CLASS – 12

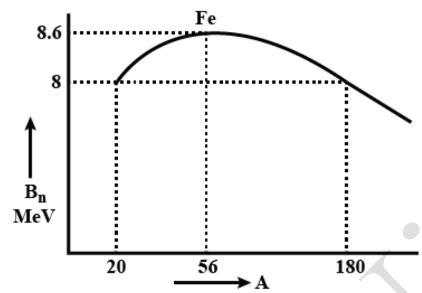
WORKSHEET- NUCLEI

A. NUCLEUS

(1Mark Questions)

1. Draw a graph between binding energy per nucleon and mass number

Sol.



2. Give the reason for the decrease of binding energy per nucleon for nuclei with high mass numbers

Sol. The decrease of the binding energy per nucleon for nuclei with high mass number is due to increased coulomb repulsion between protons inside the nucleus.

3. State two characteristic properties of nuclear force.

Sol. **Properties of Nuclear Force**

- It is attractive in nature but with a repulsive core. ...
- The range of a nuclear force is very short. ...
- The nuclear force is identical for all nucleons. ...
- At a distance of less than 0.7 Fermi, this force becomes repulsive.

4. Define atomic mass unit

Sol. An atomic mass unit is defined as accurately 1/12 the mass of a carbon-12 atom. The carbon-12 atom has six neutrons and six protons in its nucleus. It is represented as a.m.u or u (unified). 1 a.m.u is the average of the proton rest mass and the neutron rest mass.

5. Heavy stable nucle have more neutrons than protons. This is because of the fact that
(a) neutrons are heavier than protons.
(b) electrostatic force between protons are repulsive.
(c) neutrons decay into protons through beta decay.
(d) nuclear forces between neutrons are weaker than that between protons.

Sol. (b)
Heavy nuclei, which are stable contain more neutrons than protons in their nuclei. This is because electrostatic force between protons is repulsive, which may reduce stability.

6. He_2^3 and He_1^3 nuclei have the same mass number. Do they have the same binding energy?

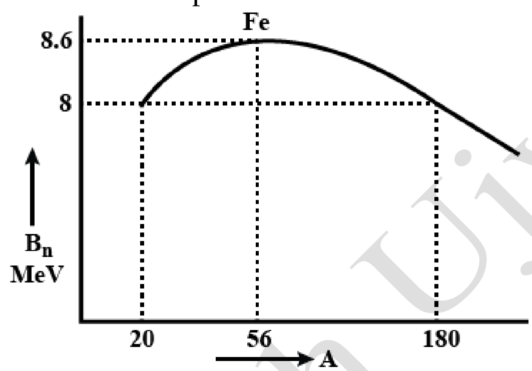
Sol. No, the binding energy of $He^3{}_1$ is greater.

(2 Marks Questions)

7. Explain binding energy between nucleons. Draw a graph between binding energy per nucleon and mass number.

Sol. The graph of the binding energy per nucleon versus mass number A is shown in figure.

The decrease of the binding energy per nucleon for nuclei with high mass number is due to increased coulomb repulsion between protons inside the nucleus.



8. Write two main interferences drawn from the graph between binding energy per nucleon and mass number.

Sol. Inferences from graph

(1) The nuclei having mass number below 20 and above 180 have relatively small binding energy and hence they are unstable.

(2) The nuclei having mass number 56 and about 56 have maximum binding energy -

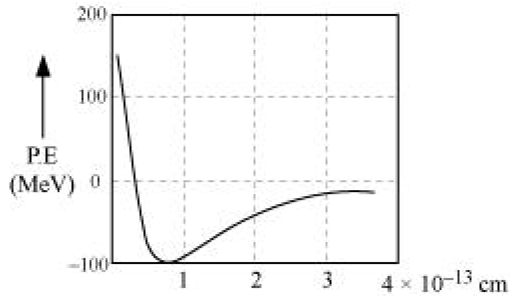
5.8MeV and so they are most stable.

9. If the nucleons bound in nucleus are separated apart from each other, the sum of their masses is greater than the mass of the nucleus. Where does this mass difference come from? Explain briefly.

Sol. When nucleons of nucleus are separated apart from each other then the sum of the nucleons is greater than the mass of nucleus. This is direct mass defeat of nucleus. Some mass has been converted into energy which holds the nucleus together.

10. Draw a plot to potential energy of a pair of nucleons as a function on their separation. What is the significance of negative potential energy in the graph drawn?

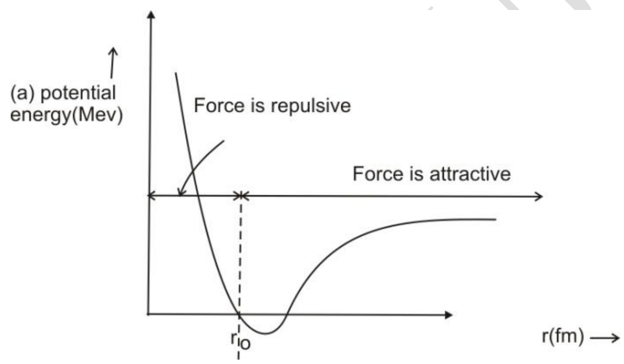
Sol.



The negative potential energy in the graph exhibits that the system bounded.

11. Draw a graph showing the variation of potential energy between a pair of nucleons as a function of their separation. Indicate the regions in which the nuclear force is (i) attractive (ii) repulsive

Sol. Plot of potential energy of a parir of nucleons as a function of their separation is givne in the figure.



Conclusions: (i) The nuclear force is much stronger than the coulomb for e acting between charges or the gravitational forces between masses. (ii) The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few fermies. (iii) For a separation greater than r_0 the force is attractive and for separation less than r_0 the force is strongly repulsive.

12. Obtain the binding energy (in MeV) of a Nitrogen nucleus $\left({}^{14}_{7}N\right)$, given $m\left({}^{14}_{7}N\right)$ =14.00307 u

Sol. m_H = 1.00783 amu; m_n = 1.0087 amu; m_N = 14.0030u amu

In $_7N^{14}$, these are 7 protons and 7 neutrons.

$\therefore$ mass defect, ,$\Delta = (7m_H + 7m_n) - m_N$

= (7×1.00783) + (7×1.00807) - 14.00307 = 0.11243 amu

Binding energy of nitrogen nucleus

= Δ×931 MeV =0.11243 × 931 =104.67 MeV

13. Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:

$m\left(^{56}_{26}Fe\right) = 55.934939$ u, $m\left(^{209}_{83}Bi\right) = 208.980388$ u

Sol. (a) $^{56}_{26}Fe$ contains 26 protons and 30 neutrons.

Mass of 26 protons=26×1.007825u=26.26345u

Mass of 30 neutrons=30×1.008665u=30.25995u

Mass of 2$^{56}_{26}$**Fe** nucleus=55.934939u

Thus, mass defect Δm=26.26345u+30.25995u−55.934939u=0.528461u.

Thus, binding energy of $^{56}_{26}$**Fe** nucleus is 0.528461×931.5MeV=492.26MeV.

Binding energy per nucleon=492.26MeV/56 = 8.790MeV

(b) $^{209}_{83}Bi$ contains 83 protons and 126 neutrons.

Mass of 83 protons=83×1.007825u=83.649475u

Mass of 126 neutrons=126×1.008665u=127.09170u

Mass of $^{209}_{83}Bi$ nucleus=208.986388u

Thus, mass defect Δm=83.649475u+127.09170u−208.986388u=1.760872u.

Thus binding energy of $^{209}_{83}Bi$ nucleus is 1.760872×931.5MeV=1640.26MeV.

Binding energy per nucleon=1640.26MeV/209 =7.848MeV

14. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 u).

Sol. We are given –

Mass of a copper coin, given by m' = 3 g

Atomic mass of $^{63}_{29}Cu$ atom, given by m = 62.92960 u

The total number of $^{63}_{29}Cu$ atoms in the coin is given by the relation $N = \frac{N_A \times m}{\text{mass number}}$

Where, N_A is the Avogadro's number, given by 6.023 × 10^{23} atoms /g

Mass number of $^{63}_{29}Cu$, given by = 63 g, So the relation gives us

$N = \frac{6.023\times10^{23}\times3}{63} = 2.868\times10^{22}$ atoms

We know that $^{63}_{29}Cu$ nucleus has 29 protons and (63 − 29) = 34 neutrons

Now, the mass defect of this nucleus is given by

$\Delta m' = 29 \times m_H + 34 \times m_n - m'$

Where,

Mass of a proton is given by m_H = 1.007825 u

Mass of a neutron is given by m_n = 1.008665 u

So, the relation becomes

$\Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$

$\Delta m' = 0.591935$ u

We have calculated the mass defect of one atom. Now we will find the mass defect of all the atoms by applying following relation –

Mass defect of all the atoms present in the coin

$\Delta m = 0.591935 \times 2.868 \times 10^{22}$

$= 1.69766958 \times 10^{22}$ u

We know that – 1 u = 931.5 MeV/c^2

$E_b = 1.69766958 \times 931.5 \times (MeV/c^2) \times c^2 = 18.581 \times 10^{25}$MeV

We know that 1 MeV = 1.6×10^{-13} J

$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$

Or $E_b = 2.5296 \times 10^{12}$J

Therefore, 2.5296×10^{12} J is the energy required to separate all the neutrons and protons from the given coin.

15. From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A).

Sol. We know that the nuclear radius is given as

$R = R_0A^{1/3}$

R_0 is a constant value

The nucleus' mass number is A.

Nuclear matter density is defined as the mass of the nucleus divided by the volume of the nucleus.

mA gives us the mass of the nucleus.

