CLASS – 12

WORKSHEET- ATOMS

A. EARLY MODELS AND RUTHERFORD'S MODEL OF ATOM

(1Mark Questions)

1. According to Rutherford's atomic model, the electrons inside an atom are
(a) stationary (b) centralized (c) non stationary (d) none of these

Ans. (c)

2. Choose the correct alternative from the clues given at the end of each statement:
(a) The size of the atom in Thomson's model is ………. the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

Ans. no different from

(b) In the ground state of ………. electrons are in stable equilibrium, while in ………. electrons always experience a net force. (Thomson's model/ Rutherford's model.)

Ans. Thomson's model, Rutherford's model

(c)A classical atom based on ………. is doomed to collapse. (Thomson's model/ Rutherford's model.)

Ans. Rutherford's model

(d) An atom has a nearly continuous mass distribution in a ……….but has a highly non-uniform mass distribution in ……….(Thomson's model/ Rutherford's model.)

Ans. Thomson's model, Rutherford's model

(e)The positively charged part of the atom possesses most of the mass in ………. (Rutherford's model/both the models.)

Ans. both the models

(2 Marks Questions)

3. Estimate the radius of a Gold nucleus when α-particle of energy 10MeV is scattered by it through 180°. Given: (i)Z_{gold} = 79, (ii) Z_α = 2 and (iii) $1/4\pi\varepsilon_0 = 9\times10^9$ nm^2 C^{-2}

Sol. Atomic number of gold = 79

Kinetic energy of α-particle = KE = 10MeV

At closest approach, all the kinetic energy of the alpha particle has converted into potential energy i.e. KE = PE

Potential energy at closest approach, PE = $\frac{kZe(2e)}{r}$

But $\frac{2kZe^2}{r} = 10\text{meV}$

$\therefore \frac{2(9\times10^9)\times79\times(1.6\times10^{-19})^2}{r} = 10\times1.6\times10^{-13} \Rightarrow r \approx 9.10\times10^{-14}$ m.

4. A Helium nucleus of energy 10 MeV collides on with a ${}_{29}Cu^{64}$ nucleus and retraces its path. Calculate the radius of the Cu nucleus.

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5. Explain how Rutherford's experiment on scattering of α-particles led to the estimation of the size of the nucleus.

Sol. The electrons and nucleus are held together by the electrostatic force of attraction. This is how the alpha ray scattering experiment of Rutherford led to the estimation of the site of the nucleus.

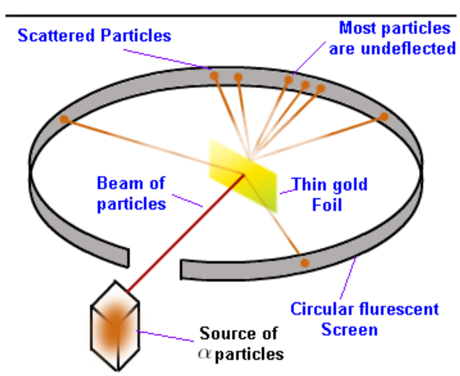
6. Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

Sol. In the alpha-particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough. This is because the mass of hydrogen (1.67×10^{-27}) is less than the mass of incident α−particles (6.64×10^{-27}).

(5 Marks Questions)

7. Draw a labeled diagram for α-particle scattering experiment. Give Rutherford's observations and discuss the significance of this experiment. Obtain the expression which helps us to get an idea of the size of a nucleus, using these observations

Sol. Alpha particles experiment led Rutherford the discovery of atomic nucleus.



Observations: (i) Most of the alpha particles pass straight through the gold foil. It means that they do not suffer any collision with gold atoms. (ii) Only about 0.14% of incident alpha particles are scattered by more than 1°. (iii) About 1 alpha particle in every 8000 alpha particles deflect by more than 90°

Significance: His experiment led him to conclude that, the entire positive charge of the atom must be concentrated in a tiny central core of the atom. The tiny massive central core of the atom was named as atomic nucleus.

The first experimental determination of a size of a nucleus was made from the results of Rutherford scattering of α particles. Distance of closest approach was found to be read into 3×10^{-14} m for 7.7 MeV energetic α particles. This fact indicated that the size of the nucleus should be less than 3×10^{-14} m. For α particles having a kinetic energy of more than 7.7 MeV, the distance of the closest approach will be smaller.

At K.E more than 5.5 MeV distance of closest approach will be smaller. At K.E more than 5.5 MeV, attractive nuclear forces start affecting the Coulomb's repulsive force between α particles and gold nucleus. The size of the nucleus can be measured by using fast electrons instead of α particles for the scattering experiment. The nuclear size was found to vary linearly with the mass number (A). Since the nucleus is supposed to be spherical, having radius R.

$$R = R_0 A^{1/3} \text{ where } R_0 = 1.2 \times 10^{-15} \text{ m}$$

B. BOHR MODEL

(1Mark Questions)

1. The ionization potential of the hydrogen atom is 13.6eV. What will be its energy in n = 2 state?

Sol. Energy of the electron in the first orbit of the atom, E_1 = - 13.6 eV. Its energy in the second orbit, $E_2 = 13.6\text{eV}/(2)^2 = -3.4$eV.

2. Taking the Bohr radius as a_0 = 53pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about

(a) 53 pm (b) 27 pm (c) 18 pm (d) 13 pm

Ans. (c)

According to Bohr's model of atom reading of an atiom in ground state is $r = r_0/z$ where r_0 is Bohr radius and z is a atomic number. Given r_0 = 53 pm.

