

CLASS – 11

WORKSHEET- KINETIC THEORY

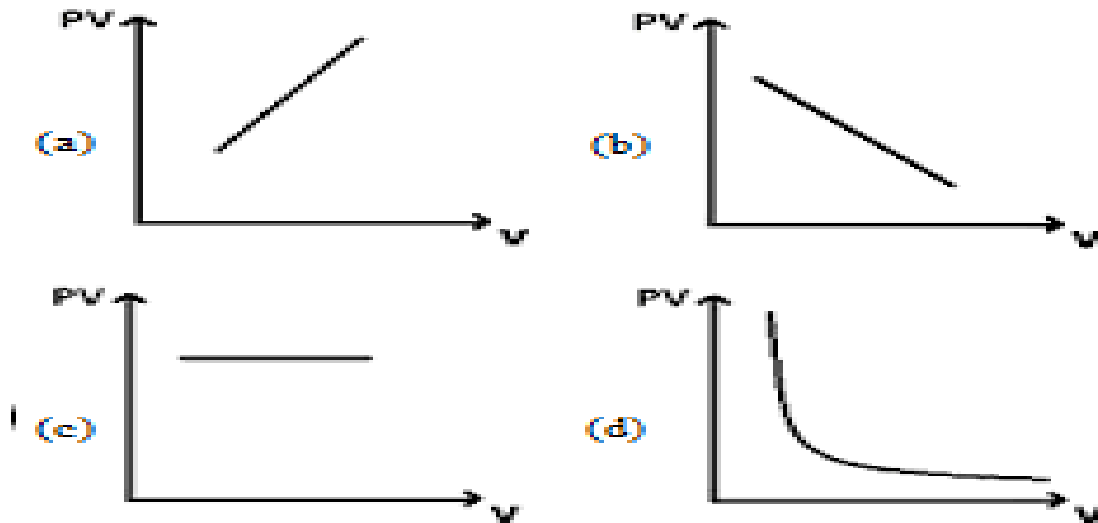
A. IDEAL GAS THEORY AND GASES LAW:

(1 Mark Questions)

1. The real gas behaves like an ideal gas if its  
(a) both pressure and temperature are high (b) both pressure and temperature are low  
(c) pressure is high and temperature is low (d) pressure is low and temperature is high

Ans. (d)

2. Which of the following graphs represent the behavior of an ideal gas?



Ans. (c)

3. Pressure of a gas at constant volume is proportional to  
(a) total internal energy of the gas (b) average kinetic energy of the molecules  
(c) average potential energy of the molecules (d) total energy of the gas

Sol. (a)

$$\text{Pressure, } p = \frac{1}{3} \frac{M}{V} v_{\text{rms}}^2 = \frac{2}{3} E$$

Here,  $E = \frac{1}{2} M v_{\text{rms}}^2$  is total internal energy of the gas.

4. The temperatures of 2 mole of an ideal monatomic gas is raised to 15K at constant volume. The work done by the gas is

(a) zero (b) 30J (c) 420J (d) 50J

Sol. (a)

The work done is zero in this process since volume is kept fixed.

5. Air pressure in a car tyre increases during driving. Explain.

Sol. A car tyre has a fixed volume (V). When it is driven, the temperature (T) of air in its tyre increases. So according to Charles' law,  $P \propto T$  at constant V, pressure of the air in the car tyre increases.

6. How is the volume of gas related to absolute zero temperature at constant pressure?

Sol. The volume of a gas is directly proportional to its absolute temperature. More specifically, for a fixed mass of gas at a constant pressure, the volume (V) is directly proportional to the absolute temperature (T). This is Charles' Law.

$$V \propto T$$

$$V = kT, \text{ where } k \text{ is a proportionality constant.}$$

$$V/T = k$$

Since k is a constant,

$$V_1/T_1 = V_2/T_2$$

7. Why is it not possible to increase the temperature of a gas while keeping its volume and pressure constant?

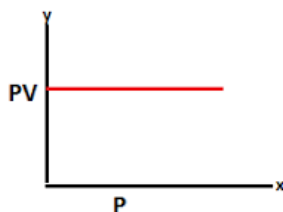
Sol. The temperature of a gas is raised while keeping its volume constant. The pressure exerted by the gas on the walls of the container increases because its molecules lose more kinetic energy to the wall.

8. Is Boyle's law perfectly obeyed at all temperatures and pressures?

Sol. No Boyle's law is perfectly obeyed at high temperature and low pressure of a gas.

9. Plot a graph between (PV) and V for a given mass of a gas at fixed temperature.

Sol.



10. What is Avogadro's number? What is its value?

Sol. The number of molecules in one gram mole of a substance is called Avogadro's number ( $N_A$ ).  $N_A = 6.023 \times 10^{23}$  per gram mole.

11. A vessel consists of three types of gas molecules A, B and C with mass  $m_A > m_B > m_C$ . Arrange the three types of gases in decreasing order of average KE.

Sol. The average K.E will be the same as conditions of temperature and pressure are the same

12. A container has equal number of molecules of hydrogen and carbon dioxide, If a fine hole is made in the container, then which of the two gases shall leak out rapidly?

Sol. Graham's law of diffusion rate of leakage is inversely proportional to square root of density of gas. density of hydrogen is smaller than density of carbon dioxide. so, hydrogen leak more rapidly than oxygen.

13. At what temperature is average speed of oxygen gas molecule is equal to rms velocity of the same gas at 27°C?

Sol. Average velocity =  $\sqrt{\frac{8RT}{\pi M}}$

Root mean square velocity =  $\sqrt{\frac{3RT}{M}} = \sqrt{\frac{3R \times 300K}{M}}$

For equal values,  $\sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{3R \times 300}{M}}$  or  $\frac{8RT}{\pi M} = \frac{3R \times 300}{M}$  or  $\frac{8T}{\pi} = 900$

or  $T = 353.57 \text{ K} = (353.57 - 273)^\circ\text{C} = 80.57^\circ\text{C}$

14. Derive Boyle's law using kinetic theory of gases.

Sol. The pressure exerted by a gas according to kinetic theory of gases is,

$$P = \frac{1}{3} \rho v^2$$

$\rho$  is the density and  $v$  is the root mean square velocity of the gas molecules

$$\rho = M/V,$$

$$P = M/3V \cdot v^2$$

$$PV = \frac{1}{3} Mv^2 = \frac{2}{3} E_k$$

Where  $M$  is mass,  $V$  is volume and  $E_k$  is the kinetic energy of the gas.

At a constant temperature, the velocity will be constant and hence the kinetic energy will be constant.