Density of the nucleus is then given by

$= (4/3)\pi R^3 = (4/3)\pi(R_0A^{1/3})^3 = (4/3)\pi R_0^3A$

$\rho = mA/[(4/3)\pi R_0^3A]$

$= 3mA/(4\pi R_0^3A)$

$\rho = 3m/(4\pi R_0^3)$

Nuclear matter density is nearly a constant and does not depend upon A.

16. Define atomic mass unit. Calculate the energy equivalent to one atomic mass unit in MeV.

Sol. Atomic mass unit (amu) is a unit of mass used to express mass of atomic particles. 1 atomic mass (1 amu) is defined as one twelfth 1/12 of the mass of an atom of carbon 12.

A amu = 1.66×10^{-27}kg

According to Einstein's mass energy equivalence relation, $E = (\Delta m)c^2$

So we need to find energy equivalent of amu, $E = (1.66 \times 10^{27}kg) \times (3 \times 10^8 m/s)^2$

$= 1.49 \times 10^{-10}$ joules

But 1 MeV = 1.6×10^{-13}J

$E = 1.49 \times 10^{-10} / 1.6 \times 10^{-13}$ MeV = 931MeV

Thus 1 amu of mass is equivalent to 931MeV of energy.

(3 Marks Questions)

17. You are given two nuclides $_3X^7$ and $_3Y^4$. (i) Are they the isotopes of the same element? Why? (ii) Which one of two is likely to be more stable? Give reason.

Sol. $_3X^7$ Nucleus :

Number of protons $p=3$

Number of neutrons $n=7-3=4$

Neutron to proton ratio $n/p=4/3$

$_3Y^4$ Nucleus :

Number of protons $p=3$

Number of neutrons $n=4-3=1$

Neutron to proton ratio $n/p=1/3$

Neutron to proton ratio is more in $_3X^7$, so it is more stable.

18. The binding energies of deuteron ($_1H^2$) and α-particle ($_2He^4$)are 1.25 and 7.2Mev/ nucleon respectively. Which nucleus is more stable? Calculate binding energies per nucleon of $_{26}Fe^{56}$. [m($_{26}Fe^{56}$) = 55.934939 amu; m(proton) = 1.007825 amu; m(neutron) = 1.008665 amu]

Sol. (a) α-particle ($_2He^4$) is more stable, because a nucleus is more stable when value of binding energy per nucleon is larger.

(b) $_{26}Fe^{56}$ nucleus contains 26 protons .

Number of neutrons =(56−26)=30 neutrons

Now,

Mass of 26 protons=26 × 1.007825=26.20345u

Mass of 30 neutrons=30 × 1.008665=30.25995u

Total mass of 56 nucleons=56.46340u

Mass of $_26Fe^{56}$ nucleus=55.934939u

Therefore,

Mass defect, $\Delta m = 56.46340-55.934939=0.528461$u

Total Binding Energy=0.528461 X 931.5Mev=492.26MeV

19. Calculate binding energy per nucleon of $_{17}Cl^{35}$ nucleus. [Given mass of $_{17}Cl^{35}$= 34.980000u, mass of proton = 1.007825u, mass of neutron = 1.008665u, 1 atomic mass unit (1u) = 931 MeV]

Sol. $\Delta m = (17\times1.007825) + (18\times1.008665) - 34.9 = 0.308995amu$
$=0.308995\times931.478MeV/c^2$.......[using $1amu=931.478MeV/c^2$]
$=287.82MeV/c^2$
$E=\Delta mc^2 \quad =287.82MeV$

20. (a) Two stable isotopes of lithium ${}^{6}_{3}Li$ and ${}^{7}_{3}Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.
(b) Boron has two stable isotopes, ${}^{10}_{5}B$ and ${}^{11}_{5}B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_{5}B$ and ${}^{11}_{5}B$.

Sol. (a) The atomic weight of lithium is m(Li) = $\frac{7.5\times6.01512+92.5\times7.01600}{100} = \frac{45.1134+648.98}{100}$
$= \frac{694.0934}{100} = 6.941amu$
(b) Suppose the natural boron contains x% of ${}^{10}_{5}B$ isotope and (100 – x)% of ${}^{11}_{5}B$ isotope. Then
Atomic mass of natural boron = weighted average of the masses of two isotopes
Therefore, $10.811 = \frac{x\times10.01294+(100-x)\times11.00931}{100}$
Or 1081.1 = 0.99637x + 1100.931 or $x = \frac{19.831}{0.99637} = 19.9$
Therefore Relative abundance of ${}^{10}_{5}B$ isotope = 19.9% and relative abundance of ${}^{11}_{5}B$ = 80.1%.

21. The three stable isotopes of Neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of Neon.

Sol. The average atomic mss of neon is m(Ne) = $\frac{90.51\times19.99+0.27\times20.99+9.22\times21.99}{100}$
$= \frac{1736.89+5.67+202.75}{100} = \frac{1945.31}{100} = 19.45amu.$

22. The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by
$Q = [\, m_A + m_b - m_C - m_d]c^2$ where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.
(i) ${}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{2}_{1}H + {}^{2}_{1}H$
(ii) ${}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{20}_{10}Ne + {}^{4}_{2}He$
Atomic masses are given to be

$m\left({}_1^2\text{H}\right) = 2.014102$ u

$m\left({}_1^3\text{H}\right) = 3.016049$ u

$m\left({}_6^{12}\text{C}\right) = 12.000000$ u

$m\left({}_{10}^{20}\text{Ne}\right) = 19.992439$ u

Sol. (i) ${}_1^1\text{H} + {}_1^3\text{H} \rightarrow {}_1^2\text{H} + {}_1^2\text{H}$

Q = [m(${}_1^2$H) + m(${}_1^3$H) – [m(${}_1^2$H) + m(${}_1^2$H)]]c^2 = (4.023874 – 4.028204) × 931.5 MeV

= 0.00433×931.5 MeV = - 4.033 MeV

Negative value of Q indicates that the reaction is endothermic.

(ii) ${}_6^{12}\text{C} + {}_6^{12}\text{C} \rightarrow {}_{10}^{20}\text{Ne} + {}_2^4\text{He}$

Q = [2m(${}_6^{12}$C) – [m(${}_{10}^{20}$Ne) + m(${}_2^4$He)]]c^2 = [2×12.000000 – (19.992439+4.002603)] ×931.5 MeV = (24 – 23.995042) × 931.5 MeV = 0.004958 × 931.5 MeV = 4.618 MeV

Positive value of Q indicates that the reaction is exothermic.

23. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on earth. The three isotopes and their masses are ${}_{12}^{24}$Mg (23.98504u), ${}_{12}^{25}$Mg (24.98584u) and ${}_{12}^{26}$Mg (25.98259u). The natural abundance of ${}_{12}^{24}$Mg is 78.99% by mass. Calculate the abundances of other two isotopes.

Sol. Average atomic mass of magnesium, m = 24.312 u

Mass of magnesium isotope ${}_{12}^{24}$Mg, m_1 = 23.98504 u

Mass of magnesium isotope ${}_{12}^{25}$Mg, m_2 = 24.98584 u

Mass of magnesium isotope ${}_{12}^{26}$Mg, m_3 = 25.98259 u

Abundance of ${}_{12}^{24}$Mg, η_1= 78.99%

Abundance of ${}_{12}^{25}$Mg, $\eta_2 = x$%

Hence, abundance of ${}_{12}^{26}$Mg, $\eta_3 = 100 - x - 78.99\% = (21.01 - x)\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.99 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$2431.2 = 1894.5783096 + 24.98584x + 545.8942159 - 25.98259x$$

$$0.99675x = 9.2725255$$

$$\therefore x \approx 9.3\%$$

$$\text{And } 21.01 - x = 11.71\%$$

Hence, the abundance of ${}^{25}_{12}Mg$ is 9.3% and that of ${}^{26}_{12}Mg$ is 11.71%.

24. The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}Ca$ and ${}^{27}_{13}Al$ from the following data:

$m\left({}^{40}_{20}Ca\right) = 39.962591$ u

$m\left({}^{41}_{20}Ca\right) = 40.962278$ u

$m\left({}^{26}_{13}Al\right) = 25.986895$ u

$m\left({}^{27}_{13}Al\right) = 26.981541$ u

Sol. For ${}^{41}_{20}Ca$ nucleus:

= Neutron separation energy =$[\{m({}^{40}_{20}\mathbf{Ca})+mn\}-m({}^{41}_{20}\mathbf{Ca}) \times 931.5MeV$

$=[(39.962591+1.008665)-40.962278]\times 931.5MeV=0.008978\times 931.5=8.36MeV$

For ${}^{27}_{13}Al$

= Neutron separation energy =$[\{m({}^{26}_{13}Al)+mn\}-m({}^{27}_{13}Al)]\times 931.5MeV$

$=[(25.986895+1.008665)-26.981541]\times 931.5MeV=13.06MeV$

B. RADIOACTIVITY

1Mark Questions)

1. Write the nuclear decay process of β-decay of ${}^{32}_{15}P$.

Sol. ${}^{32}_{15}P \rightarrow {}^{32}_{16}S + {}^{0}_{-1}e + \bar{\nu}$

2. The radioactive isotope D decays according to the sequence: $D \xrightarrow{\beta^-} D_1 \xrightarrow{\alpha-\text{particle}} D_2$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what is (i) the mass number, (ii) atomic number of D?

Sol. D mass number and atomic number are 176,71.

Alpha Particle is ${}_2He^4$

So D_1 will have 172 as the mass number and 69 as the atomic number

$\beta-$ particle is ${}_{-1}e^0$.

So D will have 172 as the mass number and 70 as atomic number.

3. Will neutron to proton ratio increase or decrease in a nucleus when (i) an electron, (ii) a positron is emitted?

Sol. In emission of an electron, a neutron is converted into a proton. Therefore, number of neutrons decreases and the number of protons increases. The neutron to proton ratio decreases. In the emission of a positron, a proton is converted into a neutron.

