2.4 \times 10^{-17}}} = \frac{6.63 \times 10^{-11}}{28.35} \text{ m} = 2.33 \times 10^{-12} \text{ m}$$

As the interatomic spacing ($1 \text{ \AA} = 10^{-10} \text{ m}$) is about 100 times greater than this wavelength, so a neutron beam of 150 eV energy is not suitable for diffraction experiments.

(b) Average kinetic energy of a neutron at absolute temperature T is $\frac{1}{2} mv^2 = \frac{3}{2} kT$

$$\text{Or } \frac{p^2}{2m} = \frac{3}{2} kT \text{ [since } p = mv] \text{ or } p = \sqrt{3mkT}$$

$$\text{Therefore de Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}$$

Given $m_n = 1.675 \times 10^{-27} \text{ kg}$, $k = 1.38 \times 10^{-23} \text{ J mol}^{-1} \text{ K}^{-1}$, $T = 27 + 273 = 300 \text{ K}$, $h = 6.63 \times 10^{-34} \text{ Js}$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}} = \frac{6.63 \times 10^{-10}}{4.56} \text{ m} = 1.45 \times 10^{-10} \text{ m} = 1.45 \text{ \AA}$$

As this wavelength is comparable to interatomic spacing ($\approx 1 \text{ \AA}$) in a crystal, so thermal neutrons can be used for diffraction experiments. A high energy neutron beams should be first thermalised before using it for diffraction.

32. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de Broglie wavelength associated with the electrons. If other factors (such as numerical aperture, etc.) are taken to be roughly the same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light?

Sol. Here $V = 50\text{kV} = 5 \times 10^4 \text{ V}$, $m_e = 9.11 \times 10^{-31} \text{ kg}$

KE of an electron, $K = 50\text{eV} = 1.6 \times 10^{-19} \times 5 \times 10^4 \text{ J} = 8 \times 10^{-15} \text{ J}$

Therefore de Broglie wavelength of electron is $\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 8 \times 10^{-15}}} \text{ m}$
 $= \frac{6.63 \times 10^{-11}}{12.07} \text{ m} = 5.5 \times 10^{-12}$,

Wavelength of yellow light, $\lambda_y = 5.9 \times 10^{-7} \text{ m}$

Resolving power of a microscope $\propto 1/\lambda$.

Therefore $\frac{\text{Resolving power of electron microscope}}{\text{Resolving power of optical microscope}} = \frac{\lambda_y}{\lambda} = \frac{5.9 \times 10^{-7}}{5.5 \times 10^{-12}} \times 10^5$

Thus the resolving power of an electron microscope is about 10^5 times greater than that of optical microscope.

33. Find the typical de Broglie wavelength associated with a He atom in helium gas at room temperature (27°C) and 1 atm pressure; and compare it with the mean separation between two atoms under these conditions.

Sol. Mass of atom is given by, $m = \frac{\text{Atomic weight of He}}{\text{Avogadro's number}}$

$$= \frac{4}{6 \times 10^{23}} \text{ g} = \frac{4 \times 10^{-3}}{6 \times 10^{23}} \text{ kg} = \frac{2}{3} \times 10^{-26} \text{ kg}$$

$$T = 27 + 273 = 300 \text{ K}$$

Average KE of He atom at absolute temperature T is $\frac{1}{2} mv^2 = \frac{3}{2} kT$

Therefore $m^2 v^2 = 3mkT$ or $p^2 = 3mkT$ or $p = \sqrt{3mkT}$

$$\text{Therefore } \lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times \frac{2}{3} \times 10^{-26} \times 1.38 \times 10^{-23} \times 300}} = 0.73 \times 10^{-10} \text{ m}$$

Kinetic gas equation for one mole of a gas can be written as $PV = RT$

Or $PV = kNT$ [since $k = R/N$]

Or $V/N = kT/P$

$$\text{Therefore mean separation, } r = \left[\frac{\text{Molar volume}}{\text{Avogadro's number}} \right]^{1/3} = \left[\frac{kT}{P} \right]^{1/3}$$

Given $T = 300 \text{ K}$, $P = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$, $k = 1.38 \times 10^{-23} \text{ J mol}^{-1} \text{ K}^{-1}$

$$\text{Hence } r = \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5} \right]^{1/3} \text{ m} = \left[\frac{138 \times 30}{101} \right]^{1/3} \times 10^{-9} \text{ m} = 3.4 \times 10^{-9} \text{ m}$$

We must find that $r \gg \lambda$, i.e, the wave packets associated with He atoms do not overlap and hence He atoms can be distinctly seen.