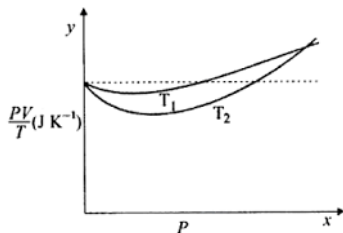
$$PV = PV = \text{constant.}$$

### (2 Marks Questions)

15. What is an ideal gas? Does such gas really exist?

Sol. A gas which obeys the ideal gas equation,  $PV = nRT$  at all temperature and pressure is called an ideal gas. No such gas does not really exists. An ideal gas is a simple theoretical model of a gas.

16. Figure shows plot of  $PV/T$  versus  $P$  for  $1.00 \times 10^{-3} \text{ kg}$  of oxygen gas at two different temperatures.



(a) Which is true  $T_1 > T_2$  or  $T_1 < T_2$ ?

(b) What is the value of  $PV/T$  where the curves meet on the y-axis?

Sol. (a) Curve at temperature  $T_1$  is more close to the dotted plot than the curve at temperature  $T_2$ . Since the behavior of a real gas approaches the perfect gas behavior, as the temperature is increased therefore  $T_1 > T_2$ .

(b) If the amount of gas under consideration is 1 mole, then the value of  $PV/T$  where the curves meet  $PV/T$  axis will be  $R (=8.31 \text{ J mole}^{-1} \text{ K}^{-1})$ . For oxygen, molecular mass is 320g i.e.  $32.0 \times 10^{-3} \text{ kg}$ . Since mass of oxygen gas under considerations is  $1.0 \times 10^{-3} \text{ kg}$ , the value of  $PV/T$ , where the curves meet the  $PV/T$  axis is given by

$$\frac{PV}{T} = \frac{8.31}{32.0 \times 10^{-3}} \times 1.0 \times 10^{-3} = 0.26 \text{ JK}^{-1}.$$

17. Why gases at high pressure and low temperature show large deviation from ideal gas behavior?

Sol. When temperature is low and pressure is high the intermolecular forces become appreciable. Thus, the volume occupied by the molecule is not negligibly small as compared to volume of gas.

18. Calculate the volume occupied by 3.2g of oxygen at 76cm of mercury at  $27^\circ\text{C}$ .

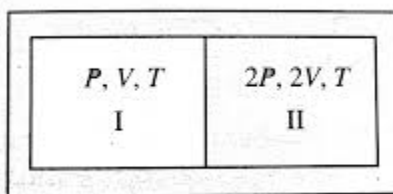
Sol.  $P = 76 \times 13.6 \times 980 \text{ dyne/cm}^2$ ,  $T = 300\text{K}$ ,  $R = 8.31 \times 10^7 \text{ erg mole}^{-1} \text{ K}^{-1}$

$$\text{Therefore } V = \frac{RT}{P} = \frac{8.31 \times 10^7 \times 300}{76 \times 13.6 \times 980} \text{ cm}^3 \text{ mol}^{-1} = 24610 \text{ cm}^3 \text{ mol}^{-1}$$

Since 1 mole of oxygen corresponds to  $24610/32 \text{ cm}^3$

$$\text{Therefore 3.2g of oxygen corresponds to } \frac{24610 \times 3.2}{32} = 2461 \text{ cm}^3$$

19. A partition divides a container having insulated walls into two compartments I and II. The same gas fills the two compartments. What is the ratio of the number of molecules in compartments I and II?



Sol. Let the respective number of mole of gases in compartment I and II be  $n$  and  $n'$ .

$$n = \frac{PV}{k_B T}; n' = \frac{2P_2V}{k_B T} \text{ or } \frac{n}{n'} = \frac{1}{4}$$

20. State and prove Avogadro's law.

Sol. Avogadro's law: A statement that under the same conditions of temperature and pressure, equal volumes of different gases contain an equal number of molecules. This empirical relation can be derived from the kinetic theory of gases under the assumption of a perfect (ideal) gas.

Consider two gases 1 and 2. We can write,  $P_1 V_1 = \frac{2}{3} N_1 \bar{K}_1$ ;  $P_2 V_2 = \frac{2}{3} N_2 \bar{K}_2$ .

If their pressure, volume and temperatures are the same then  $P_1 = P_2$ ,  $V_1 = V_2$ ,  $\bar{K}_1 = \bar{K}_2$ .

Clearly  $N_1 = N_2$ . Thus equal volumes of all ideal gases existing under the same conditions of temperature and pressure contain equal number of molecules which is Avogadro's law.

21. A balloon has 5g of helium at 7°C. Calculate: (a) the number of atoms of helium in the balloon (b) the total internal energy of the system.

Sol. (a) Here  $n = 5\text{gm mole}$ ,  $T = 7^\circ\text{C} = 280\text{K}$ ,

Therefore no. of atoms of He,  $N = nN_A = 5 \times 6.023 \times 10^{23} = 30.115 \times 10^{23}$

(b) Average kinetic energy per mole =  $\frac{3}{2} k_B T$

Total internal energy =  $\left(\frac{3}{2} k_B T\right) N = \frac{3}{2} \times (1.38 \times 10^{-23}) \times 280 \times 30.115 \times 10^{23} = 1.74 \times 10^4 \text{J}$ .

22. State the law of equipartition of energy.

Sol. According to the law of equipartition of energy, for any dynamic system in thermal equilibrium, the total energy for the system is equally divided among the degree of freedom.

### (3 Marks Questions)

23. Explain how does the behavior of real gas differ from that of an ideal gas.

Sol. Since no real gas fulfils the characteristics of an ideal gas. Hence the behavior fo a real gas differs from that of an ideal gas.

24. Two monatomic gases A and B occupying the same volume V are at same temperature T and pressure P. If they are mixed, the resultant mixture has volume V and temperature T. Calculate the pressure of the mixture.

Sol. As number of moles is conserved.

Therefore  $n_A + n_B = n_{\text{mix}}$  where  $n_A$ ,  $n_B$  and  $n_{\text{mix}}$  represent the number of moles of gas A and B and their mixture respectively. Using the ideal gas equation, we get

$$\frac{P_A V_A}{RT_A} + \frac{P_B V_B}{RT_B} = \frac{P_{\text{mix}} V_{\text{mix}}}{RT_{\text{mix}}}$$

$$\frac{PV}{RT} + \frac{PV}{RT} = \frac{P_{\text{mix}}V}{RT} \Rightarrow P_{\text{mix}} = 2P \text{ [Since } P_A = P_B = P \text{ or } V_A = V_B = V_{\text{mix}} = V]$$

25. An air bubble of volume  $1.0\text{cm}^3$  rises from the bottom of the lake 40m deep at a temperature  $12^\circ\text{C}$ . To what volume does it grow when it reaches the surface which is at a temperature of  $35^\circ\text{C}$ ?