4. Define half life of a radioactive substance.

Sol. Half-life: The time-interval in which the mass of a radioactive substance or the number of its atoms, is reduced to half its initial value is called the half life of that substance.

5. How is the half life of a radioactive substance related to its decay constant?

Sol. The time required for half of the original population of radioactive atoms to decay is called the half-life. The relationship between the half-life, $T_{1/2}$, and the decay constant is given by $T_{1/2} = 0.693/\lambda$.

6. Define the term 'activity' of a radionuclide. Write its SI unit.

Sol. Activity of radioactive substance is defined as the number of disintegration takes place in the given sample per second. Its SI unit is Becquerel (Bq). 1 Bq is one disintegration per second.

7. State the reason why heavy water is generally used as a moderator in a nuclear reactor.

Sol. Moderator is required in a Nuclear reactor to slow down the neutrons produced during the fission reaction so that the chain reaction can be sustained. Heavy Water is an excellent moderator due to its high moderating ratio and low absorption cross section for neutrons.

8. Name the absorbing material used to control the reaction rate of neutrons in a nuclear reactor.

Sol. Cadmium control rods are used to regulate the reaction rate of nuclear fission or reactors in a nuclear reactor by absorbing neutrons.

9. Suppose we consider a large number of containers each containing initially 10000 atoms of a radioactive material with a half life of 1 year. After 1 year,
(a) all the containers will have 5000 atoms of the material.
(b) all the containers will contain the same number of atoms of the material but that number will only be approximately 5000.
(c) the containers will in general have different numbers of the atoms of the material but their average will be close to 5000.
(d) none of the containers can have more than 5000 atoms.

Sol. (c)
Radioactivity is a process due to which a radioactive material spontaneously decays. Half life time for a radioactive remain average half of its or of radioactive atoms will decay. So, the containers will in general have different number of atoms of the material, after one year means half life i.e. average atom of radioactive substance remain after 1 year, in each container is equal to ½ of 10000 = 5000 atoms

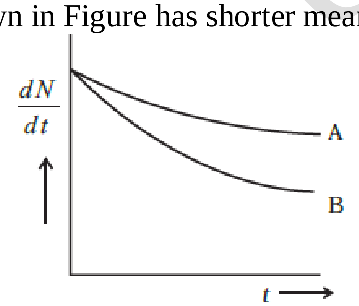
10. When a nucleus in an atom undergoes a radioactive decay, the electronic energy levels of the atom
(a) do not change for any type of radioactivity .
(b) change for α and β radioactivity but not for γ-radioactivity.
(c) change for α-radioactivity but not for others.
(d) change for β-radioactivity but not for others.

Sol. (b)
β-particle carries one unit of negative charge and α-particle carries 2 units of positive charge and γ-particle carries no charge, so electronic energy levels of the atom charges in emission of α and β particle, but not in γ decay.

11. In a nuclear reactor, moderators slow down the neutrons which come out in a fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose because
(a) they will break up.
(b) elastic collision of neutrons with heavy nuclei will not slow them down.
(c) the net weight of the reactor would be unbearably high.
(d) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature.

Ans. (b)

12. Which sample, A or B shown in Figure has shorter mean-life?



Sol. B has shorter mean life as λ is greater for B.

(2 Marks Questions)

13. With the help of an example, explain how the neutron to proton ratio changes during alpha decay of a nucleus.

Sol. Alpha decay involves the ejection of helium nucleus which is just two protons and two neutrons. This means that the atomic mass decreases by 4 and atomic number decreases by 2.

14. In the series of radioactive disintegration of $^{Z}_{A}X$, first and α-particle and then a β-particle is emitted. What is the atomic number and mass number of the new nucleus formed by these successive disintegrations?

Sol. When an alpha particle is emitted, mass number decreases by 2 and atomic number decreases by 2, and when beta particle is emitted, mass number remains same and atomic number increases by 1.

$\therefore\ {}^{A}_{1}X \xrightarrow{\alpha} {}^{A-4}_{Z-2}X \xrightarrow{\beta} {}^{A-4}_{Z-1}X$

So, mass number becomes (A – 4) and atomic number becomes (Z – 1).

15. Write an example, whether the neutron-proton ratio in a nucleus increases or decreases due to beta (β) decay.

Sol. This ratio generally increases with increase in atomic number, in a stable nuclei. Beta decay decreases the ratio of neutron to proton. It is clearly visible that the neutron-proton decreases. (of mass 62.92960 u).

16. The half life of ${}_{6}C^{14}$ is 5700 years. What does it mean?

Sol. Half life means time after which the given amount of substance reduces to half its weight or volume. so, C-14 atom has the 5700 years of half life. Again this carbon atom disintegrate half of its present value in 5700 years.

17. Two radioactive nuclei X and Y initially contain an equal number of atoms. Their half life is 1 hour and 2 hours respectively. Calculate the ratio of their rates of disintegration after two hours.

Sol. Given, nuclei X and Y contain equal number of atoms.

Half-life of X, $T_1 = 1$ hr

Half-life of Y, $T_2 = 2$ hr

Ratio of radioactive sample of X left after 2 hours,

$$N_1 = \left(\frac{1}{2}\right)^{2/1} N_0 = \frac{1}{4} N_0$$

Ratio of radioactive sample of Y left after 2 hours,

$$N_2 = \left(\frac{1}{2}\right)^{2/2} N_0 = \frac{1}{2} N_0$$

Now, decay rate is given by,

$$\boxed{R = \lambda N = \frac{0.693\,N}{T_{1/2}}}$$

$$R \propto \frac{N}{T_{1/2}}$$

$$\frac{R_1}{R_2} = \frac{(T_{1/2})_2}{(T_{1/2})_1} \times \frac{N_1}{N_2}$$

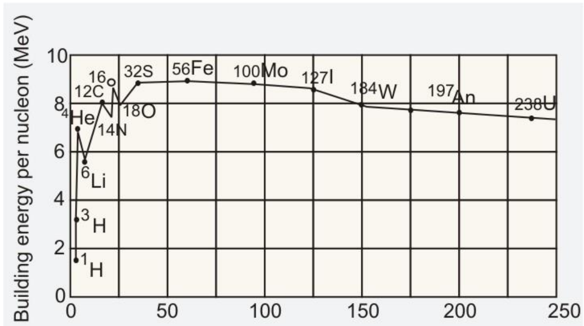
$$= \frac{2}{1} \times \frac{\frac{N_0}{4}}{\frac{N_0}{2}}$$

$$= 1$$

Hence, the activity of the two samples will be equal after 2 hours.

18. Mark the region where the nuclei are prone to fission in the graph between binding energy per nucleon and mass number.

Sol.



19. State with reason why light nuclei usually undergo nuclear fusion.

Sol. Lighter nuclei can easily undergo Nuclear fusion because of greater short-range nuclear force in comparison to coulombic repulsion. So two lighter nuclei can easily combine and fuse together to form a heavier and stable nucleus.

20. Given one point of difference between ‘nuclear fission’ and ‘nuclear fusion’.

Sol. In the case of nuclear fission, an atom divides into two or more smaller or lighter atoms. Nuclear fusion occurs when two or more atoms join or fuse together to form a large or heavier atom.

21. Name the reaction which takes place when a slow neutron beam strikes $^{235}_{92}U$ nuclei. Write the nuclear reaction involved.

Sol. When a slow neutron beam strikes $_{92}U^{235}$ then the neutron gets absorbed by the nucleus of the uranium. Due to this absorption, the formation of a new unstable $_{92}U^{236}$ takes place, and this unstable nucleus breaks apart into two different atoms.
Thus, the reaction taking place is nuclear fission.

The nuclear reaction involved is given as follows.

$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3^{1}_{0}n + E$

Here, Uis the uranium, n is the neutron, Ba represents barium, K represents krypton and E is the energy released.

22. A nucleus $^{23}_{10}Ne$ undergoes β-decay and becomes $^{23}_{11}Na$. Calculate the maximum kinetic energy of electrons emitted assuming that the daughter nucleus and anti neutrino carry negligible kinetic energy. Mass of $^{23}_{10}Ne$ =22.994466u; mass of $^{23}_{11}Na$ =22.989770u; 1u = 931.5 MeV/c^2.

Sol. The equation of β -decay of 2310 $^{23}_{10}Ne$ is Changing nuclear masses into atomic masses = 22.994466 – 22.989770 = 0.004696 u

∴ Maximum K.E., Q = 0.004696 u × 931.5 MeV/u = 4.37 MeV

23. Calculate the energy released in MeV in the following nuclear reaction: $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q$. [Mass of $^{238}_{92}U$ = 238.05079u; mass of $^{234}_{90}Th$ = 234.04363u; mass of $^{4}_{2}He$ = 4.002600u; 1u = 931.5 MeV/c^2]

Sol. The process is $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He + Q$

The energy released (α–particle)

$Q = (M_U - M_{TH} - M_{He})c^2$

$= (238.05079 - 234.04363 - 4.00260)u \times c^2 = (0.00456u) \times c^2$

$= 0.00456 \times (\frac{931-MeV}{c^2}).c^2 = 4.25MeV$

Yes, the decay is spontaneous (since Q is positive).