The atomic number of lithium is 3, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about 1/3 times to that of Bohr radius.

So, the radius of lithium ion is r_0/z = 53/3 = 18 pm

3. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(a) of the electrons not being subject to a central force.

(b) of the electrons colliding with each other

(c) of screening effects

(d) the force between the nucleus and an electron will no longer be given by Coulomb's law.

Sol. (a)

The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. So the nuclear the electrons not being subject to a central force.

4. Two H atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is
(a) 10.20 eV (b) 20.40 eV (c) 13.6 eV (d) 27.2 eV

Sol. (a)
We know that. Electron on the lowest sate of atom called the ground state have the lowest energy and the electron revolving in the orbit of smallest radius the Bohr radius, r. The energy of this state (n = 1) E_1 is – 13.6eV. Total energy of two H atoms in the ground state collide in elastically = 2 × (-13.6eV) = -27.2eV.
The maximum amount by which their combined kinetic energy is reduced when any one H atom goes into first excited state after the inelastic collision. So that the total energy of the two H atoms after the inelastic collision = $(13.6/2^2)$ + (132.6) = 17.0eV. [Since for excited state (n = 2)].
So maximum loss of their kinetic energy due to inelastic collision
= 27.2 – 17.0 = 10.2 eV

5. The mass of a H-atom is less than the sum of the masses of a proton and electron. Why is this?

Sol. According to mass energy equivalence established by Einstein, If B represents binding energy of hydrogen atom (= 13-6), the equivalent mass of this energy = B/c^2. Hence, mass of a H-atom It is less than sum of the masses of a proton and an electron.

(2 Marks Questions)

6. Calculate the speed of electron revolving around the nucleus of hydrogen atom in order that it may not be pulled into the nucleus by electrostatic attraction.

Sol. The speed of electron in 'stable' orbits of hydrogen-like atoms in given by

$$v = \frac{Ze^2}{2h\varepsilon_0}\frac{1}{n},\ n = 1, 2, 3, \ldots$$

For hydrogen atom (Z = 1) and in the first orbit (n = 1), the electron speed is

$$v = \frac{e^2}{2h\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{h}$$

Substituting the given values :

$$v = (9.0 \times 10^9\ \mathrm{Nm^2C^{-2}}) \times \frac{2\times 3.14\times(1.6\times10^{-19}\ C)^2}{6.6\times10^{-34}\ Js} \quad = 2.19 \times 10^6\ \mathrm{m\,s^{-1}}.$$

7. Wavelength of first line Balmer series in hydrogen spectrum is 6563Å. Calculate the wavelength of second line in this series.

Sol. $\frac{1}{\lambda_1} = R_H\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$ and $\frac{1}{\lambda_2} = R_H\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$

$\frac{\lambda_2}{\lambda_1} = \frac{5}{36} \times \frac{16}{30} = \frac{20}{27}$

$\lambda_2 = \frac{20}{27} \times 6563 \times 10^{-10} = 4861 \times 10^{-10}$m = 4861 Å

8. Calculate the radius of the smallest orbit of a H-atom

Sol. The radius of the smallest orbit of the electron(aO) in hydrogen atom is 0.053 nm.
The radius of the nth orbit of a hydrogen atom is given by

$$r = \frac{n^2h^2}{4\pi^2 mKe^2}$$

Radius of innermost orbit, called Bohr's radius, is obtained by putting n = 1. It is denoted by r_0 =

$$r_0 = \frac{h^2}{4\pi^2 mKe^2} = \frac{(6.6\times10^{-34})^2}{4\times(3.14)^2\times9.1\times10^{-31}\times9\times10^9\times(1.6\times10^{-19})^2}$$

r_0 = 0.53 x 10-10 m = 0.53nm.

9. The electron in a hydrogen atom having energy – 0.85eV makes a transition to a state with energy – 3.4eV. Calculate the wavelength of the emitted photon.

Sol. From the energy data, we see that the 'H' atom transits from binding energy of 0.85 e V to excitation energy of 102 e V.
= Binding Energy of - 3.4 e V
So, n = 4 to n =2
We know that $1/\lambda = 1.097 \times 10^7 \times 10^7 \left(\frac{1}{4} - \frac{1}{16}\right)$ or $\lambda = \frac{16}{1.097\times3\times10^7} = 4.8617\times10^{-7}$
= 487nm.

10. Calculate the frequency of the photon radiated by a hydrogen atom when it de-excites from the first excited state to the ground state.

Sol. Given system de excites from its excited state to ground state
So, n_1 = 1 and n_2 = 2
$1/\lambda = \left(\frac{1.097\times10^7}{2}\right)\left(1 - \frac{1}{4}\right) \Rightarrow \lambda = 2.43\times10^{-7}$m = 2430Å

11. State Bohr's postulates for explaining the spectrum of hydrogen atom.

Sol. It states that 'Whenever an electron jumps from one of its specified non-radiating orbit to another such orbit, it emits or absorbs a photon whose energy is equal to the energy difference between the initial and final states'.

12. The value of ground state energy of hydrogen atom is – 13.6eV. (i) What does the negative sign signify? (ii) How much energy is required to take an electron in this atom from the ground state to the first excited state?

Sol. (i) Negative sign shows that electron in ground state is bound in H-atom due to attractive force between electron and nucleus.