Sol. Given  $V_1 = 10^{-6}\text{m}^3$ ,  $T_1 = 285\text{K}$ ,  $T_2 = 308\text{K}$

Pressure of bubble  $P_1 = \text{water pressure} + \text{atmospheric pressure}$

$$= \rho gh + P_{\text{atm}} = 4.93 \times 10^5 \text{Pa}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \text{ or } V_1 = \frac{4.93 \times 10^5 \times 1 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} = 5.3 \times 10^{-6} \text{m}^3$$

26. We have 0.5g of hydrogen gas in a cubic chamber of size 3cm kept at NTP. The gas in the chamber is compressed keeping the temperature constant till a final pressure of 100atm. Is one justified in assuming the ideal gas law, in the final state (Hydrogen molecules can be considered as spheres of radius  $1\text{\AA}$ ).

Sol. Her  $V_g = (3 \times 10^{-2})^3 = 27 \times 10^{-6} \text{m}^3$ ;  $P_i = 1\text{atm}$ ,  $P_f = 100\text{atm}$ ,  $T = \text{constant}$ ,  $V_f = ?$ , mass of hydrogen,  $m = 0.5\text{g}$

$$V_f = \frac{P_i V_i}{P_f} = \frac{1 \times 27 \times 10^{-6}}{100} = 2.7 \times 10^{-7} \text{m}^3$$

Number of molecules in 0.5g of  $\text{H}_2$ ,  $N = \frac{N_A}{M} m$ . Her  $N_A = 6.022 \times 10^{23} \text{mol}^{-1}$ ,  $M = 2\text{g mol}^{-1}$

$$\text{Therefore } N = \frac{6.022 \times 10^{23}}{2} \times 0.5 = 1.5 \times 10^{23}$$

Radius of hydrogen molecule =  $1 \text{\AA} = 10^{-10}\text{m}$ ; Volume of a hydrogen molecule =  $(10^{-10})^3 = 10^{-30}\text{m}^3$ ; Molecular volume =  $(1.5 \times 10^{23}) \times 10^{-30} = 1.5 \times 10^{-7} \text{m}^3$ .

We get  $V_f$  is of same order as the molecular volume. Hence intermolecular forces cannot be ignored as the molecules lie very close to each other i.e. gas cannot be treated as an ideal gas. Therefore one is not justified in assuming the ideal gas law.

27. A vessel is filled with gas at a pressure of 76cm of mercury at a certain temperature. The mass of the gas is increased by 50% by introducing more gas in the vessel at the same temperature. Find out the resultant pressure of the gas.

Sol. According to kinetic theory of gases,  $PV = \frac{1}{3} m v_{\text{rms}}^2$

At constant temperature  $v_{\text{rms}}^2$  is constant. As  $v$  is also constant,  $P \propto m$ .

When the mass of the gas increase by 50% pressure also increase by 50% at constant volume.

$$\text{Therefore final pressure} = 76 + \frac{50}{100} \times 76 = 114 \text{cm of Hg.}$$

28. A vessel A contains hydrogen and another vessel B whose volume is twice of A contains same mass of oxygen at the same temperature. Compare (a) average kinetic energies of

hydrogen and oxygen molecules and (b) pressures of gases in A and B. Molecular weights of hydrogen and oxygen are 2 and 32 respectively.

Sol. (a) For all gases at the same temperature, average kinetic energy per molecule is same and is  $E = \frac{3}{2} k_B T$

As the gases in both the vessels are at the same temperature, so the ratio of their average kinetic energy per molecule = 1:1.

(b) According to kinetic theory,  $P = \frac{1}{3} \frac{m}{V} v_{rms}^2$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\text{Therefore } \frac{v_{H_2}}{v_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4:1$$

Where M is the mass and V is the volume of the gas. Since masses of both gases are equal. So the ratio of their pressure is

$$\frac{P_{H_2}}{P_{O_2}} = \left[ \frac{v_{H_2}}{v_{O_2}} \right]^2 \times \frac{V_{O_2}}{V_{H_2}} = \frac{16}{1} \times \frac{2}{2} = 32:1$$

29. There are N molecules of a gas in a container. If the number of molecules is increased to 2N then what will be (a) pressure of the gas (b) rms speed of the gas?

Sol.  $P = \frac{1}{3} mn\overline{v^2} = \frac{1}{3} \frac{mN}{V} \overline{v^2} \quad \left[ n = \frac{N}{V} \right]$

(a) As  $P \propto N$ , so the pressure of the gas is doubled when the number of molecules is increased from N to 2N

(b) The rms speed remains same because it does not depend on the number of molecules.

30. A box contains equal number of molecules of hydrogen and oxygen. If there is a fine hole in the box, which gas will leak rapidly? Why?

Sol. According to Graham's law of diffusion, rate of diffusion  $V_{rms} = 1/\sqrt{M}$ .

Hence hydrogen gas will leak more rapidly because its mass is smaller than oxygen.

31. An insulated container containing monatomic gas of molar mass m is moving with a velocity  $v_0$ . If the container is suddenly stopped, find the change in temperature.

Sol. Let container contains n number of moles of gas, at temperature  $T_1$  moving with velocity  $v_0$ .

$$\text{Initial internal energy, } U_1 = n \times \frac{f}{2} RT_1 = \frac{3}{2} nRT_1$$

Where f = degree of freedom = 3, for monatomic gas. Final internal energy,  $U_f = \frac{3}{2} nRT_2$

$$\Delta U = \Delta KE$$

$$\text{Or } \frac{3}{2} nR\Delta T = \frac{1}{2} mv_0^2 \times n \Rightarrow \Delta T = \frac{mv_0^2}{3R}$$

32. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15atm and a temperature of 27°C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17°C. Estimate the mass of oxygen taken out of the cylinder.  $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ , molecular weight of oxygen = 32.

Sol. Initial volume,  $V_1 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$ ; Initial pressure,  $P_1 = 15 \text{ atm} = 15 \times 1.013 \times 10^5 \text{ Nm}^{-2}$ ; Initial temperature,  $T_1 = 27 + 273 = 300 \text{ K}$

$$\text{Initial number of moles, } n_1 = \frac{P_1 V_1}{RT_1} = \frac{15 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 300} = 18.3$$

Final pressure,  $P_2 = 11 \text{ atm} = 11 \times 1.013 \times 10^5 \text{ Nm}^{-2}$ ; Final volume,  $V_2 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$ ; Final temperature,  $T_2 = 17 + 273 = 290 \text{ K}$

$$\text{Final number of moles, } n_2 = \frac{P_2 V_2}{RT_2} = \frac{11 \times 1.013 \times 10^5 \times 30 \times 10^{-3}}{8.31 \times 290} = 13.8$$

Number of moles of oxygen taken out =  $18.3 - 13.9 = 4.4$

Mass of gas taken out of cylinder =  $4.4 \times 32 \text{ g} = 140.8 \text{ g} = 0.141 \text{ kg}$ .