24. A radioactive isotope has a half-life of *T* years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

Sol. (a) $\frac{R}{R_0} = \frac{N}{N_0} = \frac{3.125}{100} = \frac{1}{32}$

Or $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^5$ or $n = 5$

Therefore t = nT = 5T years

(b) $\frac{R}{R_0} = \frac{N}{N_0} = \frac{1}{100}$

Required time, t = $\frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303T}{0.693}\log 100 = \frac{2.303\times 2\times T}{0.693} = 6.65T$ years

25. Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

Sol. Here R = 8.0mCo = $8.0\times10^{-3}\times3.7\times10^{10}$ dis s^{-1} = 29.6×10^{7} dis s^{-1}

$T_{1/2}$ = 5.3 yrs = $5.3\times3.16\times10^{7}$ s

But R = λN = $\frac{0.693}{T_{1/2}}.N$

Therefore N = $\frac{RT_{1/2}}{0.693} = \frac{29.6\times10^7\times5.3\times3.16\times10^7}{0.693}$ = 7.15×10^{16} atoms

At 60g of cobalt contains 6.023×10^{23} atoms, so the amount necessary to obtain a source of the required strength = $\frac{60\times7.15\times10^{16}}{6.023\times10^{23}} = 7.123\times10^{-6}$g

26. The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Sol. Here $T_{1/2}$ = 28 yrs = $28\times3.154\times10^{7}$s, m = 15mg = 0.015g, M = 90

Number of atoms in 0.015g samples of $^{90}_{38}Sr$,

$N = \frac{m}{M} \times Avogadro's number = \frac{0.015 \times 6.023 \times 10^{23} atoms}{90}$

Activity of the sample,

$R = \lambda N = \frac{0.693}{T_{1/3}}.N = \frac{0.693 \times 0.015 \times 6.023 \times 10^{23}}{28 \times 3.154 \times 10^{7} \times 90}$

$= 7.877 \times 10^{10}$ disintegrations/sec $= 7.877 \times 10^{10}$Bq $= \frac{7.877 \times 10^{10}}{3.7 \times 10^{10}} Ci = 2.13 Ci$

27. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Sol. As $R = R_0 A^{1/3}$, where $R_0 = 1.1 \times 10^{-15}$ m

Therefore, $\frac{R(107_{Au})}{R(107_{Ag})} = \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.23$

Since the nuclear mass density is independent of the size of the nucleus, so $\frac{\rho_{nu}(Au)}{\rho_{nu}(Ag)}=1$

28. How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^2_1H + ^2_1H \longrightarrow ^3_1He + n + 3.27 \text{ MeV}$$

Sol. The given fusion reaction is:

$$^2_1H + ^2_1H \longrightarrow ^3_2He + n + 3.27 \text{ MeV}$$

Amount of deuterium, m = 2 kg

1 mole, i.e., 2 g of deuterium contains 6.023×10^{23} atoms.

$\therefore$2.0 kg of deuterium contains $= \frac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released. Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$= 1.576 \times 10^{14} \text{ J}$$

Power of the electric lamp, P = 100 W = 100 J/s

Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$\frac{1.576\times10^{14}}{100}$ s

$\frac{1.576\times10^{14}}{100\times60\times60\times24\times365}\approx4.9\times10^{4}$ years

29. Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Sol. Given, the radius of deuteron nucleus is 2 fm.Let the distance between the centres of two deuterons be d.

Formula for the distance d between the two deuterons is, d=radius of 1 deuteron +radius of 2 deuteron

Substitute the values in the above equation.

$d=2\times10^{-15}+2\times10^{-15}=4\times10^{-15}$ m

Charge on a deuteron nucleus is $e=1.6\times10^{-19}$ C

Potential energy of the two-deuteron system is given by,

$V= e^2\ 4\pi\ \varepsilon_0 d$

Here, ε_0 is the permittivity of free space..

Substituting the values in the above equation, we get:

$V= e^2\ 4\pi\ \varepsilon_0\ d = (1.6\times10^{-19})\ 2\ 4\times\pi\times8.85\times10^{-12}\times4\times10^{-15}$ =360 keV

Thus, the height of the potential barrier of the two-deuteron system is 360 keV.

30. A source contains two phosphorous radio nuclides $^{32}_{15}P$ ($T_{1/2}$ = 14.3d) and $^{33}_{15}P$ ($T_{1/2}$ = 25.3d). Initially, 10% of the decays come from $^{33}_{15}P$. How long one must wait until 90% do so?

Sol. Let initially there be N_1 atoms of $^{32}_{15}P$ and N_2 atoms of $^{33}_{15}P$ and let their decay constants be λ_1 and λ_2 respectively

Since initially the activity of $^{33}_{15}P$ is 1/9 times that of $^{32}_{15}P$ we have

$$N_1\lambda_1=\frac{N_2\lambda_2}{9} \qquad \text{(i)}$$

Let after time t the activity of $^{33}_{15}P$ be 9 times that of $^{32}_{15}P$

$$N_1\lambda_1e^{-\lambda_1 t}=9N_2\lambda_2e^{-\lambda_2 t} \qquad \text{(ii)}$$

Dividing equation (ii) by (i) and taking the natural log of both sides we get

$$-\lambda_1 t = ln81 - \lambda_2 t$$
$$t = \frac{ln81}{\lambda_2 - \lambda_1}$$
where $\lambda_2 = 0.048/day$ and $\lambda_1 = 0.027/day$
t comes out to be 208.5 days

31. Calculate and compare the energy released by a) fusion of 1.0 kg of hydrogen deep within Sun and b) the fission of 1.0 kg of ^{235}U in a fission reactor.

Sol. (a) Amount of hydrogen, *m* = 1 kg = 1000 g

1 mole, i.e., 1 g of hydrogen ($^{1}_{1}H$) contains 6.023 × 10^{23} atoms.

∴1000 g of $^{1}_{1}H$ contains 6.023 × 10^{23} × 1000 atoms.

Within the sun, four $^{1}_{1}H$ nuclei combine and form one $^{4}_{2}He$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg $^{1}_{1}H$ is:

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^3}{4}$$
$$= 39.1495 \times 10^{26} \text{ MeV}$$

(b) Amount of $^{235}_{92}U$ = 1 kg = 1000 g

1 mole, i.e., 235 g of $^{235}_{92}U$ contains 6.023 × 10^{23} atoms.

∴1000 g of $^{235}_{92}U$ contains $\frac{6.023 \times 10^{23} \times 1000}{235}$ atoms

It is known that the amount of energy released in the fission of one atom of $^{235}_{92}U$ is 200 MeV.

Hence, energy released from the fission of 1 kg of $^{235}_{92}U$ is:

$$E_2 = \frac{6 \times 10^{23} \times 1000 \times 200}{235}$$
$$= 5.106 \times 10^{26} \text{ MeV}$$

$$\therefore \frac{E_1}{E_1} = \frac{39.1495 \times 10^{26}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

32. Explain the source of energy in the sun.

Sol. Solar energy is created by nuclear fusion that takes place in the sun. Fusion occurs when protons of hydrogen atoms violently collide in the sun's core and fuse to create a helium atom. This process, known as a PP (proton-proton) chain reaction, emits an enormous amount of energy.

33. What is nuclear holocaust?

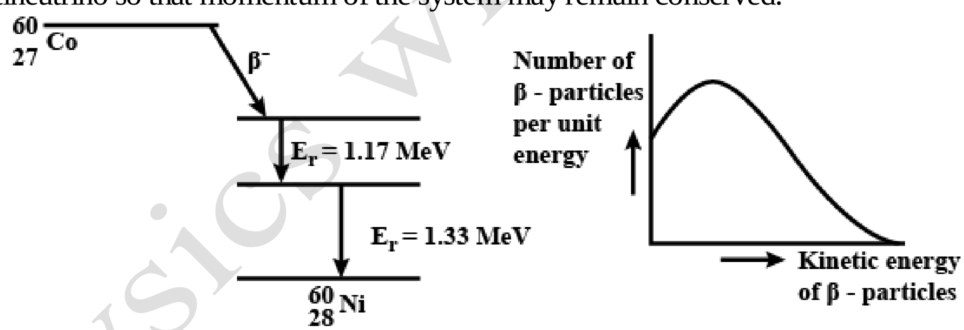
Sol. A nuclear holocaust, also known as a nuclear apocalypse, nuclear Armageddon or atomic holocaust, is a theoretical scenario where the mass detonation of nuclear weapons causes globally widespread destruction and radioactive fallout. Such a scenario envisages large parts of the Earth becoming uninhabitable due to the effects of nuclear warfare, potentially causing the collapse of civilization and, in the worst case, extinction of humanity and/or termination of life on Earth.

(3 Marks Questions)

34. (a) Draw the energy level diagram showing the emission of β-particles followed by γ-rays by a $^{60}_{27}Co$ nucleus.

(b) Plot the distribution of kinetic energy of β-particles and state why the energy spectrum is continuous?

Sol. The energy spectrum of b-particles is continuous because an antineutrino is simultaneously emitted in β-decay: the total energy released in b-decay is shared by b-particle and the antineutrino so that momentum of the system may remain conserved.



(a) Energy level diagram (b) Energy distribution of β - particles

35. Explain with the help of a nuclear reaction in each of the following cases, how the neutron to proton ratio changes during (i) alpha decay (ii) beta decay?

Sol. During alpha decay, the number of protons and neutrons of the daughter nucleus decreases by two. But during beta decay, a neutron is converted into a proton and thus the atomic number increases. That is, the number of protons increases.

Alpha decay occurs when the nucleus of an atom spontaneously ejects an alpha particle. The alpha particle is the same as a helium nucleus with 2 protons and 2 neutrons. This means the number of protons in the nucleus is reduced by 2 and the total number of nucleons is reduced by 4.

$^{241}Am_{95} \rightarrow {}^{Z}X_{A} + {}^{4}He_{2}$
A = number of protons = 95 - 2 = 93
X = the element with atomic number = 93
According to the periodic table, X = neptunium or Np.
β^- decay occurs when a neutron converts into a proton and ejects an energetic electron called the beta particle. This means the the number of neutrons, N, is reduced by 1 and the number of protons, A, is increased by 1 on the daughter atom.
$^{138}I_{53} \rightarrow {}^{Z}X_{A} + {}^{0}e_{-1}$
A = number of protons = 53 + 1 = 54
X = the element with atomic number = 54
According to the periodic table , X = xenon or Xe
The mass number, A, remains unchanged because the loss of one neutron is offset by the gain of a proton.