34. Compute the typical de Broglie wavelength of an electron in a metal at 27 °C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m.

Sol. Mass of an electron is $m = 9.11 \times 10^{-31}$ kg and $T = 27+273 = 300$ K

$$\text{Therefore de Broglie wavelength of electrons is } \lambda = \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}} \text{ m} = \frac{6.63 \times 10^{-8}}{\sqrt{3 \times 9.11 \times 1.38 \times 3}} = \frac{6.63 \times 10^{-8}}{10.64} = 6.2 \times 10^{-9} \text{ m}$$

Mean separation between two electrons in a metal, $r = 2 \times 10^{-10}$ m

$$\text{Therefore } \frac{\lambda}{r} = \frac{6.2 \times 10^{-9}}{2 \times 10^{-10}} = 31$$

Thus the de Broglie wavelength is much greater than the given inter electron separation.

(5 Marks Questions)

35. What is the de Broglie wavelength of
 (a) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s,
 (b) a ball of mass 0.060 kg moving at a speed of 1.0 m/s, and
 (c) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s?

Sol. de Broglie wavelength, $\lambda = \frac{h}{mv}$

$$(a) \lambda_{\text{bullet}} = \frac{6.63 \times 10^{-34}}{0.040 \times 1.0 \times 10^3} = 1.7 \times 10^{-35} \text{ m}$$

$$(b) \lambda_{\text{ball}} = \frac{6.63 \times 10^{-34}}{0.060 \times 1.0} = 1.1 \times 10^{-32} \text{ m}$$

$$(c) \lambda_{\text{particle}} = \frac{6.63 \times 10^{-34}}{1.0 \times 10^{-9} \times 2.2} = 3.0 \times 10^{-25} \text{ m}$$

36. Answer the following questions:

(a) Quarks inside protons and neutrons are thought to carry fractional charges $[(+2/3)e ; (-1/3)e]$. Why do they not show up in Millikan's oil-drop experiment?

(b) What is so special about the combination e/m ? Why do we not simply talk of e and m separately?

(c) Why should gases be insulators at ordinary pressures and start conducting at very low pressures?

(d) Every metal has a definite work function. Why do all photoelectrons not come out with the same energy if incident radiation is monochromatic? Why is there an energy distribution of photoelectrons?

(e) The energy and momentum of an electron are related to the frequency and wavelength of the associated matter wave by the relations:

$$E = hv, \quad p = h/\lambda$$

But while the value of λ is physically significant, the value of v (and therefore, the value of the phase speed $v\lambda$) has no physical significance. Why?

- Sol. (a) Quarks are thought to be confined within a proton or neutron by forces which grow stronger if one tries to pull them apart. It therefore seems that though fractional charges may exist in nature, observable charges are still integral multiples of electronic charge e .
- (b) Electric fields needed in the experiment with much bigger drops will be impractically high.
- (c) Stokes' formula is valid for motion through a homogenous continuous medium. The size of the drop should be much larger than the intermolecular separation in the medium for this assumption to be valid, otherwise the drop 'sees' inhomogeneties in the medium, i.e. there is concentrated mass density in the molecules, and voids in between the molecules.
- (d) Work function merely indicates the minimum energy required for the electrons in the highest level of the conduction band to get out of the metal. Not all electrons in the metal belong to this level. They occupy a continuous band of levels. Consequently for the same incident radiation, electrons knocked off from different levels come out with different energies.
- (e) The de Broglie wavelength $\lambda = h/p$ of the matter wave of an electron has a fixed value and λ has physical significance. But the absolute value of energy E of any particle is arbitrary to within an additive constant. Consequently, absolute value of frequency ν of a matter wave of an electron has no direct physical meaning. Likewise the phase speed $v_p = v\lambda$ is not physically significant. However, the group speed of the matter wave is physically significant and equals the speed of the particle as proved below.

$$\text{As } v_p = v\lambda = \omega/k$$

$$\text{Therefore group speed} = \frac{d\omega}{dk} = \frac{d\nu}{d(\frac{1}{\lambda})} = \frac{d(h\nu)}{d(\frac{h}{\lambda})} = \frac{dE}{dp} = \frac{d}{dp} \left(\frac{p^2}{2m} \right) = \frac{p}{m} = \text{particle speed.}$$

37. Describe Davisson and Germer experiment to establish that wave nature of electrons. Draw a labeled diagram of the apparatus used

Sol. **Davisson and Germer Experiment:** In 1927 Davisson and Germer performed a diffraction experiment with electron beam in analogy with X-ray diffraction to observe the wave nature of matter.