33. An air bubble of volume  $1.0 \text{ cm}^3$  rises from the bottom of a lake 40m deep at a temperature of 12°C. To what volume does it grow, when it reaches the surface, which is at a temperature of 35°C? Given  $1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$

Sol. Temperature at 40m depth,  $T_1 = 12 + 273 = 285 \text{ K}$ ; Volume of the air bubble at 40m depth,  $V_1 = 1.0 \text{ cm}^3 = 1.0 \times 10^{-6} \text{ m}^3$ ; Pressure at 40m depth,  $P_1 = 1 \text{ atm} + h\rho g = 1.01 \times 10^5 + 40 \times 10^3 \times 9.8 = 4.93 \times 10^5 \text{ Pa}$ ; Temperature at the surface of water,  $T_2 = 35 + 273 = 308 \text{ K}$ ; Pressure at the surface of the lake,  $P_2 = 1 \text{ atm} = 1.01 \times 10^5 \text{ Pa}$

Let  $V_2$  be the volume of air bubble at the surfaces of the lake.

$$\text{As } \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\text{Therefore } V_2 = \frac{P_1 V_1 T_2}{T_1 P_2} = \frac{4.93 \times 10^5 \times 1.0 \times 10^{-6} \times 308}{285 \times 1.01 \times 10^5} = 5.275 \times 10^{-6} \text{ m}^3$$

34. Estimate the total number of molecules inclusive of oxygen, nitrogen, water vapour and other constituents in a room of capacity  $25.0 \text{ m}^3$  at a temperature of 27°C and 1 atmospheric pressure. [Given  $k_B = 1.38 \times 10^{-23} \text{ JK}^{-1}$ ]

Sol. As Boltzmann's constant,  $k_B = \frac{R}{N} \therefore R = k_B N$

Now  $PV = nRT = nk_B NT$

$\therefore$  The number of molecules in the room

$$= nN = \frac{PV}{Tk_B} = \frac{1.013 \times 10^5 \times 25.0}{300 \times 1.38 \times 10^{-23}} = 6.117 \times 10^{26}$$

35. Estimate the average energy of a helium atom at (i) room temperature (27°C) (ii) the temperature on the surface of the sun (6000K) and (iii) the temperature of  $10^7 \text{ K}$ . Given  $k_B = 1.38 \times 10^{-23} \text{ Jmol}^{-1} \text{ K}^{-1}$

Sol. Here Given  $k_B = 1.38 \times 10^{-23} \text{ Jmol}^{-1} \text{ K}^{-1}$



Average kinetic energy per molecule is given by  $\bar{E} = \frac{3}{2}k_B T$

(i)  $T = 27+273 = 300\text{K}$

Therefore  $\bar{E} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.21 \times 10^{-21}\text{J}$

(ii)  $T = 6000\text{K}$

$\bar{E} = \frac{3}{2} \times 1.38 \times 10^{-23} \times 6000 = 1.242 \times 10^{-19}\text{J}$

(iii)  $T = 10^7\text{K}$

$\bar{E} = \frac{3}{2}k_B T = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7 = 2.07 \times 10^{-16}\text{J}$

36. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). (i) Do the vessels contain equal number of molecules? (ii) Is the root mean squared speed of molecules same in the three cases? If not, in which case  $v_{\text{rms}}$  is the largest?

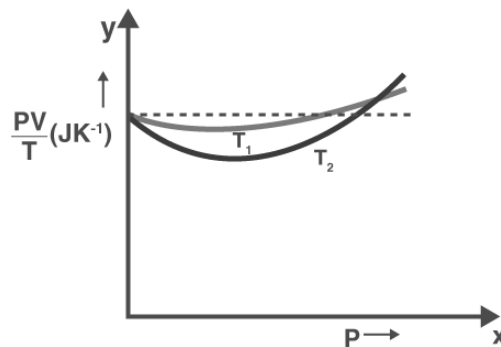
Sol. (i) Yes, the vessels contain equal number of molecules. This is in accordance with Avogadro's hypothesis that equal volumes of all gases under similar conditions of temperature and pressure contain equal number of molecules.

(ii)  $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$  i.e.  $v_{\text{rms}} \propto \frac{1}{\sqrt{M}}$

As the molecule masses of the three gases are different, so the rms speeds of the molecules will be different in the three cases. Moreover, as the value of  $M$  is smallest for neon, so  $v_{\text{rms}}$  is largest for neon.

**(5 Marks Questions)**

37. Figure shows plot of  $PV/T$  versus  $P$  for  $1.00 \times 10^{-3}$  kg of oxygen gas at two different temperatures.



- (a) What is the dotted plot signify?  
 (b) Which is true:  $T_1 > T_2$  or  $T_1 < T_2$ ?  
 (c) What is the value of  $PV/T$  where the curves meet on the y-axis?  
 (d) If we obtained similar plots for  $1.00 \times 10^{-3}$  kg of hydrogen, would we get the same value of  $PV/T$  at the point where the curves meet on the y-axis? If not, what mass of

hydrogen yields the same output of PV/T (for low pressure high temperature region of the plot)? [Molecular mass of H<sub>2</sub> = 2.02u, of O<sub>2</sub> = 32.0u, R = 8.31 J mol<sup>-1</sup> K<sup>-1</sup>]

Sol. (a) As the dotted line is parallel to P axis, it shows that PV/T remains same when the pressure is increased. It thus indicates the ideal behavior of the gas.

(b) The curve at temperature T<sub>1</sub> is more close to the dotted line than the curve at temperature T<sub>2</sub>. Since the behavior of the ideal gas behavior at high temperature so, T<sub>1</sub> > T<sub>2</sub>.

(c) Number of moles in 1.00×10<sup>-3</sup>kg or 1g of oxygen,

$$n = \frac{\text{Mass of oxygen in grams}}{\text{Molecular mass}} = \frac{1}{32} \text{ mole}$$

$$\text{As } PV = nRT, \text{ therefore } \frac{PV}{T} = nR = \frac{1}{32} \times 8.31 = 0.26 \text{ JK}^{-1}$$

(d) No, this is because PV/T depends upon the volume of the gas and volumes of equal masses of different gases are different.