36. Define the terms half life period and decay constant of a radioactive substance. Write their SI units. Establish the relationship between the two.

Sol. The time required for half of the original population of radioactive atoms to decay is called the half-life. The relationship between the half-life, $T_{1/2}$, and the decay constant is given by $T_{1/2} = 0.693/\lambda$.

37. Which of the following radiations, alpha rays, beta rays, gamma rays (i) are similar to X-rays (ii) are easily absorbed by matter? (iii) travel with greatest speed? (iv) are similar in nature to cathode rays?

Sol. (i) gamma rays
(ii) alpha rays
(iii) gamma rays
(iv) beta rays

38. (a) Show that the decay rate 'R' of a sample of a radionuclide is related to the number of radioactive nuclei 'N' at the same instant by the expression $R = \lambda N$.
(b) The half life of $^{238}_{92}U$ against α-decay is 1.5×10^{17}s. What is the activity of a sample of $^{238}_{92}U$ have 25×10^{20} atoms?

Sol. (a) According to radioactive decay law. $N = N_0\, d^{-\lambda t}$
Rate of decay, $R = -\frac{dN}{dt} = \frac{-dN}{dt}\left(N_0 e^{-\lambda t}\right) = \lambda N_0 e^{-\lambda t} = \lambda N.$
(b) Here, $T = 1.5 \times 10^{17}$s, R=?
$N = 2.5 \times 10^{21}$
$R = \lambda N = \frac{0.693N}{T} = \frac{0.693\times2.5\times10^{21}}{1.5\times10^{17}}$ = 11550 disintegrations/sec.

39. A radioactive sample contains 2.2 mg of pure ${}^{11}_{6}C$ which has half life period of 1224 seconds. Calculate: (i) the number of atoms present initially. (ii) the activity when 5mg of the sample life will be left

Sol. Here, number atoms present in 11 gram of sample = $6.023 \times 10^{23} = 6.023 \times 10^{23}$

$\therefore$ No. of atoms present in 2.2 mg of sample initially

$N_0 = 6.023 \times 10^{23} \times 2.2 \times 10^{-3}/11 = 1.2 \times 10^{20}$

No. of atoms present in 5μg, $N = 6.023 \times 10^{23} \times 5 \times 10^{-6}/11 = 2.74 \times 10^{17}$

Activity of the sample, $A = \lambda N = 0.693/T.\ N = 0.693 \times 2.74 \times 10^{17}/1224$

$A = 1.55 \times 10^{14}$ disintegration/sec.

40. The half life of ${}^{238}_{92}U$ against α-decay is 4.5×10^{9} years. Calculate the activity of 1g sample of ${}^{238}_{92}U$.

Sol. $T_{1/2} = 4.5 \times 10^{9}$ years $= 4.5 \times 10^{9} \times 365 \times 24 \times 60 \times 60$

$T^{1/2} = 1.419 \times 10^{17}$nsec.

Number of atoms in lg of U-238

$$N = \frac{N_A \times m}{A}$$

$$= \frac{6.023 \times 10^{23} \times 1}{238}$$

$$= 0.0253 \times 10^{23}$$

Decay rate or activity $A = \lambda N$ but

$$T_{1/2} = \frac{0.693}{\lambda},$$

$$\text{then } \lambda = \frac{0.693}{T_{1/2}}$$

then,

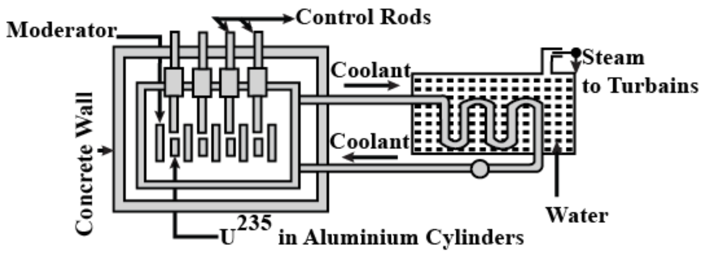
$$A = \frac{0.693}{T_{1/2}} N$$

$$= \frac{0.693 \times 2.53 \times 10^{21}}{1.419 \times 10^{17}}$$

$A = 1.235 \times 104$ Bq

41. With the help of a labeled diagram, describe the construction and working of a nuclear reactor.

Sol.



Principle:

A nuclear reactor works on the principles of generating great amounts of energy by achieving controlled chain reaction of Uranium ^{238}U enriched with ^{235}U. This is made possible by:

1. Slowing down the fission neutrons to thermal neutrons using a moderator. Thermal neutrons initiate the fission of ^{235}U.

2. By using control rods of a material which can absorb neutrons. This is important to control the rate of reaction and maintain it so that the value of neutron multiplication factor K remains 1.

Working:

- Fuel rods are filled with Uranium. These are placed in aluminum cylinders.
- In between the fuel cylinders, the graphite moderator is placed.
- Control rods made up of Cadmium, Beryllium or Boron are placed in the holes of the block of graphite
- When neutrons undergo fission, fast neutrons are released. On passing through the surrounding graphite moderators, these fast neutrons lose their energy and become thermal neutrons.
- ^{235}U captures these thermal neutrons.
- Coolants such as water are converted to steam through this heat energy.
- This steam is then used to rotate a steam turbine which drives a generator to produce electricity

42. Explain the term 'chain reaction'. What are the functions of moderator and control rods in a nuclear reactor?

Sol. A chain reaction refers to a process in which neutrons released in fission produce an additional fission in at least one further nucleus. This nucleus in turn produces neutrons, and the process repeats. The process may be controlled (nuclear power) or uncontrolled (nuclear weapons).The moderator helps slow down the neutrons produced by fission to sustain the chain reaction. Control rods can then be inserted into the reactor core to reduce the reaction rate or withdrawn to increase it. The heat created by fission turns the water into steam, which spins a turbine to produce carbon-free electricity.

43. When a deuteron of mass 2.0141u and negligible kinetic energy is absorbed by a lithium

($^{6}_{3}Li$) nucleus of mass 6.0155u, the compound nucleus disintegrates spontaneously into two alphas particles, each of mass 4.0026u. Calculate the energy in joules carried by each alpha particle. (1u = 1.66×10^{-27} kg).

44. A neutron is absorbed by a $^{6}_{3}Li$ nucleus with the subsequent emission of an alpha particle. (i) Write the corresponding nuclear reaction. (ii) Calculate the energy released, in MeV, in this reaction. Given mass ($^{6}_{3}Li$) = 6.01512126u; mass (neutron) = 1.0086654u; mass (alpha particle) = 4.0026044u and mass (triton) = 3.0100000u. Take 1u = 931 MeV/c^2.

Sol. (i) Nuclear reaction is given as

$^{6}_{3}Li + ^{1}_{0}n \rightarrow ^{4}_{2}He + ^{3}_{1}H + Q$

(ii) Mass defect (Δm) = [m($^{6}_{3}Li$) + m($^{1}_{0}n$) – m($^{4}_{2}He$) – m($^{3}_{1}H$)

= [6.015126 + 1.0086654 – 4.0026044 – 3.01]

= [7.0237914u – 7.0126044u)] = 0.0111870u

Therefore energy released = 0.0111870 ×931 = 10.415 MeV.

45. Calculate the amount of energy released during the α-decay of $^{238}_{92}U \rightarrow ^{234}_{90}Th + ^{4}_{2}He$

Given: atomic mass of $^{238}_{92}U$ = 238.05079u; atomic mass of $^{234}_{90}Th$ = 234.04363u; atomic mass of $^{4}_{2}He$ = 4.00260u; 1u = 931.5 MeV/c^2. Is this decay spontaneous? Give reason.

Sol. The process is

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + \underset{(\alpha-\text{particle})}{^{4}_{2}He} + Q$$

The energy released Q = (M_U – M_{TH} — M_{He}) c^2

= (238.05079 – 234.04363 – 4.00260)u × c^2 = (0.00456 u) × c^2

= 0.00456 × (931.5 mev/c^2)c^2 = 4.25 MeV Y

es, the decay is spontaneous (since Q is positive)

46. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive $^{14}_{6}C$ present with the stable carbon isotope $^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of $^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is

the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Sol. Given normal activity, R_0 = 15 decays min^{-1}

Present activity, R = 9 decays min^{-1}, $T_{1/2}$ = 5730 years

Since activity is proportional to the number of radioactive atoms, therefore

$\frac{R}{R_0} = \frac{N}{N_0} = \frac{N_0 e^{-M}}{N_0} = e^{-M}$ or $\frac{9}{15} = e^{-\lambda T}$ or $e^{\lambda t} = \frac{15}{9}$

Taking natural logarithms, $\log_e e^{\lambda t} = \log_e \frac{15}{9}$

Or $\lambda t \log e = 2.303 \log_{10}\frac{5}{3} = 2.303 \times 0.2215$

Or $t = 0.5109/\lambda$ [since $\log_e e = 1$]

As $t_{1/2} = 0.693/\lambda$

Therefore $t = \frac{0.5109}{0.693/T_{1/2}} = \frac{0.5109}{0.639} \times T_{1/2} = \frac{0.5109 \times 5730}{0.693}$ years $= 4224$ years

47. The fission properties of ${}^{239}_{94}Pu$ are very similar to those of ${}^{235}_{92}U$.

The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}Pu$ undergo fission?