(ii) Energy of electron in H-atom in nth orbit is

$$E_n = -\frac{Rhc}{n^2} = -\frac{13\cdot 6}{n^2}$$

For first excited state n= 2

$$E_2 = -\frac{13\cdot 6}{4}\text{ eV} = -3\cdot 4\text{ eV}$$

Energy required to take electron from ground state to first excited state

$$\Delta E = E_2 - E_1$$
$$= -13\cdot 6\text{ eV} - (-3\cdot 4\text{ eV})$$
$$= \mathbf{10\cdot 2\ eV}$$

13. What is the shortest wavelength present in the Paschen series of spectral lines?

Sol. For shortest wavelength of Paschen series, $n_1 = 3$, $n_2 = \infty$

Therefore, $\frac{1}{\lambda_s} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = \frac{R}{9}$

Or $\lambda_s = \frac{R}{9} = \frac{9}{1.097\times10^7} = 8.2041\times10^{-7}\text{m} = 8204.1$ Å

14. A 12.5eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

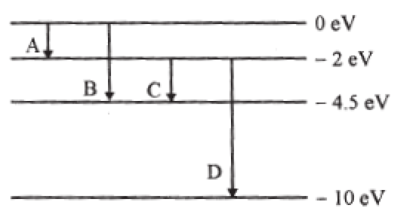
Sol. In the above case if 12.5eV electron beam is used to bombard gaseous hydrogen at room temperature, the Lyman series will be emitted.

(3 Marks Questions)

15. A muonic hydrogen atom in bound state of a negatively charged muon (denoted by μ) of mass 207m, and a proton, and the muon orbits around the proton. Obtain (i) radius of its first Bohr orbit and (ii) its ground state energy.

Sol. Same as 30.

16. The energy levels of an atom are as shown below. Which one of the transitions will result in the emission of a photon of wavelength 275nm?



Sol. The energy E of a photon of wavelength 275nm is given by $E = \frac{hc}{\lambda}$

$$= \frac{6.626\times10^{-34}\times3\times10^8}{275\times10^{-9}}\text{J} = \frac{6.626\times10^{-34}\times3\times10^8}{275\times10^{-9}\times1.6\times10^{-19}}\text{eV} = 4.5\text{eV}$$

This energy corresponds to the transition B for which the energy change from 0 – (- 4.5) = 4.5eV.

17. The ground state energy of hydrogen atom is – 13.6eV. (i) What is the kinetic energy of an electron in the 3^{rd} excited state? (ii) If the electron jumps to the ground state from the 3^{rd} excited state, calculate the wavelength of the photon emitted.

Sol. (i) 2^{nd} excited state means 3^{rd} normal state of n = 3state

total energy of nthlevel is $E_n=E_0/n^2$,

given that E0=−13.6eV

So total energy for 3rd state will be $E_3 = -13.6/9 = -1.51$eV

We know that total energy= - kinetic energy so kinetic energy

= - (-1.51eV) = 1.51eV

(ii) Energy of the transition will be the difference in energy levels so

E=−1.51−(−13.6)=12.09eV

Wavelength $\lambda = \frac{1237.5}{E(ev)}\,\mathbf{nm} = \frac{1237.5}{12.09}\,\mathbf{nm}$ = 102.3 nm.

18. The ground state energy of hydrogen atom is – 13.6eV. If an electron makes a transition from an energy level – 0.85eV to – 3.4eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?

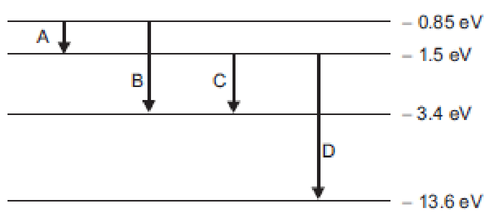
Sol. When energy is – 0.85eV then n = 3, when energy is – 3.4ev then n = 2

Therefore $1/\lambda = R\left(\frac{1}{2^2-3^2}\right) = 1.1\times 10^7 \times \frac{5}{36}$

$\lambda = \frac{36}{5\times 1.1}\times 10^{-7} = 6.545\times 10^{-7}$ = 6545Å

Thus it belongs to Balmer series.

19. The energy level diagram of an element is given below. Identify by doing necessary calculations which transition corresponds to the emission of a spectral line of wavelength 102.7 nm.



Sol.

$$\Delta E = \frac{hc}{\lambda} = \frac{6\cdot6\times10^{-34}\times3\times10^{8}}{102\cdot7\times10^{-9}}\text{ J}$$

$$= \frac{6\cdot6\times10^{-34}\times3\times10^{8}}{102\cdot7\times10^{-9}\times1\cdot6\times10^{-19}}\text{ eV} = \frac{66\times3000}{1027\times16} = 12\cdot04\text{ eV}$$

Now, $\Delta E = |-13\cdot6-(-1\cdot50)| = 12\cdot1$ eV

Hence, transition shown by arrow D corresponds to emission of l =102×7nm.

20. A difference of 2.3eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Sol. Here E = 2.3eV = $2.3\times1.6\times10^{-19}$J

As E = hv

So, frequency, v = $\frac{E}{h} = \frac{2.3\times1.6\times10^{-19}}{6.6\times10^{-34}}$ = 5.6×10^{14}Hz

21. The ground state energy of hydrogen atom is –13.6 eV. What are the kinetic and potential energies of the electron in this state?

Sol. Total energy, E = - 13.6eV, KE = - E = - (- 13.6) = 13.6eV,

PE = - 2KE = - 2 × 13.6 = - 27.2eV

22. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon.