If we wish to have the curve of hydrogen gas to start from the same point, the PV/T of hydrogen should be equal to PV/T of oxygen.

$$\text{From part (c), } \frac{PV}{T} = nR = 0.26$$

$$\text{Therefore } n = \frac{0.26}{R} = \frac{0.26}{8.31} = \frac{1}{32} \text{ mole}$$

$$\text{Mass of 1 mole of H}_2 = 2.02\text{g}$$

$$\text{Therefore mass of } 1/32 \text{ moles of H}_2 = 2.02 \times \frac{1}{32} = 0.063\text{g} = 6.3 \times 10^{-5} \text{ kg.}$$

## B. MICROSCOPIC VIEW OF GASES

1. Which one of the following is not an assumption of kinetic theory of gases?

- (a) The volume occupied by the molecules of the gas is negligible.
- (b) The force of attraction between the molecules is negligible
- (c) The collision between the molecules are negligible
- (d) All molecules have same speed

Ans. (d)

2. If three molecules have velocities 0.5 km s<sup>-1</sup>, 1 km s<sup>-1</sup> and 2 km s<sup>-1</sup>, the ratio of the rms speed and average speed is

- (a) 2.15
- (b) 1.14
- (c) 0.53
- (d) 3.96

Ans. (b)

$$V_{\text{rms}} = \sqrt{\{(V_1^2 + V_2^2 + V_3^2)/3\}} = \sqrt{[(0.5)^2 + (1)^2 + (2)^2]/3} = \sqrt{\{5.25/3\}}$$

$$V_{\text{rms}} = 1.32 \text{ km/s}$$

$$\text{Also } V_{\text{average}} = \{(V_1 + V_2 + V_3)/3\} = \{(0.5 + 1 + 2)/3\} = (3.5/3) = 1.16$$

$$\text{Hence ratio of rms speed \& average speed is } (1.32/1.16) = 1.137 = 1.14.$$

3. In the kinetic theory of gases, why do we not take into account the changes in gravitational potential energy of the molecule?

Sol. If the molecule converted its kinetic energy into potential energy by moving from the bottom to the top of the container, the molecule's kinetic energy, and hence its speed, would not noticeably change at all. This is why we could ignore the effects of gravity.

4. In terms of kinetic theory of gases, explain why the pressure of a gas in a container increases when a gas is heated?

Sol. The volume of the gas therefore becomes larger as the temperature of the gas increases. As the number of gas particles increases, the frequency of collisions with the walls of the container must increase. This, in turn, leads to an increase in the pressure of the gas.

5. What would be the effect on the rms velocity of gas molecule if the temperature of the gas is increased by a factor of 4?

Sol. The rms velocity is directly proportional to the square root of temperature and inversely proportional to the square root of molar mass. Thus quadrupling the temperature of a given gas doubles the rms velocity of the molecules.

6. In a monatomic gas, total degree of freedom are due to

(a) translational motion

(b) rotational motion

(c) vibrational motion

(d) oscillatory motion

Sol. (a)

7. The average translational kinetic energy of  $O_2$  at a particular temperature is  $0.768\text{eV}$ . The average translational kinetic energy of  $N_2$  molecules in eV at the same temperature is

(a) 0.0015

(b) 0.0030

(c) 0.048

(d) 0.768

Ans. (c)

8. Name two factors on which the degree of freedom of gas depends.

Sol. (i) Atomicity of gas molecules (ii) shape of the molecule (iii) temperature of the gas.

9. Define degree of freedom.

Sol. Each of a number of independently variable factors affecting the range of states in which a system may exist, in particular any of the directions in which independent motion can occur.

10. The ratio of molar heat capacities of a diatomic gas at constant pressure to that at constant volume is

(a)  $7/5$

(b)  $3/2$

(c)  $3/2$

(d)  $5/2$

Sol. (a)

For a diatomic gas,

Molar heat capacity at constant pressure,  $C_p=7/2R$

Molar heat capacity at constant volume,  $C_v=5/2R$

$$\therefore C_p/C_v = 7/2R / 5/2R = 7/5$$

11. The average degree of freedom per molecule for a gas is 6. The gas performs 25J of work when it expands at constant pressure. What is the heat absorbed by the gas?

Sol.  $\Delta u = f/2 RT = 3RT$

$$\Delta w = nR\Delta T = 25.5$$

$$\Delta Q = \Delta V + \Delta W$$

$$= 3nR\Delta T + nR\Delta T = 4nR\Delta T$$

$$= 4 \times 25 = 100 \text{ J}$$

12. Calculate the internal energy of 1g of oxygen gas at STP.

Sol. Since oxygen is a diatomic gas,  $C_v = 8/2R$

$$U(\text{internal energy per mole}) = 5/2 RT$$

$$\text{Internal energy of 1g of oxygen,} = \frac{1}{32} \left[ \frac{5}{2} RT \right] = \left( \frac{5}{64} \times 8.31 \times 273 \right) \text{ J} = 177.2 \text{ J}$$

13. Should the specific heat of monatomic gas be less than, equal to or greater than that of a diatomic gas at room temperature? Justify your answer.

Sol. Specific heat of a gas at constant volume is equal to  $f/2R$ . For monoatomic gases  $f = 3$  so  $C_v = 3/2 R$ . For diatomic gases  $f = 5$  so  $C_v = 5/2 R$ . Hence the specific heat for monoatomic gas is less than that for a diatomic gas.

14. The mean free path of a gas varies with the density of gas according to the following relation:

(a)  $\lambda \propto \rho$

(b)  $\lambda \propto \sqrt{\rho}$

(c)  $\lambda \propto 1/\rho$

(d)  $\lambda \propto \rho^2$

Sol. (c)

As density of gas increase, the molecule become closer to each other. Therefore they are more likely to run into each other, no mean free path decreases.

15. The mean free path of the molecule of a gas having number density  $0.167 \times 10^{28} \text{ m}^{-3}$  and molecular diameter  $1 \text{ \AA}$  is

(a)  $1.51 \times 10^{-6} \text{ m}$

(b)  $2.0 \times 10^{-9} \text{ m}$

(c)  $1.34 \times 10^{-8} \text{ m}$

(d)  $1.1 \times 10^{-4} \text{ m}$

Sol. (c)

Given  $d = 1 \text{ \AA}$

Since mean free path,  $\lambda = \frac{1}{\sqrt{2}n\pi d^2} = \frac{1}{\sqrt{2} \times 0.167 \times 10^{28} \times 3.14 \times (1 \times 10^{-10})^2} = 1.34 \times 10^{-8} \text{m}$

16. Calculate the ratio of the mean free path of the molecules of two gases having molecular diameters  $1\text{\AA}$  and  $2\text{\AA}$ . The gases may be considered under identical conditions of temperature, pressure and volume.

Sol. As  $\lambda = 1/d^2$ .

Given  $d_1 = 1 \text{\AA}$  and  $d_2 = 2 \text{\AA}$  or  $\lambda_1:\lambda_2 = 4:1$ .

17. What is mean free path of gases?

Sol. The mean free path of a gas molecule is defined as the average distance travelled by a molecule between two successive collisions.

18. How does mean free path depend on number density of the gas?

Sol. The mean free path is inversely proportional to the number density of the gas.

19. What is the order of mean free path ( $\lambda$ ) of the gas molecule?

Sol.  $\lambda$  is order of micron.  $1 \text{ micron} = 10^{-6} \text{m}$ .