Apparatus: It consists of three parts

- (i) **Electron Gun:** It gives a fine beam of electrons, de Broglie used electron beam of energy 54eV. de Broglie wavelength associated with this beam

$$\lambda = 2mEKh$$

Here m = mass of electron = 9.1×10^{-31} kg

$E-K$ = Kinetic energy of electron = 54 eV

$$= 54 \times 1.6 \times 10^{-19} \text{ joule} = 86.4 \times 10^{-19} \text{ joule}$$

$$\therefore \lambda = 2 \times 9.1 \times 10^{-31} \times 86.4 \times 10^{-19} \times 1.6 \times 10^{-24}$$

$$= 1.66 \times 10^{-10} \text{ m} = 1.66 \text{ \AA}$$

(i) Nickel Crystal: The electron beam was directed on nickel crystal against its (111) face.

The smallest separation between nickel atoms is 0.914 \AA . Nickel crystal behaves as diffraction grating.

(ii) Electron Detector: It measures the intensity of electron beam diffracted from nickel crystal. It may be an ionisation chamber fitted with a sensitive galvanometer. The energy of electron beam, the angle of incidence of beam on nickel crystal and the position of detector can all be varied.

Method: The crystal is rotated in small steps to change the angle (α) between incidence and scattered directions and the corresponding intensity (I) of scattered beam is measured. The variation of the intensity (I) of the scattered electrons with the angle of scattering α is obtained for different accelerating voltages.

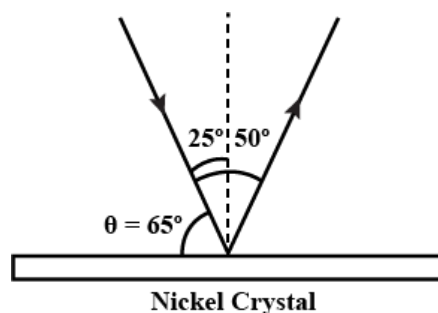
The experiment was performed by varying the accelerating voltage from 44V to 68V. It was noticed that a strong peak appeared in the intensity (I) of the scattered electron for an accelerating voltage of 54V at a scattering angle $\alpha = 50^\circ$

\therefore From Bragg's law

$$2d \sin \theta = n\lambda$$

Here $n=1$, $d=0.914 \text{ \AA}$, $\theta=65^\circ$

The measured wavelength is in close agreement with the estimated de Broglie wavelength. Thus the wave nature of electron is verified. Later on G.P. Thomson demonstrated the wave nature of fast electrons. Due to their work Davission and G.P. Thomson were awarded Nobel prize in 1937. Later on experiments showed that not only electrons but all material particles in motion (e.g., neutrons, α particles, protons etc.) show wave nature.



38. Draw a schematic diagram of the experimental arrangement used by Davisson and Germer to establish the wave nature of electrons. Express the de Broglie wavelength associated with electron in terms of the accelerating voltage V .
An electron and proton have the same kinetic energy. Which of the two will have larger wavelength and why?

D. CASE STUDY

1. **Photocell:** A photocell is a technological application of the photoelectric effect. It is a device whose electrical properties are affected by light. It is also sometimes called an electric eye. A photocell consists of semi cylindrical photo sensitive metal plate C (emitter) and a wire loop A (collector) supported in an evacuated glass or quartz bulb. It is connected to the external circuit having a high tension battery B and micro ammeter (μA).

Sometimes instead of the plate C, a thin layer of photosensitive material is pasted on the inside of the bulb. A part of the bulb is left clean for the light to enter it. When light of suitable wavelength falls on emitter C, photoelectrons are emitted. These photoelectrons are drawn to the collector A. Photocurrent of the order of a few microampere can be normally obtained from a photo cell. A photocell converts a change in intensity of illumination into a change in photocurrent. This current can be used to operate control systems and in light measuring devices.

- (i) Photocell is an application of
 (a) thermoelectric effect (b) photoelectric effect
 (c) photo resistive effect (d) none of these

Sol. (b)
 Photocell is a technological application of the photoelectric effect.