Sol. Energy of an electron in the nth orbit of H atom, $E_n = -\frac{13.6}{n^2}$ eV

Energy in the ground state (n – 1) level, $E_1 = -\frac{13.6}{1^2}$ = - 13.6eV

Energy in fourth level (n 4) level, $E_4 = -\frac{13.6}{4^2}$ = - 0.85eV

$\Delta E = E_4 - E_1$ = - 0.85 – (- 13.6) = 12.75eV = $12.75 \times 1.6 \times 10^{-19}$J

As hv = ΔV

Therefore frequency, v = $\frac{\Delta E}{h} = \frac{12.75\times1.6\times10^{-19}}{6.6\times10^{-34}}$ = 3.078×10^{15}Hz

Wavelength, $\lambda = \frac{c}{v} = \frac{3\times10^8}{3.078\times10^{15}}$ = 0.9744×10^{-7}m = 974.4 Å

23. (a) Using the Bohr’s model, calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

Sol. (a) Speed of the electron in Bohr’s nth orbit is $v_n = \frac{2\pi ke^2}{nh} = \frac{v_1}{n}$

Speed of electron in Bohr’s first (n = 1) orbit is $v_1 = \frac{2\pi ke^2}{h}$

$= \frac{2\times3.14\times9\times10^9\times(1.6\times10^{-19})^2}{6.63\times10^{-34}}$= 2.186×10^6ms^{-1}

$v_2 = \frac{v_1}{2} = 1.093 \times 10^6$ms^{-1}

$v_3 = \frac{v_1}{3} = 0.729\times10^6$ ms^{-1}

(b) Orbital period of electron in Bohr’s first orbit is $T_1 = \frac{2\pi r_1}{v_1}$

$= \frac{2\times3.14\times0.53\times10^{-10}}{2.186\times10^{6}}\mathbf{s}$ = 1.52×10^{-16}s
As $T_n = n^3T_1$
Therefore $T_2 = (2)^3 \times 1.52 \times 10^{-16} = 12.16 \times 10^{-16} = 1.22 \times 10^{-15}$s
$T_3 = (3)^3 \times 1.52 \times 10^{-16} = 41.04 \times 10^{-16} = 4.10\times 10^{-15}$s

24. The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the n = 2 and n =3 orbits?

Sol. Here $r_1 = 5.3 \times 10^{-11}$m
As $r_n = n^2r_1$
Therefore $r_2 = 2^2 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}$m
$R_3 = 3^2 \times 5.3 \times 10^{-11} = 4.77\times 10^{-10}$m

25. In accordance with the Bohr's model, find the quantum number that characterizes the earth's revolution around the sun in an orbit of diameter 3×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)

Sol. According to Bohr's quantization condition of angular momentum
Angular momentum of the earth around the sun, mvr = $\frac{nh}{2\pi}$
Therefore n = $\frac{2\pi mvr}{h} = \frac{2\times3.14\times6.0\times10^{24}\times1.5\times10^{11}\times3\times10^{4}}{6.6\times10^{-34}}$ = 2.57×10^{74}.

26. The gravitational attraction amongst proton and electron in a hydrogen atom is weaker than the coulomb attraction by a component of around 10^{-40}. Another option method for taking a gander at this case is to assess the span of the first Bohr circle of a hydrogen particle if the electron and proton were bound by gravitational attraction. You will discover the appropriate response fascinating.

Sol. The radius of the first orbit in Bohr's model is given by $v_0 = \frac{h^2}{4\pi^2 m_e k e^2}$
If instead of electrostatic attraction between electron and proton, we consider the atom bound by gravitational force $\frac{Gm_pm_e}{r^2}$, then the term ke^2 should b replaced by Gm_pm_e. The radius of the first Bohr orbit in a gravitationally bound hydrogen atom will be
$$r_0^G = \frac{h^2}{4\pi^2 G m_p m_e^2} = \frac{(6.6\times10^{-34})^2}{4\times9.87\times6.67\times10^{-11}\times1.625\times10^{-27}\times(9.1\times10^{-31})^2} = 1.194\times10^{29}\text{m}$$
$\approx 1.2 \times 10^{29}$ m
This radius is much greater than the estimated size of the whole universe.

27. Obtain an expression for the frequency of a radiation emitted when a hydrogen atom de-excites from level n to level (n – 1). For a large n, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Sol. From Bohr's theory, the frequency v of the radiation emitted when an electron de excites from level n_2 to level n_1 is given by
$$v = \frac{2\pi^2mk^2Z^2e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$
Given $n_1 = n - 1$, $n_2 = n$
Therefore $v = \frac{2\pi^2mk^2Z^2e^4}{h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$

$= \frac{2\pi^2mk^2Z^2e^4}{h^3}\left[\frac{n^2-(n-1)^2}{(n-1)^2n^2}\right] = \frac{2\pi^2mk^2Z^2e^4}{h^3}\frac{(2n-1)}{(n-1)^2n^2}$

For large n, 2n – 1 – 2n and n – 1 – n and for hydrogen Z = 1

Therefore v = $\frac{2\pi^2mk^2e^4}{h^3} \times \frac{2n}{n^2.n^2} = \frac{4\pi^2mk^2e^4}{n^3h^3}$ …(i)

Now in Bohr's model,

Velocity of electron in nth orbit = $\frac{nh}{2\pi mr}$ and radius of nth orbit = $\frac{n^2h^2}{4\pi^2mke^2}$

Thus orbital frequency of electron in nth orbit is

$v_C = \frac{v}{2\pi r} = \frac{1}{2\pi r} \times \frac{nh}{2\pi mr} = \frac{nh}{4\pi^2 m} \times \left(\frac{4\pi^2mke^2}{n^2h^2}\right)^2 = \frac{4\pi^2mk^2e^4}{n^3h^3}$

which is same as obtained in equation (i).