### (2 Marks Questions)

20. The root mean square (rms) speed of oxygen molecule at certain temperature  $T$  is  $V$ . If temperature is doubled, oxygen gas dissociates into atomic oxygen. What is the rms speed of atomic oxygen?

Sol.  $v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$ ; Given  $T' = 2T$  and  $M' = M/2$

$$\frac{v'_{\text{rms}}}{v_{\text{rms}}} = \sqrt{\frac{T'}{T} \times \frac{M}{M'}} = \sqrt{\frac{2T}{T} \times \frac{M}{M/2}} = 2$$

Therefore  $v'_{\text{rms}} = 2v_{\text{rms}}$

21. From the expression for pressure of a gas on the basis of kinetic theory find an expression for rms speed of a gas molecule

Sol. Same as 31

22. Write any four fundamental postulates of the kinetic theory of an ideal gas.

Sol. (i) All gases consist of molecules, The molecules are rigid, elastic spheres and identical in all respects for a given gas and different for different gases.

(ii) The size of a molecule is negligible as compared to the average distance between molecules.

(iii) The molecules are in a state of continuous random motion, moving in all directions with all possible velocities.

(iv) The molecules exert no force on each other or on the walls of the container except during collision.

23. Calculate the molecular kinetic energy and translation of a mole of hydrogen atom at NTP. Given  $R = 8.31 \text{ J/mol/K}$ .

Sol. KE per mole of gas =  $\frac{3}{2} RT = (1.5 \times 8.31 \times 373) = 3.40 \times 10^3 \text{ J}$ .

24. Derive Avogadro's law using kinetic theory of gases.

Sol. Same as Qs. 20 sec A.

25. Calculate the number of degrees of freedom of molecules of hydrogen in  $1 \text{ cm}^3$  of hydrogen gas at NTP.

Sol. At NTP, volume occupied by 1g mole of gas =  $22400 \text{ cm}^3$

$$\text{Number of molecules in } 1 \text{ cm}^3 \text{ of H}_2 = \frac{6.023 \times 10^{23}}{22400} = 2.688 \times 10^{19}$$

Since each diatomic molecule has 5 degrees of freedom.

$$\text{Therefore total number of degrees of freedom} = 5 \times 2.688 \times 10^{19} = 1.344 \times 10^{20}.$$

26. Equal masses of helium and oxygen gases are given equal quantities of heat. Which gas will undergo a greater temperature rise?

Sol. Helium is a monatomic gas, while oxygen is diatomic. Therefore the heat given to helium will be totally used up in increasing the translational kinetic energy of its molecules, whereas the heat given to oxygen will be used up in increasing the translational kinetic energy of its molecules and also in increasing the kinetic energy of rotation and vibration. Hence there will be a greater rise in the temperature of helium.

27. Calculate specific heat of water using law of equiproportion of energy.

Sol. Consider water molecule as a solid made up of 3 atoms (2 hydrogen and 1 oxygen) and each atom is free to vibrate in three dimensions about its mean position.

In 3 dimension, the average energy per atom of water molecule =  $3k_B T$

Therefore total energy of one molecule of water is  $3 \times (3k_B T) = 9 k_B T$

Total energy of 1 mole of water,  $U = 2 \times 3k_B T \times N_A = 9RT$

$$\text{Therefore specific heat of water } C_V = \frac{\partial U}{\partial T} = 9R$$

28. The molar specific heats of an ideal gas at constant pressure and volume are denoted by  $C_P$  and  $C_V$  respectively. If  $\gamma = C_P/C_V$  and  $R$  is the universal gas constant, then find  $C_V$ .

Sol. For an ideal gas,  $C_P - C_V = R$

Divide  $C_V$  on both sides we get

$$\frac{C_P}{C_V} - 1 = \frac{R}{C_V}$$

$$\text{As } \gamma = \frac{C_p}{C_v}, \therefore \gamma - 1 = \frac{R}{C_v} \text{ or } C_v = \frac{R}{\gamma - 1}$$

29. A cylinder of fixed capacity 44.78 litres contains helium gas at standard temperature and pressure. What is the amount of heat needed to raise the temperature of the gas in the cylinder by 15.0°C? ( $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$ ).

Sol. Using the gas law,  $PV = nRT$ , you can easily show that 1 mol of any (ideal) gas at standard temperature (273K) and pressure (1atm =  $1.01 \times 10^5 \text{ Pa}$ ) occupies a volume of 22.4litres. This universal cylinder contains 2 mol of helium. Further, since helium is monatomic its predicted volume,  $C_v = (3/2)R$  and molar specific heat at constant pressure,  $C_p = (3/2)R + R = (5/2)R$ . Since the volume of the cylinder is fixed the heat required is determined by  $C_v$ . Therefore heat required =  $nC_v\Delta T = 2 \times 1.5R \times 15.0 = 45R = 45 \times 8.31 = 374 \text{ J}$ .

30. What is mean free path of a gas molecule? On which factors does the mean free path depend?

Sol. The mean free path of a gas molecule is defined as the average distance travelled by a molecule between two successive collisions.

$$\text{Since mean free path, } \lambda = \frac{m}{4\sqrt{2}\pi d^2 \rho} \text{ or } \lambda = \frac{k_B T}{\sqrt{2}\pi d^2 \rho}$$

So it ( $\lambda$ ) depends on mass ( $m$ ), diameter ( $d$ ), density ( $\rho$ ), and temperature ( $T$ ) for the given gas.

### (3 Marks Questions)

31. Show that the average kinetic energy of a gas molecule is directly proportional to the temperature of the gas. Hence give the kinetic interpretation of temperature.

Sol. According to kinetic theory, the pressure exerted by the gas is

$$P = \frac{1}{3} \rho \overline{v^2} = \frac{1}{3} \frac{M}{V} \overline{v^2} \text{ or } PV = \frac{1}{3} M \overline{v^2} = \frac{2}{3} \left( \frac{1}{2} \right) M \overline{v^2}$$

But  $\frac{1}{2} M \overline{v^2}$  is the average kinetic energy  $E$  of one mole of the gas.

$$\text{Therefore } PV = 2/3 E$$

The ideal gas equation for one mole of a gas is  $PV = RT$

$$\text{Therefore } 2/3 E = RT \text{ or } E = 3/2 RT$$

The above equation gives the average kinetic energy of one mole of the gas. If  $N$  is the Avogadro's number, then the average kinetic energy per molecule is given as

$$\bar{E} = \frac{E}{N} = \frac{3}{2} \frac{R}{N} T \text{ or } \bar{E} = \frac{3}{2} k_B T$$

Where  $k_B$  is called Boltzmann's constant. The average kinetic energy of a molecule depends on the absolute temperature of the gas. For temperature = 0K, average energy = 0. Thus at 0K, the velocity of molecules becomes zero.

32. (a) Using the kinetic theory expression for pressure, show that the average translational kinetic energy per gram molecules equal  $(3/2)RT$ .