- (ii) Photosensitive material should be connected to
 (a) –ve terminal of the battery (b) +ve terminal of the battery
 (c) any one of (a) or (b) (d) connected to ground

Sol. (a)
 Photosensitive material used as emitter should be connected to –ve terminal of the battery so that the emitted electrons are repelled by emitter and collected by collector.

- (iii) Which of the following statement is true?
 (a) The photocell is totally painted black (b) A part of the photocell is left clean
 (c) The photo cell is completely transparent (d) A part of the photocell is made black.

Sol. (b)
 A part of the bulb is left clean for the light to enter in it.

- (iv) The photocurrent generated s in the order of
 (a) ampere (b) milliampere (c) microampere (d) none of these

Sol. (c)
 Photocurrent of the order of a few microampere can be mormalluy obtained from a photocell.

- (v) A photocell converts a change in ____ of incident light into a change in ____.
 (a) Intensity, photo-voltage (b) Wavelength, photo-voltage
 (c) Frequency, photo-current (d) Intensity, photo-current

Sol. (d)
 A photocell converts a change in the intensity of illumination into a change in photocurrent.

2. A photon is the smallest discrete amount or quantum of electromagnetic radiation. It is the basic unit of all light.

According to Einstein, photons have energy equal to their frequency times Planck's constant. The intensity of the light corresponds to the number of photons.

The basic properties of photons are:

- (i) They have zero mass and rest energy. They only exists as moving particles.
- (ii) They are elementary particles despite lacking rest mass.
- (iii) They have no electric charge.
- (iv) They are stable.
- (v) They carry energy and momentum which are dependent on the frequency.
- (vi) They can have interactions with other particles such as electrons, such as the Compton effect.

(vii) They can be destroyed or created by many natural processes, for instance when radiation is absorbed or emitted.

(viii) In free space, they travel at the speed of light.

(i) Photons have energy equal to their frequency times

- (a) Rydberg's constant (b) Planck's constant
(c) Avogadro's constant (d) Boltzmann constant

Sol. (b)

Photons have energy equal to their frequency times Planck's constant, $E = h\nu$.

(ii) The intensity of the light correspond to

- (a) Number of photons (b) Speed of photons
(c) Energy of photons (d) Frequency of photons

Sol. (a)

(iii) Charge of a photon is

- (a) $-e$ (b) $+e$ (c) 0 (d) none of these

Sol. (c)

(iv) Which of the following statement is wrong?

- (a) Photons only exist as moving particles (b) Photons carry energy and momentum
(c) Mass of a photon is equal to mass of electron
(d) Photons travel at the speed of light.

Sol. (c)

Photons have zero mass.

(v) Which of the following statements is wrong?

- (a) Photons can neither be destroyed nor created.
(b) Photons can have interactions with other particles.
(c) Photons are elementary particles (d) Photon is the basic unit of light.

Sol. (a)

Photons can be destroyed or created by many natural processes, for instance when radiation is absorbed or emitted.

E. ASSERTION-REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false (d) If both assertion and reason are false

(e) If assertion is false but reason is true.

1. Assertion: Cathode rays get deflected by electric or magnetic fields but light rays remain unaffected by these fields.

Reason: Light rays are electromagnetic waves and not charged particles.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.

Cathode rays contain electrons, hence they are deflected by both electric and magnetic fields. On the other hand, light waves are electromagnetic in nature. They do not consist of charged particles. Hence on passing through the fields, light rays remain unaffected.

2. Assertion: An electron is not deflected on passing through certain region of space. This observation confirms that there is no magnetic field in that region.

Reason: The deflection of electron depends on angle between velocity of electron and direction of magnetic field.

- Ans. (e) Assertion is false but reason is true.

If electron is moving parallel to the magnetic field, then the electron is not deflected i.e. if electron is not deflected we cannot be sure that there is no magnetic field in that region.

3. Assertion: Light is produced in gases in the process of electric discharge through them at high pressure.

Reason: At high pressure electrons does not take place at very low pressure in gases.

- Ans. (d) Both assertion and reason are false.

Light is produced in gases in the process of electric discharge at low pressure. When accelerated electrons collide with atoms of the gas, atoms get excited. The excited atoms return to their normal state and in this process light radiation are emitted.

4. Assertion: Electric discharge does not take place at very low pressure in gases.

Reason: At low pressure the gas starts converting itself into the liquid state.