Hence for large value of n, the classical frequency of revolution of electron in nth orbit is same as that obtained from Bohr's theory.

28. The total energy of an electron in the first excited state of the hydrogen atom is about -3.4 eV
(a) What is the kinetic energy of the electron in this state?
(b) What is the potential energy of the electron in this state?
(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Sol KE of an electron in nth orbit, T = $\frac{1}{2}\frac{kZe^2}{r^2}$

PE of an electron in nth orbit, V = $\frac{kZe^2}{r} = \mathbf{2T}$

Total energy, E = T +V = T – 2T = - T

Therefore kinetic energy, T = - E = - (- 3.4) = 3.4eV

Potential energy, V = - 2T = - 2 × 3.4 = - 6.8eV

(c) If the zero of the potential energy is chosen differently, kinetic energy does not change. Potential energy and hence total energy will be affected.

29. If Bohr's quantization postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion as well. Why then do we never speak of quantization of orbits of planets around the sun?

Sol. Angular momenta associated with planetary motion are incomparably large relative to $h/2\pi$. For example, angular momentum of the earth in its orbital motion is of the order to 10^{70} $h/2\pi$. In terms of the Bohr's quantization postulate, thus corresponds to a very large value of h (of the order of 10^{70}). For such a large value of $h/2\pi$ the differences in the successive energies and angular momenta of the quantized levels of the Bohr model are so small compared to the energies and angular momenta respectively of the levels that on ecan, for all practical purposes, consider the levels continuous.

30. Obtain the first Bohr's radius and the ground state energy of a muonic hydrogen atom [i.e., an atom in which a negatively charged muon ($\mu-$) of mass about 207me orbits around a proton].

Sol. In Bohr's model, the radius of nth orbit is r = $\frac{n^2h^2}{4\pi^2mkZe^2}$ i.e r $\propto \frac{1}{m}$

Now in a muonic hydrogen atom, a negatively charged muon (μ^-) of mass 207 m_e revolves around a proton.
Therefore we can write $\frac{r_\mu}{r_e} = \frac{m_e}{m_\mu} = \frac{m_e}{207m_e}$
Therefore $r_\mu = \frac{1}{207} \times r_e = \frac{1}{207} \times 0.53 \times 10^{-10}m = 2.5\times10^{-13}m$
Energy of electron in nth orbit, $E = \frac{2\pi^2mk^2Z^2e^4}{n^2h^2}$
When all other factors are
Therefore $\frac{E_\mu}{E_e} = \frac{m_\mu}{m_e} = \frac{207m_e}{m_e}$
Or $E_\mu = 207\ E_e = -\ 207 \times 13.6eV = -\ 2.8$ keV

31. Derive Rydberg's formula to derive the wavelength of spectral lines of hydrogen from n_1 orbit to n_2 orbit.

Sol. The Rydberg formula is the mathematical formula to determine the wavelength of light emitted by an electron moving between the energy levels of an atom. When an electron transfers from one atomic orbital to another, it's energy changes. When an electron shift from an orbital with high energy to a lower energy state, a photon of light is generated. A photon of light gets absorbed by the atom when the electron moves from low energy to a higher energy state. The Rydberg Formula applicable to the spectra of the different elements and is it is expressed as

$$\bar{v} = \frac{1}{\lambda} = R(\frac{1}{n_1^2} - \frac{1}{n^{2_2}}$$

Where, n and n_2 are integers and n_2 is always greater than n_1.
R is constant, called Rydberg constant and formula is usually written as

$$\bar{v} = R_H(\frac{1}{n_1^2} - \frac{1}{n_2^2})$$

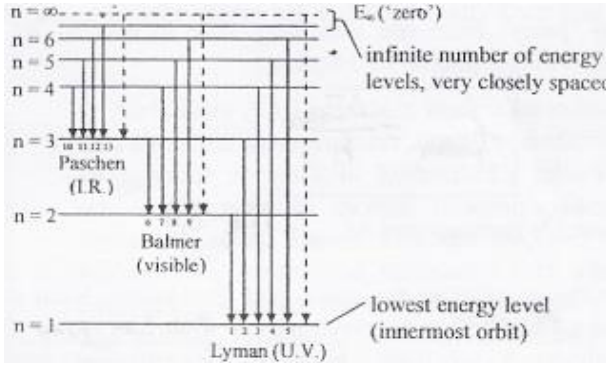
The modern value of Rydberg constant is known as 109677.57 cm^{-1} and it is the most accurate physical constant.

$$\bar{v} = \frac{1}{\lambda} = 109680(\frac{1}{n_1^2} - \frac{1}{n_2^2})cm^{-1}(n_2 > n_1)$$

(5 Marks Questions)

32. (a) Draw the energy diagram showing the ground state, and the next few excited states for hydrogen (H) atom. Mark the transition, which corresponds of the emission of spectral lines for the Balmer series. (b) Calculate the wavelength of the first spectral line in this series.