(b) At what temperature would the rms velocity of gas become  $n$  times its value at  $0^\circ\text{C}$ , pressure remaining constant.

Sol. (a) same as 31

(b) Let  $v$  be the rms velocity at  $0^\circ\text{C}$ . Let  $t$  be the temperature at which rms velocity becomes  $n$  times.

Therefore  $v_1 = v$ ,  $T_1 = 273+0=273\text{K}$ ,  $v_2 = nv$ ,  $T_2 = (273+t)\text{K}$

$$\text{As } \frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}; \frac{nv}{v} = \sqrt{\frac{273+t}{273}}$$

$$n = \sqrt{\frac{273+t}{273}}; n^2 = \frac{273+t}{273}$$

$$273n^2 = 273+t \text{ or } 273n^2 - 273 = t \text{ or } 273(n^2 - 1) = t$$

$$\text{Or } t = [273(n^2 - 1)]^\circ\text{C}$$

33. Show that the average kinetic energy of a gas molecule is directly proportional to absolute temperature of the gas.

Sol. Same as 31.

34. Give a formula for mean free path of the molecule of a gas. Briefly explain how its value is affected by (i) change in temperature and (ii) change in pressure.

Sol. The value of mean free path of the molecules of a given gas is given by  $\lambda = \frac{1}{\sqrt{2}n\pi d^2}$

Here  $n$  = number of gas molecules present in unit volume of given gas and  $d$  = molecular diameter.

(i) As temperature of gas is increased at constant pressure, volume of a gas increases and hence  $n$ , the number of molecules per unit volume decreases. In fact

$$n \propto 1/V \text{ and } V \propto T. \text{ Thus } n \propto 1/T.$$

Due to decrease in molecular number density, the value of mean free path of the gas increased i.e.  $\lambda \propto 1/n \propto T$ . Thus constant pressure, the mean free path of a gas is directly proportional to its absolute temperature.

(ii) At constant temperature, on increasing pressure, the volume  $V$  decreases the molecular number density  $n$  increases and consequently the mean free path decreases i.e.  $\lambda \propto 1/V \propto 1/n$ .

$$\text{Therefore } \lambda \propto 1/n \text{ or } \lambda \propto 1/P$$

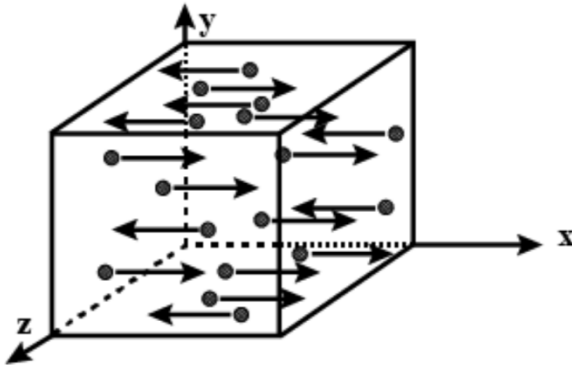
Thus at a constant temperature the mean free path of a gas is inversely proportional to its pressure.

### (5 Marks Questions)



35. Show that the pressure exerted by an ideal gas is  $p = \frac{1}{3} \rho \overline{v^2}$  where  $\rho$  is the density and  $v$  is the root mean square velocity.

Sol.



Let us consider a particle in the cube of side  $a$ , to bounce between the walls along distance  $a$  and it has velocity  $v_x$  and mass  $m$ . It would then have momentum.

$$M = mv_x$$

It takes time  $dt$  to go between the two walls

$$dt = \frac{a}{v_x}$$

When the particle strikes the wall, it transfers momentum to the wall. Thus the force going into the wall is given by Newton's Second law as rate of change of momentum,  $F = \frac{M}{dt} = \frac{mv_x^2}{a}$

$$\text{Pressure, } P = \frac{\text{Force}}{\text{Area}} = \frac{mv_x^2}{a^3} = \frac{mv_x^2}{V}$$

where  $V$  is the volume of cube.

Molecules are going in all directions, not just 'x', and the average velocity in all directions is

$$\sum v^2 = \sum v_x^2 + \sum v_y^2 + \sum v_z^2$$

Where  $v$  is the root mean square speed.

Since there is an equal probability for going all ways,

$$v_x^2 = \frac{v^2}{3}$$

Substituting this value in the above equation for  $n$  molecules of gas .

$$PV = \frac{\text{Force}}{\text{Area}} = \frac{nmv_x^2}{a^3} = \frac{nmv^2}{3}$$

From equation of state for an ideal gas.

$PV = nRT$ ,  $R$  is universal gas constant and  $T$  is the absolute temperature.

Equating the two

$$\frac{nmv^2}{3} = nRT$$

$$v = \sqrt{\frac{3RT}{m}}$$

Thus root mean square velocity of gas molecule is directly proportional to the square root of its absolute temperature.

36. Derive an expression for pressure of a gas in a container. Using it, relate KE with pressure.

Sol. (a) Consider a cubical vessel (of side  $l$ ) with walls perfectly elastic as in the figure. Let the vessel have one gram molecule of a gas with its molecules in random motion.

Consider a molecule (mass  $m$ ) moving with a velocity  $c$ , that can be resolved into three components  $u, v$ , and  $w$  in the direction of the edges of the cube along  $x, y, z$  axes.

$$\therefore c^2 = u^2 + v^2 + w^2 \quad \dots\dots(1)$$

Let us consider two faces of the cube, say P and Q, normal to  $x$ -axis. When the molecule collides with the side Q with a velocity  $u$ , it rebounds with  $-u$ , while its other components remain unchanged.

Change in momentum of the molecule due to this collision  $= -2mu$ , which is imparted to the wall per collision.

The time taken by molecule to cover distance  $l = 1/u$

$\therefore$ , after every interval of time  $2l/u$ , the molecule will again collide with the wall Q, and the number of collisions per unit time with the wall Q is equal to  $u/2l$ .

$\therefore$ , Momentum imparted to the wall per unit time  $= 2mu \times u/2l = mu^2/l$  (ignoring the negative sign).

The pressure exerted on the wall Q due to one molecule  $= mu^2/l \times 1/l^2 = mu^2/l^3$

If the vessel contains  $N$  molecules with velocities  $u_1, u_2, u_3, \dots, u_n$  along  $x$ -axis, then the pressure exerted by them on Q, say

$$P = m/l^3 (u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2) = mN/V \cdot (\bar{u}^2)$$

where  $V$  is the volume of the vessel and  $(\bar{u}^2)$  is the average value of  $u^2$  of all  $N$  molecules.

$$(\bar{u}^2) = (u_1^2 + u_2^2 + u_3^2 + \dots + u_n^2)/N$$

As the molecules move randomly,

$$(\bar{u}^2) = (\bar{v}^2) = (\bar{w}^2) = (\bar{c}^2)/3 = \text{where } \bar{c} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 + \dots + c_n^2}{N}} \quad \dots\dots(2)$$

$\bar{c}$  is called the root mean square velocity of the molecules.

$$\therefore P = \frac{1}{3} \frac{mN(\bar{c})^2}{V} \quad \dots\dots(3)$$

This equation gives the pressure exerted by the gas on the walls of the vessel. It has the same value in any direction because the molecules have no preference for direction.