Sol. Number of atoms present in 239g of ${}^{239}_{94}Pu = 6.023 \times 10^{23}$

Therefore number of atoms present in 1kg or 1000g of ${}^{239}_{94}Pu$

$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$

Energy released per fission = 180MeV

Total energy released = $2.52 \times 10^{24} \times 180$ MeV = 4.54×10^{26} MeV

48. A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}^{235}_{92}U$ and that this nuclide is consumed only by the fission process.

Sol. Power of the reactor, P = 1000MW = 10^9 W; Time of power generation, T = 5y = 5 × 365 × 24 × 60 × 60s Total energy generated in 5y = Pt = 10^9 × 5 × 365 × 24 × 60 × 60s; Energy generated in each fission of ${}^{235}_{92}U$ = 200MeV = $200 \times 1.6 \times 10^{-13}$J; Number of atoms in 235g of ${}^{235}_{92}U = 6 \times 10^{23}$

Number of atoms in 1 g of ${}^{235}_{92}U = \frac{6 \times 10^{23}}{235}$

Energy generated per gram of ${}^{235}_{92}U = \frac{200 \times 1.6 \times 10^{-13} \times 6 \times 10^{23}}{235} J\,g^{-1}$

The amount of ${}^{235}_{92}U$ consumed in 5y = $\frac{\text{Total enrgy generated}}{\text{Energy generated per gram}}$

$= \frac{10^9 \times 5 \times 365 \times 24 \times 60 \times 60 \times 235}{200 \times 1.6 \times 10^{-13} \times 6 \times 6 \times 10^{23}}$ g = 1929937.5g = 1930kg

This amount is half the fuel taken initially.

Therefore mass of $^{235}_{92}U$ taken initially = 3860kg.

49. For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the K−shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + ^A_Z X \rightarrow ^A_{Z-1} Y + \nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice−versa

Sol. Consider the two competing processes:

Positron emission: $^A_Z X \rightarrow {}_{Z-1}^{\ A}Y + e^+ + v + Q_1$

Electron emission: $e^- + {}^A_Z X \rightarrow {}_{Z-1}^{\ A} + v + Q_2$

The energy changes is the two processes are

$Q_1 = [m_N(^A_Z X) – m_N(_{Z-1}^{\ A}Y) - m_c]c^2$

$=[m_N(^A_Z X)- Zm_e =- m\ (_{Z-1}^{\ A}Y) + (Z – 1)m_e = m_e]c^2$

$= [m(^A_Z X) - m\ (_{Z-1}^{\ A}Y)- 2m_e]c^2$

$Q_2 = [m_N(^A_Z X) + m_e + m_N(_{Z-1}^{\ A}Y)]c^2 = [m(^A_Z X) – m\ (_{Z-1}^{\ A}Y)]c^2$

This means $Q_1>0$ implies $Q_2>0$ does not necessarily imply $Q_1>0$. Thus if positron emission is energetically allowed, electron rupture is necessarily allowed but not vice versa.

50. Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}Ra \longrightarrow ^{209}_{82}Pb + ^{14}_{6}C$$

$$^{223}_{88}Ra \longrightarrow ^{219}_{86}Rn + ^{4}_{2}He$$

Calculate the *Q*-values for these decays and determine that both are energetically allowed.

Sol. The Q value for the first decay process is given by $Q = m(^{223}_{88}Ra) - m(^{209}_{82}Pb) - m(^{14}_{6}C)$

= [122.01850 – 208.981107 – 14.000324] amu × c^2 = 0.034109 × 931.5 MeV

= 31.85 MeV

For the second process, $Q = m(^{223}_{88}Ra) - m(^{219}_{86}Rn) - m(^{4}_{2}He)$

= [223.01850 – 219.00948 – 4.00260] amu × c^2 = 0.00642×931.5

= 5.98MeV

As the Q value is positive in both cases, so both decay processes are energetically possible.

51. Consider the fission of $^{238}_{92}U$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments,

are ${}^{140}_{58}Ce$ and ${}^{99}_{44}Ru$. Calculate Q for this fission process. The relevant atomic and particle masses are

$m\left({}^{238}_{92}U\right)$ =238.05079 u

$m\left({}^{140}_{58}Ce\right)$ =139.90543 u

$m\left({}^{99}_{44}Ru\right)$ = 98.90594 u

Sol. The fission may be represented as

$${}^{238}_{92}U + {}^{1}_{1}n \rightarrow {}^{140}_{58}Ce + {}^{99}_{44}Ru + 10\,{}^{0}_{-1}e + Q$$

The Q value for the process is

$$Q = [m({}^{238}_{92}U) + m_n - m_N({}^{140}_{88}Ce) - m_N({}^{99}_{44}Ru - 10m_e)]c^2$$

In terms of atomic masses, we can write

$$Q = [\{m({}^{238}_{92}U) - 92m_e\} + m_n - \{m({}^{140}_{88}Ce) - 58m_e\} - \{m({}^{99}_{44}Ru) - 44m_e\}10m_e]c^2$$

$$= [m({}^{238}_{92}U) + m_n - m({}^{140}_{88}Ce) - m({}^{99}_{44}Ru)]c^2$$

= [238.05079+1.00867 – 139.90543 – 98.90594]amu × c^2

= (239.05946 – 238.81137)amu × 931.5 MeV/amu

= 0.24809 × 931.5 = 231.09 MeV = 231.1 MeV.

(5 Marks Questions)

52. Explain what is meant by radioactive decay.

A radioactive nucleus is represented by the symbol ${}_bZ^a$. How is the new nucleus represented after the emission of (i) an alpha particle (ii) a beta particle (ii) a gamma particle.

The activity of a source undergoing a single type of decays is I_0 at time t = 0, obtain an expression in terms of the half life $T_{1/2}$ for the activity I at any subsequent time t.

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53. (a) Derive the relation connecting decay constant and half life of a radioactive sample.

(b) How many disintegrations per second will occur in one gram of ${}^{238}_{92}U$, if its half life against alpha decay is 1.42×10^{17}s?

Sol. (a) The time required for half of the original population of radioactive atoms to decay is called the half-life. The relationship between the half-life, $T_{1/2}$, and the decay constant is given by $T_{1/2} = 0.693/\lambda$.

(b) Given half life period (T) = $0.693/\lambda = 1.42\times10^{17}$s

Therefore, $\lambda = 0.693/(1.42\times10^{17}) = 4.88\times10^{-18}$

Avogadro's number = 6.023×10^{23}

n is the no. of atoms present in 1 g of ${}^{238}_{92}U$ = N/A = $(0.623\times10^{23})/238 = 25.30\times10^{20}$

No. of disintegrations is given by, $dN/dt = \lambda N = 4.88\times10^{-8}\times25.30\times10^{20}$

= 1.2346×10^{4} disintegrations per sec.

54. Write nuclear reaction equations for

(i) α-decay of ${}^{226}_{88}Ra$ **(ii)** α-decay of ${}^{242}_{94}Pu$

(iii) β^{-}-decay of ${}^{32}_{15}P$ **(iv)** β^{-}-decay of ${}^{210}_{83}Bi$

(v) β^{+}-decay of ${}^{11}_{8}C$ **(vi)** β^{+}-decay of ${}^{97}_{43}Tc$

(vii) Electron capture of ${}^{120}_{54}Xe$

Sol. (i) ${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He$

(ii) ${}^{242}_{94}Pu \rightarrow {}^{238}_{92}U + {}^{4}_{2}He$

(iii) ${}^{32}_{15}Pu \rightarrow {}^{32}_{16}S + e^{-} + \bar{v}$

(iv) ${}^{210}_{83}B \rightarrow {}^{210}_{84}PO + e^{-} + \bar{v}$

(v) ${}^{11}_{8}C \rightarrow {}^{11}_{8}B + e^{+} + v$

(vi) ${}^{97}_{43}Tc \rightarrow {}^{97}_{42}Mo + e^{+} + v$

(vii) ${}^{120}_{54}Xe + e^{+} \rightarrow {}^{120}_{53}I + v$

55. Explain laws of Radioactivity and derive the relation: $N = N_0 e^{-\lambda t}$.

Sol. The elements or isotopes which emit radiation and undergo the phenomena of radioactivity known as radioactive elements. These elements undergo three types of radioactive decay such as: Alpha decay, Beta decay and Gamma decay. Radioactive elements have unstable nuclei. This process is a random process at the level of single atoms. The SI unit of the radioactivity is becquerel (Bq). One Bq is defined as one disintegration per second.

The law of radioactive decay states that for a particular time, the rate of radioactive disintegration is directly proportional to the number of nuclei of the elements present at that time. This law can be expressed as follows:

$\frac{dN}{dt} \propto N$ or $\frac{dN}{dt} = -\lambda N$, where λ is the proportionality constant known as radioactive decay constant.

If $N_0$is the number of nuclei presents at the initial time and NN is the number of nuclei presents at the time t then,

$$\int_{N_0}^{N} \frac{dN}{N} = -\int_0^t \lambda dt = -\lambda \int_0^t dt$$

$\log_e N - \log_e N_0 = -\lambda t$

$\log_e (N/N_0) = -\lambda t$

$N/N_0 = e^{-\lambda t}$

$N = N_0 e^{-\lambda t}$, hence the expression for the radioactive decay is derived.

56. Explain the working of Nuclear reactor with the help of diagram and explain the following terms: (a) Control Rods (b) Moderator (c) Critical Size or ratio

Sol.

Principle:

A nuclear reactor works on the principles of generating great amounts of energy by achieving controlled chain reaction of Uranium 238U238U enriched with 235U235U. This is made possible by:

1. Slowing down the fission neutrons to thermal neutrons using a moderator. Thermal neutrons initiate the fission of 235U235U.
2. By using control rods of a material which can absorb neutrons. This is important to control the rate of reaction and maintain it so that the value of neutron multiplication factor K remains 1.