Sol. (a)



(b) We know that Balmer series terminates at n = 2. Hence the first spectral line of Balmer series corresponds to transition from $n_1 = 3$ to $n_2 = 2$. Hence we have

$$\frac{1}{\lambda_{H_1}} = R \times 1^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\lambda_{H_1} = \frac{36}{5R}$$

$$\text{or} \quad 6561 = \frac{36}{5R}$$

The second line of Balmer series of ionzied helium atom corresponds to transition from $n_1 = 4$ to $n_2 = 2$

$$\text{Hence } \frac{1}{\lambda_{He}(2)} = R \times 2^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$= 4R\left(\frac{1}{4} - \frac{1}{6}\right) = 4R\left(\frac{3}{16}\right) = \frac{3R}{4}$$

$$\lambda_{He_{(2)}} = \frac{4}{3R}$$

$$\frac{\lambda_{He_{(2)}}}{\lambda_{H_1}} = \frac{4}{3R} \times \frac{5R}{36} = \frac{5}{27}$$

$$\lambda_{He_{(2)}} = \frac{6561 \times 5}{27} = 1215\,\text{Å}$$

33. State Bohr's postulates for atomic model. Derive the expression for total energy of an electron bound to hydrogen atom.

Sol. Postulates of Bohr's Model of an Atom: In an atom, **electrons (negatively charged) revolve around the positively charged nucleus in a definite circular path called orbits or shells**. Each orbit or shell has a fixed energy and these circular orbits are known as orbital shells.

According to Rutherford's model we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_o}\frac{ze^2}{r^2}$$

$$\Rightarrow mv^2 = \frac{1}{4\pi\epsilon_o}\frac{ze^2}{r}$$

Therefore,

Total energy = P.E + K.E

$$T.E. = -\frac{1}{4\pi\epsilon_0}\frac{ze^2}{r} + \frac{1}{2}mv^2$$

$$= -\frac{1}{2}.\frac{1}{4\pi\epsilon_0}\frac{ze^2}{r}$$

$$= -.\frac{1}{8\pi\epsilon_0}\frac{ze^2}{r}$$

Energy is negative implies that the electron nucleus is a bound or attractive system.

34. The electron in a given Bohr orbit has a total energy of – 1.5eV. Calculate its (i) kinetic energy (ii) potential energy and (iii) wavelength of light emitted, when the electron makes a transition to the ground state (Ground state energy = - 13.6eV).

Sol. Given E = 1.5 ev

(i) Kinetic energy of electron

K.E. = -E K.E. = -5(-1.5 ev)

K. E. = 1.5 ev

(ii) Potential energy of electron

P.E. = -2 K.E. P.E. = -2 x 1.5

P.E. = -3.0 ev

(iii) Energy of photon

$\Delta E = E_2 - E_1$

$\Delta E = -1.5 - (-13.6)$

$\Delta E = 12.1$ ev

$\Delta E = 12.1 \times 1.6 \times 10\text{-}19$ J

$\Delta E = 19.36 \times 10\text{-}19$ J

Energy of photon, $\Delta E = hc/\lambda = 19.36 \times 10^{-19}$

Wavelength of radiation, $\lambda = \frac{6.6\times10^{-34}\times3\times10^{8}}{19.36\times10^{-19}}$

Or $\lambda = 1.022 \times 10^{-7}$m

35. Derive the expression for the radius of the ground state orbit of hydrogen atom, using Bohr's postulates. Calculate the frequency of the photon, which can excite the electron to – 3.4eV from – 13.6eV

Sol. For H atom, Z = 1

E = hv

⇒ |-13.6| - |-3.4| = 4.14×10^{-15} × v

v = $\frac{10.2}{4.14\times10^{-15}}$ = 2.46 ×10^{15}Hz

36. Using Bohr postulate derive the velocity of electron in nth orbit of hydrogen atom.

Sol. According to Bohr's postulates,

$$L_n = mv_nr_n = \frac{nh}{2\pi}$$

$L_n \rightarrow$ Angular momentum

$v_n \rightarrow$ Speed of moving electron in the orbit

$r_n \rightarrow$ radius of n^{th} orbit

$h \rightarrow$ Planck's constant

$m \rightarrow$ mass of particle

For a dynamically stable orbit in a hydrogen atom,

$$F_e = F_c$$

$$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n^2}$$

$$mv_n^2r_n = ke^2$$

By Bhors postulate

$$mvr = \frac{nh}{2\pi}$$

Therefore

$$\frac{nh}{2\pi}v_n = ke^2$$

$$v_n = \frac{2\pi ke^2}{nh}$$

$$v_n = \frac{2\pi e^2}{4\pi\epsilon_0 nh}$$

$$v_n = \frac{e^2}{2\epsilon_0 nh}$$

37. In the Auger process an atom makes a transition to a lower state without emitting a photon. The excess energy is transferred to an outer electron which may be ejected by the atom. (This is called an Auger electron). Assuming the nucleus to be massive, calculate the kinetic energy of an n = 4 Auger electron emitted by Chromium by absorbing the energy from a n = 2 to n = 1 transition.

Sol. As the nucleus is massive, recoil momentum of the atom may be neglected and the entire energy of the transition may be considered transferred to the Auger electron. As there is a single valence electron in Cr, the energy states may be thought of as given by the Bohr model.

The energy of the nth state $E_n = -Z^2R_1/n^2$ where R is the Rydberg constant and Z = 24. The energy released in a transition from 2 to 1 is

$$\Delta E = Z^2R\left(1-\frac{1}{4}\right)=\frac{3}{4}Z^2R.$$

The energy required to eject a n = 4 electron is $E_4 = Z^2R_1/16$.