We have  $\rho = mN/V$

$$\therefore P = \frac{1}{3} \rho (\bar{c})^2$$

or  $P = \frac{2}{3} E$  where  $E = \frac{1}{2} \rho (\bar{c})^2$  is called the mean kinetic energy per unit volume of the gas.

(b) Mean KE of translation per unit volume of the gas is  $E = \frac{1}{2} \rho \bar{v}^2$

$$\therefore \frac{P}{E} = \frac{\frac{1}{3} \rho \bar{v}^2}{\frac{1}{2} \rho \bar{v}^2} = \frac{2}{3}$$

$$P = \frac{2}{3} E$$

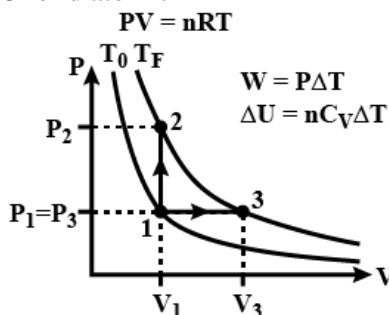
37. Using the law of equipartition of energy determine the values of  $C_p$ ,  $C_v$  and  $\gamma$  for (a) monatomic (b) diatomic (c) triatomic gases

Sol. According to the law of equipartition of energy, for any dynamic system in thermal equilibrium, the total energy for the system is equally divided among the degree of freedom.

$C_v = \frac{3}{2} R$  for monatomic;  $C_v = \frac{5}{2} R$  for diatomic;  $C_v = \frac{7}{2} R$  for triatomic

$C_p = \frac{5}{2} R$  for monatomic ;  $C_p = \frac{7}{2} R$  for diatomic;  $C_p = \frac{9}{2} R$  for triatomic

$\gamma = \frac{5}{3}$  for monoatomic;  $\gamma = \frac{7}{5}$  for diatomic



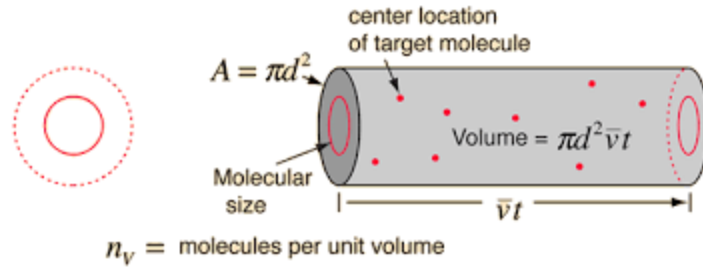
38. What is meant by mean free path of a gas molecule? Derive expression for it. On which factors does the mean free path depends?

Sol. The mean free path of a gas molecule is defined as the average distance travelled by a molecule between two successive collisions.

Let us consider the mean free path, we make use of the following assumptions:

- (i) Each molecule of the gas is a sphere of diameter  $d$ .
- (ii) All molecules of the gas except the molecule A under consideration are at rest.

As shown figure, suppose the molecule A has average speed  $\bar{v}$ , it will collide with all those molecules whose centres lie within a distance  $d$  from its path. In time  $\Delta t$ , it will collide with all those molecules in the cylinder of volume  $\pi d^2 \bar{v} \Delta t$ . Let  $n$  be the number of molecules per unit volume.



Number of collisions suffered by molecule A in time  $\Delta T =$  volume of the cylinder swept by molecule A in time  $\Delta T \times$  number of molecules per unit volume  $= \pi d^2 \bar{v} \Delta t \times n$

$$\text{Mean free path of the gas molecule, } \lambda = \frac{\text{Distance covered in time } \Delta t}{\text{no. of collisions suffered } n \text{ time } \Delta t}$$

$$= \frac{\bar{v} \Delta t}{\pi d^2 \bar{v} \Delta t n} = \frac{1}{n \pi d^2}$$

Taking into consideration the motion of all the gas molecules, the mean free path comes out to be,  $\lambda = \frac{1}{\sqrt{2} n \pi d^2}$

If  $m$  is the mass of each gas molecule, then the density of the gas is  $\rho = mn$  or  $n = \rho/m$

$$\text{Therefore } \lambda = \frac{1}{\sqrt{2} \pi d^2 \rho}$$

For one mole of a gas,  $PV = RT$  or  $P = RT/V = N/V \times R/N \Rightarrow T = nk_B T$

$$\text{Or } n = \frac{P}{k_B T} \quad \therefore \frac{k_B T}{\sqrt{2} \pi d^2 P}$$

Dependency of mean path ( $\lambda$ ): Since mean free path,  $\lambda = \frac{m}{4\sqrt{2} \pi d^2 \rho}$  or  $\lambda = \frac{k_B T}{\sqrt{2} \pi d^2 P}$

So it ( $\lambda$ ) depends on mass ( $m$ ), diameter ( $d$ ), density ( $\rho$ ), and temperature ( $T$ ) for the given gas.

### C. CHALLENGING PROBLEMS

1. A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15cm column of air. What happens if the tube is held vertically with the open end at the bottom?

Sol. Length of the narrow bore,  $L = 1 \text{ m} = 100 \text{ cm}$

Length of the mercury thread,  $l = 76 \text{ cm}$

Length of the air column between mercury and the closed end,  $l_a = 15 \text{ cm}$

Since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space is:  $100 - (76 + 15) = 9 \text{ cm}$

Hence, the total length of the air column  $= 15 + 9 = 24 \text{ cm}$

Let  $h \text{ cm}$  of mercury flow out as a result of atmospheric pressure.

$\therefore$  Length of the air column in the bore  $= 24 + h \text{ cm}$

And, length of the mercury column =  $76 - h$  cm

Initial pressure,  $P_1 = 76$  cm of mercury

Initial volume,  $V_1 = 15 \text{ cm}^3$

Final pressure,  $P_2 = 76 - (76 - h) = h$  cm of mercury

Final volume,  $V_2 = (24 + h) \text{ cm}^3$

Temperature remains constant throughout the process.

$$\therefore P_1 V_1 = P_2 V_2$$

$$76 \times 15 = h(24 + h)$$

$$h^2 + 24h - 1140 = 0$$

$$h = 23.8, -47.8 \text{ cm}$$

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and 52.2 cm of mercury will remain in it. The length of the air column will be  $24 + 23.8 = 47.8$  cm.