- Ans. (c) Assertion is true but reason is false

When the battery of gas becomes very low i.e. 10^{-4} mm of mercury or below, then the number of atoms in the gas is so much reduced that sufficient positive ions are not available. As a result of this, electrons lost are not emitted from the cathode i.e. discharge current reduces to zero. In order to convert gases to liquids very high pressure and low temperature (below critical temperature for each gas) is necessary.

5. Assertion: An electric field is preferred in comparison to magnetic field for deflecting the electron beam in a television picture tube.

Reason: Electric field require low voltage.

- Ans. (d) Both assertion and reason are false.

If electric field is used for detecting the electron beam, then very high voltage will have to be applied or very long tube will have to be taken.

F. CHALLENGING PROBLEMS

1. Light of intensity 10^{-5} W m^{-2} falls on a sodium photo-cell of surface area 2 cm^2 . Assuming that the top 5 layers of sodium absorb the incident energy, estimate time required for photoelectric emission in the wave-picture of radiation. The work function for the metal is given to be about 2 eV. What is the implication of your answer?

Sol. Suppose sodium has n_e conduction electron available per atom.

Effective atomic area = 10^{-20} m^2

Therefore number of conduction electrons in 5 layers = $\frac{5 \times \text{Area of 1 layer}}{\text{Effective atomic area}}$

$$= \frac{5 \times 2 \times 10^{-4} \text{m}^2}{10^{-20} \text{m}^2} = 10^{17}$$

$$\text{Now intensity} = \frac{\text{Energy}}{\text{Area} \times \text{Time}} = \frac{\text{Power}}{\text{Area}}$$

$$\begin{aligned} \text{Therefore incident power} &= \text{incident intensity} \times \text{area} = 10^{-5} \text{ Wm}^{-2} \times 2 \times 10^{-4} \text{ m}^2 \\ &= 2 \times 10^{-9} \text{ W} \end{aligned}$$

In terms of wave picture, incident power is uniformly absorbed by all the electrons continuously.

$$\text{Therefore energy absorbed per second (or power) per electron} = \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26} \text{ W}$$

$$\text{Time required for photoelectron emission} = \frac{\text{Energy required per electron}}{\text{Energy absorbed per second per electron}}$$

$$= \frac{2 \text{ eV}}{2 \times 10^{-26} \text{ W}} = \frac{2 \times 1.6 \times 10^{-19} \text{ J}}{2 \times 10^{-26} \text{ Js}^{-1}} = 1.6 \times 10^7 \text{ s} = 0.5 \text{ year [Given } W_0 = 2 \text{ eV]}$$

Implication: Experimentally, photoelectric emission is observed nearly instantaneously ($=10^{-9}$ s). Thus the wave picture is in gross disagreement with experiment.

2. (a) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons. The *specific charge* of the electron, i.e., its e/m is given to be $1.76 \times 10^{11} \text{ C kg}^{-1}$.

(b) Use the same formula you employ in (a) to obtain electron speed for an collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified?

- Sol. (a) Here $V = 500 \text{ V}$, $e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$, $v = ?$

$$E_k = \frac{1}{2} mv^2 = eV$$

$$\text{Therefore } v = \sqrt{\frac{2eV}{m}} = \sqrt{2 \times 1.7 \times 10^{11} \times 500} = 1.33 \text{ ms}^{-1}$$

$$(b) \text{ Here } V = 10 \text{ MV} = 10^7 \text{ V}$$

$$\text{Therefore } v = \sqrt{2 \times 1.7 \times 10^{11} \times 10^7} = 1.88 \times 10^9 \text{ ms}^{-1}.$$

This speed is not possible because no particle can have a speed greater than the speed of light ($3 \times 10^8 \text{ ms}^{-1}$). At speeds comparable to the speed of light, we need to use the relativistic formula.

$$E_k = mc^2 - m_0c^2 = (m - m_0)c^2$$

$$\text{Or } eV = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2 \text{ Or } \frac{eV}{m_0c^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \text{ Or } \frac{eV}{m_0c^2} + 1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\text{Substituting the various value in LHS we get } \frac{1.6 \times 10^{-19} \times 10^7}{9.1 \times 10^{-31} \times (3 \times 10^8)^2} + 1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\text{Or } 19.536 + 1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\text{Or } 1 - \frac{v^2}{c^2} = \frac{1}{(20.536)^2} = 0.00237$$

$$\text{Or } \frac{v^2}{c^2} = 1 - 0.00237 = 0.99763 \text{ or } v = \sqrt{0.99763}c = 0.999c$$

SPACE FOR ROUGH WORK

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SPACE FOR NOTES

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