Thus the kinetic energy of the Auger electron is

$$K.E = Z^2R\left(\frac{3}{4}-\frac{1}{16}\right) = \frac{1}{16}Z^2R$$

$$=\frac{11}{16}\times 24\times 24\times 13.6\,\text{eV}$$

$$= 5385.6 \text{ eV}$$

C. CASE STUDY

1. **The Photoelectric Effect:** It has been observed when electromagnetic radiation of short enough wavelength (or high enough frequency) falls on a metal, electrons are emitted from tis surface. This phenomenon is called photoelectric effect and the emitted electrons are called photoelectrons. Different substances emit photoelectrons only when exposed to different kind of radiations. For a given emitter illuminated by light of a given frequency, the photoelectric current is directly proportional to the intensity of the incident light. For every emitter there is a definite threshold frequency below which no photoelectrons are emitted, no matter what the intensity of light is. Above the threshold frequency, the maximum kinetic energy of the photo electrons is proportional to the frequency of the incident light. Einstein proposed a theory for photo electric effect. According to him, when photon of light falls on a metal, it is absorbed, resulting in the emission of a photoelectron. The maximum kinetic energy of the emitted electron is given by $K_{max} = hv - W_0$ where v is the frequency of incident radiation and W_0 is the work function of the metal. The constant h is called the Planck's constant and its value is $h = 6.63 \times 10^{-34}$ Js. Furthermore, $W_0 = hv_0$ where v_0 is the threshold frequency.

(i) A metal of work function is 3.3eV is illuminated by a light of wavelength 300nm. The threshold frequency of photoelectric emission is (take $h = 6.6\times10^{-34}$Js)

(a) 8.0×10^{14}Hz (b) 6.6×10^{14}Hz (c) 4.0×10^{14}Hz (d) 3.3×10^{14}Hz

Sol. (a)

Use $W_0 = hv_0$

(ii) In Qs (i) above, the maximum kinetic energy of photoelectrons is

(a) 1.32×10^{-19}J (b) 3.3×10^{-19}J (c) 6.6×10^{-19}J (d) 1.6×10^{-19}J

Sol. (a)

Use $K_{max} = h(v - v_0)$ where $v = \frac{c}{\lambda} = \frac{3\times10^8}{300\times10^{-9}} = 10 \times 10^{14}$Hz

(iii) In Qs (i) above the stopping potential is

(a) 2.475V (b) 1.65V (c) 0.825V (d) 3.3V

Sol. (c)

Use $K_{max} = eV_0$

(iv) The photoelectric emission from a metal begins at a frequency of 6×10^{14}Hz. The emitted photoelectrons are fully stopped by a retarding potential of 3.3V. The frequency of the incident radiation if (take $h = 6.6\times10^{-34}$Js)

(a) 1.0×10^{15}Hz (b) 1.2×10^{15}Hz (c) 1.4×10^{15}Hz (d) 1.6×10^{15}Hz

Sol. (c)

Use $eV_0 = h(v - v_0)$

(v) Light of wavelength 3000Å is incident on two metals A and B whose work functions are 4e and 2eV respectively. Then

(a) A will emit photoelectrons but B will not (b)B will emit photoelectrons but A will not

(c) Both A and B will emit photoelectrons (d) neither A nor B will emit photoelectrons

Sol. The energy of the incident photon is $E = hv = \frac{hc}{\lambda} = \frac{(6.6\times10^{-34})\times(3\times10^8)}{(3300\times10^{-10})} = 6\times10^{-19}$J = 3.75eV

2. **X rays**

Experimental investigations have revealed that there are two type of X rays – characteristic X rays and continuous X rays. Characteristic X rays are produced when high energy electrons strike the target in an X ray tube. The high energy electrons possess enough energy to penetrate into the atom and knock out one of the two electrons of the K shell. When an electron is missing from the K shell, an electron from the neighbouring L shell jumps into it, simultaneously emitting high frequency X rays. When this transition has taken place, an electron from the M shell jumps in to the vacancy of the L shell resulting in the emission of another X rays photon and so on.

Continuous X rays are produced by another process called Bremsstrrahlung. When an electron of energy eV (electron volts) travels towards the target, it is accelerated towards the nucleus by the Coulomb force Ze^2/r^3 where Z is the atomic number of the target atom and r is the distance at a particular time of the incident electron. Consequently the electron is accelerated, the acceleration a being given by $Ze^2/re^3 = ma$.

The electron is progressively accelerated towards the nucleus and eventually deflected to one side, the emitted X rays photon being deflected to the other side. The maximum frequency of the emitted photon is given by $eV = hv_{max}$

(i) The production of characteristic X rays is due to the

(a) consequence acceleration of incident electrons towards the nucleus
(b) continuous retardation of incident electrons towards the nucleus
(c) electron transitions between inner shells of the target atom
(d) electron transitions between outer shells of the target atom

Sol. (c)

(ii) Production of continuous X rays is due to the
(a) Acceleration of the incident electrons by the nucleus of the target atom
(b) Electron transitions between inner shells of the target atoms
(c) Electron transitions between outer shells of the target atom
(d) Annihilation of the mass of incident electrons

Sol. (a)

(iii) The study of the spectrum of characteristic X ray helps us to
(a) measure the energy of the incident electrons
(b) measure the wavelength of the incident electrons
(c) measure the energy of the emitted X rays
(d) identify the element of which the target is made

Sol. (d)

(iv) The maximum frequency limit of the continuous X rays spectrum depends upon
(a) the atomic number of the atoms of the target
(b) the kinetic energy of the incident electrons
(c) the maximum frequency limit of the characteristic X ray spectrum
(d) the degree of vacuum in the X ray tube

Sol. (b)

(v) The potential difference applied to an X ray tube is 5kV and the current through it is 3.2mA. Then the number of electrons striking the target per second is
(a) 2×10^{16} (b) 5×10^{16} (c) 1×10^{17} (d) 4×10^{15}

Sol (a)
$n = \frac{I}{e} = \frac{3.2\times10^{-3}}{1.6\times10^{-19}} = 2\times10^{16}$ per second which is choice (a).