Working:

- Fuel rods are filled with Uranium. These are placed in aluminum cylinders.
- In between the fuel cylinders, the graphite moderator is placed.
- Control rods made up of Cadmium, Beryllium or Boron are placed in the holes of the block of graphite
- When neutrons undergo fission, fast neutrons are released. On passing through the surrounding graphite moderators, these fast neutrons lose their energy and become thermal neutrons.
- 235U235U captures these thermal neutrons.
- Coolants such as water are converted to steam through this heat energy.
- This steam is then used to rotate a steam turbine which drives a generator to produce electricity.

Diagram:

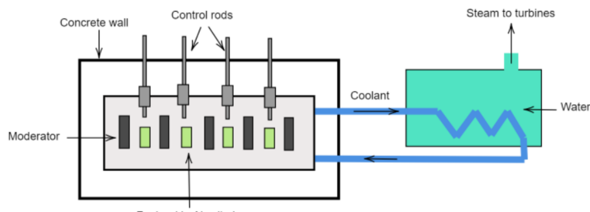


Fig 1: Diagram of a nuclear reactor

Additional information: In a Nuclear reactor station use of cold water sink after passage from turbine is necessary not only to maintain continuous flow of water into the channel but to absorb enormous heat release as it can completely change the chemical behavior of gases above and around the reactor in a radius of 10-15km.

Control rods are used in nuclear reactors to control the rate of fission of the nuclear fuel – uranium or plutonium. Their compositions include chemical elements such as boron, cadmium, silver, hafnium, or indium, that are capable of absorbing many neutrons without themselves decaying. These elements have different neutron capture cross sections for neutrons of various energies. Boiling water reactors (BWR), pressurized water reactors (PWR), and heavy-water reactors (HWR) operate with thermal neutrons, while breeder reactors operate with fast neutrons. Each reactor design can use different control rod materials based on the energy spectrum of its neutrons. Control rods have been used in nuclear aircraft engines like Project Pluto as a method of control.

Moderator is a substance that slows down the speed of neutrons. In traditional nuclear reactors, the moderator is same thing as that of a coolant like water.

A fissionable material of a particular size for which a chain reaction takes place at a constant rate is called its critical size.

C. CASE STUDY

1. **Radiocarbon Dating:** Our atmosphere contains a large number of stable isotopes. When cosmic rays strike these isotopes, a no. of radioisotopes are produced. One of these radi isotopes is carbon 14 (symbol ${}^{14}_{6}C$) which is produced by the bombardment of atmospheric nitrogen with high energy neutrons according to the equation: ${}^{14}_{7}N + {}^{1}_{0}e \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$

Radiocarbon is unstable and decays no nitrogen with a half life of 5600 years. The carbon 14 is incorporated into atmospheric carbon dioxide molecules which are taken in by plants when they breath in carbon dioxide. Animals which eat the plants also take in carbon 14. Ultimately the concentration of ${}^{14}C$ in all living organisms reaches an equilibrium value because ${}^{14}C$ decays in the organism which also takes in atmospheric ${}^{14}C$. When an organism dies, it stops taking ${}^{14}C$ from the atmosphere and the concentration of ${}^{14}C$ present in the organism decreases with time. By measuring the ratio of the concentration for ^{14}C to ^{12}C in any ancient organism, say a tree, one can determine the date when the organism died.

(i) A capsule contains 8g of $^{14}_{6}C$ whose half life is 5600 years, After 16800 years, the amount of $^{14}_{6}C$ left in the capsule will be

(a) 4g (b) 2g (c) 8/3g (d) 1g

Sol (d)

Number of half lives, n = $\frac{16800}{5600}$ = 3. Therefore the amount of C 15 left = $\frac{8g}{(2)^n} = \frac{8g}{2^3}$ = 1g.

The correct choice is (d)

(ii) Radiocarbon is produced in the atmosphere as a result of

(a) collision between fast neutrons and nitrogen nuclei.

(b) the action of cosmic rays on atmospheric oxygen

(c) the action of X rays on carbon

(d) lightning discharge in atmosphere

Sol. (a)

(iii) Select the only correct statement

(a) The amount of ^{14}C in a living plant keeps increasing indefinitely

(b) the amount of ^{14}C in a living plant keeps decreasing indefinitely

(c) the amount of ^{14}C in a living plant reaches an equilibrium value at some stage of its lifetime

(d) the amount of ^{14}C in a dead plant reaches an equilibrium value and stops decreasing after a certain length of time

Sol. (c)

(iv) The age of ancient tree is determined by measuring the

(a) Amount of ^{14}C still present in a given sample of wood

(b) Amount of ^{12}C still present in the given sample

(c) Total amount of ^{14}C and ^{12}C in the given sample

(d) Ratio of the amount of ^{14}C in that of ^{12}C still present in the given sample

Sol. (d)

(v) Choose the only incorrect statement. In radioactive decay of an element

(a) α particles may be emitted (b) β particles may be emitted

(c) γ rays may be emitted (d) the nucleus does not undergo any change

Sol (d)

2. **Alpha, Beta and Gamma Decay:** The process of emission of an alpha particle from a nucleus is called alpha decay. An alpha particle is a helium nucleus having 2 protons and 2 neutrons, Hence when a nucleus undergoes alpha decay, it loses 2 protons and 2 neutrons. As a result, the nucleus of a new element is formed. It is found that the total mass of the decay products is less than th mass of the original nucleus. This mass defect is equivalent to energy which is released in the process. The process of emission of an electron from a nucleus is called beta decay. In this process a neutron decays into a

proton with the emission of an electrons and an uncharged particles called antineutrino. When a nucleus undergoes beta decay, the mass number remains uncharged but the atomic number is increased by 1. This process of emission of gamma rays from a nucleus is called gamma decay. Gamma rays are high frequency electromagnetic radiations (i.e. photons) which do not carry any charge. Hence in this process t\no new element is formed.

(i) $^{238}_{92}U$ nucleus decays successively to form $^{234}_{90}Th$, $^{234}_{91}Pa$, $^{234}_{92}U$ and $^{230}_{90}Th$. The sequence of radiations emitted is

(a) a, b, a (b) b, a , b (c) a, b, b (d) b, b, a

Sol. (c)

In alpha decay, the mass number decreases by 4 and atomic number decreases by 2. In beta decay, the mass number remains unchanged but the atomic number increases by 1. Hence correct choice is (c).

(ii) The sequence of a step wise decays of a radioactive nucleus D is as follows:

$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\alpha} D_4$

If the mass number and atomic number of D_2 are 176 and 71 respectively, the corresponding values of nucleus D are

(a) 180, 72 (b) 176, 72 (c) 168, 67 (d) 172, 69

Sol. (a)

(iii) In Qs (ii) what are the mass and atomic number of nucleus D_4

(a) 172, 69 (b) 176, 71 (c) 180, 72 (d) 168, 67

Sol. (d)

(iv) Which of the following statements is not true about beta decay?

(a) The mass number of the nucleus remains unchanged

(b) The atomic number is increased by 1

(c) A new atom is formed

(d) All electrons emitted in beta decay have the same energy

Sol. (d)

Statement (d) is not true. In beta decay, an uncharged particle called antineutrino is emitted along with an electron. The reaction energy is shared by both these particles. When the energy of antineutrino is maximum, the energy of the electron is minimum and vice versa. Hence all electrons emitted in beta decay do not have the same energy.

(v) A neutron is captured by $^{10}_{5}B$ nucleus with the subsequent emission of an alpha particle. The mass and atomic numbers of the resulting nucleus are

(a) 7, 3 (b) 11, 5 (c) 8, 3 (d) 9, 3

Sol. (a)

The nuclear reaction can be written s $^{10}_{5}B + ^{1}_{0}n \rightarrow ^{11}_{5}X \rightarrow ^{7}_{3}Y + ^{4}_{2}He$.

Hence correct choice is (a).

D. ASSERTION REASON TYPE QUESTIONS:

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true.

1. Assertion: Isotopes of an element can be separated by using a mass spectrometer
Reason: Separation of isotopes is possible because of difference in electron number of isotopes.

Ans. (c) Assertion is true but reason is false
Number of electrons in isotopes of an element is same. If two atoms have the same number of protons but different number of neutrons, their atomic (proton) number are equal but not their mass (nucleon) numbers.
In mass spectrograph, the deflecting electric and magnetic fields were arranged so that all particles of the same mass, irrespective of their speed, were brought to a line focus. When ions of different mass war present a series of lines, ie., a mass spectrum, obtained on a photographic film. The relative intensities of the lines enabled an estimate to be made of relative amount of isotopes.

2. Assertion: For the scattering of α-particles at a large angles, only the nucleus of the atom is responsible.
Reason: Nucleus is very heavy in comparison to electrons.

Ans. (a) Both assertion and reason and reason is the correct explanation of assertion.
We know that an electron is very light particle as compared to an α-particle. Hence electron cannot scatter the α-particle at large angles, according to law of conservation of momentum. On the other hand, mass of nucleus is comparable with the mass of α-particle, hence only ht nucleus of atom is responsible for scattering of α-particle.

3. Assertion: Penetrating power of X rays increases with increasing wavelength.
Reason: The penetrating power of X rays increases with the frequency of X rays

Ans. (e) Assertion is false but reason is true.
Higher the wavelength of X ray, lesser is the frequency or penetrating power.

4. Assertion: It is necessary to keep high vacuum in Coolidge tube to produce X rays.
Reason: High vacuum is kept in Coolidge tube so that the electrons emitting from the filament of the tube may not lose their energy in colliding with the atoms of the gas in the tube.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.
The high vacuum Coolidge tube is used to generate X rays.