D. ASSERTION REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true.

1. Assertion: Bohr postulated the electrons in stationary orbits around the nucleus do not radiate.
Reason: According to e classical Physics, al moving electrons radiate.

Ans. (c) Assertion is true but reason is false

Bohr postulated that electrons around the nucleus do not radiate. This is true. According to classical Physics, the moving electrons radiate only when they jump from a higher energy orbit to lower energy orbit. So the reason is false.

2. Assertion: In the a-particle scattering experiment, most of the α-particles pass undeviated.
   Reason: Most of the space in the atom is empty

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
Most of the α- particles pass roughly in a straight line (within 1°) without deviation. This shows that no force is acting on them. So, the assertion is true. Most of the space in the atom is empty. Only 0.14% of α-particles are scattered more than 1°. So, the reason is also true and explains the assertion.

3. Assertion: The total energy of an electron in a stationary state of ht hydrogen is –ve.
   Reason: The –ve sign indicates that energy is required to remove the electron from the hydrogen atom.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
The –ve sign of the total energy of an electron moving in an orbit means that the electron is bound with the nucleus. Energy wi be required to remove the electron from the hydrogen atom to a distance infinitely far away from the nucleus.

4. Assertion: The positively charged nucleus of an atom has a radius of almost 10^{-15} m.
   Reason: The α-particle scattering experiment, the distance of closest approach for α-particles is $\cong 10^{-15}$m.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
In α-particle scattering experiment, Rutherford found a small number of α-particles which were scattered back through an angle approaching to 180°. This is possible only if the positive charges are concentrated at the centre or nucleus of the atom.

5. Assertion: According to classical theory, the proposed path of an electron in Rutherford atom model will be parabolic.
   Reason: According to electromagnetic theory an accelerated particle continuously emits radiation.

Ans. (e) assertion is false but reason is true.
According to classical electromagnetic theory, an accelerated charge continuously emits radiation. As electrons revolving in circular path\s are constantly experiencing centripetal acceleration, hence they will be losing their energy continuously and the orbital radius will go on decreasing and form spiral and finally the electron will fall on the nucleus.

E. CHALLENGING PROBLEMS

1. Choose a suitable solution to the given statements which justify the difference between Thomson's model and Rutherford's model

(a) In the case of scattering of alpha particles by a gold foil, average angle of deflection of alpha particles stated by Rutherford's model is (less than, almost the same as, much greater than) stated by Thomson's model.
(b) Is the likelihood of reverse scattering (i.e., dispersing of α-particles at points more prominent than 90°) anticipated by Thomson's model (considerably less, about the same, or much more prominent) than that anticipated by Rutherford's model?
(c) For a small thickness T, keeping other factors constant, it has been found that amount of alpha particles scattered at direct angles is proportional to T. This linear dependence implies?
(d) To calculate average angle of scattering of alpha particles by thin gold foil, which model states its wrong to skip multiple scattering?

Sol. (a) almost the same as
In both the Rutherford's model and the Thompson's model, average angle was considered for experimentation. So on these grounds, it can be established that the normal point of diversion of alpha particles in Thomson's model is about the same as in the case of Rutherford's model

(b) much less
In Rutherford's model , back/reverse scattering is more likely to happen. Reverse-scattering refers to the scattering of alpha particles at points more than 90°. The chances of back scattering anticipated by Thomson's model is considerably less than that that of Rutherford's model.

(c) Single collisions give rise to the dispersing phenomenon. The probability of a single collision is directly proportional to the amount of target molecules. And since the number of target particles increase with an expansion in thickness, the impact likelihood depends directly on the thickness of the objective.

(d) Thomson's model
Multiple scattering in Thomson's model should not be disregarded in order to figure out the average scattering angle of alpha particles by a thin gold film. This can be established on the grounds that almost no deflection is caused by a solitary collision in this model. Subsequently by considering multiple scattering, the watched normal scattering edge can be clarified.

2. Classify an electron can be in nay orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To stimulate what he might well have done before his discovery, let us play as follows with the dimensions of length that is roughly equal to the known size of an atom ($\approx 10^{-10}$ m)
(a) Construct a quantity with the dimensions of length from the fundamental constants e, m_e and c. Determine the numerical value.
(b) You will find that the length obtained in (a) in many orders of magnitude smaller than the atomic dimensions, Further, it involves c. But energies of atoms are mostly in non-realistic domain where c is not expected to play any role. That is why may have suggested Bohr to discard c and look for something else to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere, Bohr's great insight lay in recognizing that h, m_e and e will yield the right atomic size. Construct the quantity

with the dimension of length from h, m_e and e and confirm that its numerical value has indeed the correct order of magnitude

Sol. According to Coulomb's law, the force between hydrogen nucleus and electron is given as:

$F= 1/4\pi\varepsilon_0.e^2/r^2$

$\Rightarrow r=1/4\pi\varepsilon_0/e^2e^2$

Now, using the fundamental constants e, me and c, we will obtain a quantity which has the dimensions of length.

We know that F.r (force x distance) is the amount work or energy and it is also given as =mc2

$\therefore r=1/4\pi\varepsilon 0.mc^2e^2=2.8\times10^{-15}m.$

It is comparatively smaller than the size of the atom.

<u>SPACE FOR ROUGH WORK</u>

SPACE FOR NOTES