5. Assertion: The mass of a nucleus can be either less than or more than the sum of the masses of nucleons present in it.
Reason: The whole mass of the atom is concentrated in the nucleus.

Ans. (e) Assertion is false but reason is true.
The mass of a nucleus is always less than the sum of the masses of the nucleons present in it. When nucleons combine to form a nucleus, some energy is liberated, and this is

binding energy of the nucleus. The mass of the nucleus cannot be more than the total mass of the nucleons because then stable nucleus cannot be formed.

E. CHALLENGING PROBLEMS

1. Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) ${}^{226}_{88}Ra$ and (b) ${}^{220}_{86}Rn$.

Given $m\left({}^{226}_{88}Ra\right)$ = 226.02540 u, $m\left({}^{222}_{86}Rn\right)$ = 222.01750 u,

$m\left({}^{220}_{86}Rn\right)$ = 220.01137 u, $m\left({}^{216}_{84}Po\right)$ = 216.00189 u.

Sol. (a) ${}^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}He + Q$

Q = $[m({}^{226}_{88}Ra) - m({}^{226}_{86}Rn) - m({}^{4}_{2}He)]c^2$

= [226.02540 – 222.01750 – 4.00260] ×931.5 MeV = 0.0053×931.5 = 4.937 MeV

$K_\alpha = \frac{A-4}{A}Q = \frac{226-4}{226} \times 4.937 = 4.85MeV$

(b) ${}^{220}_{86}Rn \rightarrow {}^{216}_{84}Po + {}^{4}_{2}He + Q$

Q = $[m({}^{220}_{86}Rn) - m({}^{216}_{84}Po) - m({}^{4}_{2}He)]c^2$

= [220.1137 – 216.00189 – 4.00260]×931.5MeV = 0.00688×931.5 = 6.41MeV

$K_\alpha = \frac{A-4}{A}Q = \frac{220-4}{220} \times 6.41 = 6.29MeV$

2. The radionuclide ${}^{11}C$ decays according to

$${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu : \quad T_{1/2} = 20.3 \text{ min}$$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$$m\left({}^{11}_{6}C\right) = 11.011434 \text{ u and } m\left({}^{11}_{6}B\right) = 11.009305 \text{ u,}$$

calculate *Q* and compare it with the maximum energy of the positron emitted

Sol. ${}^{11}_{6}C \rightarrow {}^{11}_{5}B + e^{+} + \nu + Q$ where Q is the energy released in the decay process. It is given by Q = $[m_N({}^{11}_{6}C) - m_N({}^{4}_{5}B) - m_e]c^2$

To express Q value in terms of atomic masses, we have to subtract $6m_e$ from the atomic mass of carbon and $5m_e$ from the atomic mass of boron to get the corresponding nuclear masses. So we get

Q = $[m({}^{11}C) – 6m_e – m({}^{11}B) + 5m_e – m_e]c^2$ = $[m({}^{11}_{6}C)$ - $m({}^{11}_{2}B)$ - $2m_e]c^2$

= [11.011434 – 11.009305 – 2 × 000548]amu× c^2 = 0.001033amu ×931.5 $\frac{MeV}{amu}$

= 0.9622 MeV ∝ 0.96 MeV

3. The nucleus ${}^{23}_{10}Ne$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$m\left({}^{23}_{10}Ne\right)$ = 22.994466 u

= 22.989770 u.

Sol. The beta decay of ${}^{23}_{10}Ne$ may be represented as ${}^{23}_{10}Ne \rightarrow {}^{23}_{11}Na + {}^{0}_{-1}e + \bar{v} + Q$

Ignoring the rest mass of neutrino, the expression for the kinetic energy released may be written as

$Q = [m_N({}^{23}_{10}Ne) - m_N({}^{23}_{11}Na) - m_e]c^2$

$= [[m_N({}^{23}_{10}Ne) + 10m_e] = [m_N({}^{23}_{11}Na) + 11m_e]]c^2$

$= [m_N({}^{23}_{10}Ne) - m_N({}^{23}_{11}N1)]c^2$ [Since c^2 = 931.5 MeV/amu]

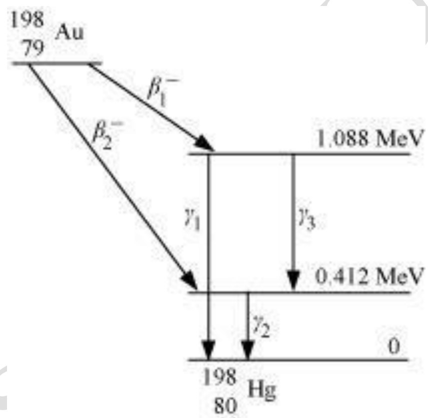
= [22.994466 – 22.989770]×931.5 MeV = 0.004696×931.5MeV = 4.374 MeV

As ${}^{23}Na$ is massive, the kinetic energy released is mainly shared by electron positron pair. When the neutrino carries no energy the electron has a maximum kinetic energy equal to 4.374 MeV.

4. Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. You are given that

m (${}^{198}Au$) = 197.968233 u

m (${}^{198}Hg$) =197.966760 u



Sol. The frequencies o g-radiation will be equal to the corresponding energy differences divided by Planck's constant h.

$v = \frac{E_2 - E_1}{h}$

Therefore $v(\gamma_1) = \frac{(1.088-0)\times 1.6\times 10^{-13}}{6.63\times 10^{-34}}$ = 2.627×10^{20}Hz

$v(\gamma_2) = \frac{(0.412-0)\times 1.6\times 10^{-13}}{6.63\times 10^{-34}}$ = 9.949×10^{19}Hz

$v(\gamma_3) = \frac{(1.088-0.412)\times 1.6\times 10^{-13}}{6.63\times 10^{-34}}$ = 1.632×10^{19}Hz

The β_1^- decay can be represented as

${}^{198}_{79}Au \rightarrow {}^{198}_{80}Hg + {}^{0}_{-1}e + Q(\beta_1^-) + Q(\gamma_1)$ where $Q(\gamma_1)$ = 1.088 MeV

Therefore maximum kinetic energy of β_1^- particle is

$K_{max}(\beta_1^-) = \left[m({}^{198}_{79}Au) - \left\{m({}^{198}_{80}Hg) + \frac{1.088}{931.5}\right\}\right] c^2$

[Neglecting the rest mass of β particle]

5. Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200MeV.

Sol. Target of power by 2020AD = 2×10^5 MW = 2×10^{11}W

Power required from nuclear power plants = 10% of 2×10^{11}W = $(10/100)\times2\times10^{11}$ = 2×10^{10}W

Therefore energy required from nuclear power plants per year = power × time

= $2\times10^{10} \times 365.25 \times 24 \times 60 \times 60$J = 6.312×10^{17}J

Energy released per fission = 200MeV

Available electrical enery per fission of ^{235}U nucleus = 25% of 200Mev

= (25/100)×200MeV = $25\times2\times1.6\times10^{-13}$J = 8×10^{-12}J [since 1MeV = 1.6×10^{-13}J]

Number of ^{235}U fissions required per year = $\frac{6.312\times10^{17}}{8\times10^{-12}} = 7.89\times10^{28}$

Required number of moles of $^{235}U = \frac{7.89\times10^{28}}{\text{Avogadro's number}} = \frac{7.89\times10^{28}}{6.023\times10^{23}} = 13.1\times10^4$

Mass of ^{235}U required = Number of moles × mass number

= $13.1\times10^4 \times 235$g = 3078.5×10^4g = 3.078×10^4kg.

6. Consider the D−T reaction (deuterium−tritium fusion)

$${}_1^2H + {}_1^3H \longrightarrow {}_2^4He + n$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m\left({}_1^2H\right) = 2.014102$ u

$m\left({}_1^3H\right) = 3.016049$ u

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event =average thermal kinetic energy available with the interacting particles = $2(3kT/2)$; k = Boltzman's constant, T = absolute temperature.)

Sol. **(a)** Take the D-T nuclear reaction: ${}_1^2H + {}_1^3H \longrightarrow {}_2^4He + n$

It is given that:

Mass of ${}_1^2H$, m_1= 2.014102 u

Mass of ${}_1^3H$, m_2 = 3.016049 u

Mass of ${}_2^4He$ m_3 = 4.002603 u

Mass of ${}_0^1n$, m_4 = 1.008665 u

Q-value of the given D-T reaction is:
$Q = [m_1 + m_2 - m_3 - m_4]\, c^2$
$= [2.014102 + 3.016049 - 4.002603 - 1.008665]\, c^2$
$= [0.018883\, c^2]$ u
But 1 u = 931.5 MeV/c^2
$\therefore Q = 0.018883 \times 931.5 = 17.59$ MeV
(b) Radius of deuterium and tritium, $r \approx 2.0$ fm $= 2 \times 10^{-15}$ m
Distance between the two nuclei at the moment when they touch each other, $d = r + r = 4 \times 10^{-15}$ m
Charge on the deuterium nucleus = e
Charge on the tritium nucleus = e
Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi \epsilon_0 (d)}$$

Where,
ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}} = 5.76 \times 10^{-14} \text{ J}$$

$$= \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} = 3.6 \times 10^5 \text{ eV} = 360 \text{ keV}$$

Hence, 5.76×10^{-14} J or 360 keV of kinetic energy (KE) is needed to overcome the Coulomb repulsion between the two nuclei.
However, it is given that:

$$\text{KE} = 2 \times \frac{3}{2} kT$$

Where,
k = Boltzmann constant $= 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
T = Temperature required for triggering the reaction

$$\therefore T = \frac{\text{KE}}{3K}$$

$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^9 \text{ K}$$

Hence, the gas must be heated to a temperature of 1.39×10^9 K to initiate the reaction.

SPACE FOR ROUGH WORK

SPACE FOR NOTES