CLASS – 11

WORKSHEET- OSCILLATIONS

A. INTRODUCTION TO PERIODIC AND OSCILLATORY MOTION

(1 Marks Questions)

1. The equation of motion of a particle is $x = A\cos(\alpha t)^2$. The motion is

(a) periodic but not oscillatory (b) periodic and oscillatory
(c) oscillatory but not periodic (d) neither periodic nor oscillatory

Sol. (c)

As the given equation is $x = A\cos(\alpha t)^2$ is a cosine function. Hence , it is an oscillatory motion.

2. Can a motion be periodic but not oscillatory?

Sol. An oscillatory motion is always periodic. A periodic motion may or may not be oscillatory. For example, the motion of planets around the Sun is always periodic but not oscillatory.

(2 Marks Questions)

3. Describe the motion of a particle acted upon by force $F = -(x - 3)^3$.

Sol. Here $F = -(x - 3)^3$

As $x = 3$, $F = 0$. Force is along negative x direction for $x > 3$ and it is along positive x direction for $x < 3$. Thus the motion of the particle is oscillatory (but not simple harmonic) above $x = 3$.

B. SIMPLE HARMONIC MOTION

(1 Marks Questions)

1. The time period of simple harmonic motion depends upon

(a) amplitude (b) energy (c) phase constant (d) mass

Ans. (d)

2. What is the (a) distance moved (b) displacement of a particle executing SHM in one vibration?

Sol. A body while executing S.H.M. completes one vibration in time equal to its period, so the body reaches its initial position after a time equal to its period. Thus the total distance traveled is 4A and displacement is zero.

3. Simple Harmonic motion is the projection of uniform motion on

(a) x-axis (b) y-axis (c) reference circle

(d) any diameter of reference circle

Sol. (d)

Simple harmonic motion is the projection of uniform circular motion on any diameter of reference circle.

4. A particle executing SHM. The phase difference between acceleration and displacement is

(a) 0 (b) $\pi/2$ (c) π (d) $1/2\ \pi$

Sol. (c)

5. Can velocity and acceleration be in the same direction in a SHM?

Sol. In an SHM the velocity and acceleration cannot be in the same direction.

6. What is phase relationship between displacement, velocity and acceleration in SHM?

Sol. In SHM, the velocity leads the displacement by a phase $\pi/2$ radians and acceleration leads the velocity by a phase by $\pi/2$ radians.

7. The equation of motion in a simple harmonic motion is

(a) $\frac{d^2x}{dt^2} = -\omega^2 x$ (b) $\frac{d^2x}{dt^2} = -\omega^2 t$ (c) $\frac{d^2x}{dt^2} = -\omega x$ (d) $\frac{d^2x}{dt^2} = -\omega t$

Ans. (a)

8. Which of the following relationships between the acceleration a and the displacement x of a particle executing simple harmonic motion?

(a) $a = 2x^2$ (b) $a = -2x^2$ (c) $a = 2x$ (d) $a = -2x$

Sol. (d)

In SHM, acceleration a is related to displacement, x by the relation $a - \omega^2 x$ which is for option d.

9. The total energy of a simple harmonic oscillator is proportional to

(a) amplitude (b) square of amplitude (c) frequency (d) velocity

Ans. (b)

10. The amplitude of a simple harmonic oscillator is doubled. How does this affect: (a) the period (b) the total energy (c) the maximum velocity of the oscillator?

Sol. (a) The time period of a simple harmonic oscillator is independent of its amplitude and as such remains unaffected.

(b) The total energy (E) of the oscillator is given by $E = \frac{1}{2} m\omega^2 A^2$ where A is the amplitude. Obviously when A is doubled, E becomes four times its previous value.

(c) As $v_{rnms} = \omega A \Rightarrow v_{max} = A$ i.e when A is doubled, v_{rms} is also doubled.

11. Write an expression for PE of a harmonic oscillator at any point.

Sol. $E_p = \frac{1}{2} ky^2 = \frac{1}{2} m\omega^2 y^2$

12. The girl sitting on swing stands up. What will be the effect on periodic time of swing?

Sol. We have, $v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

So keeping mass constant $v \propto \sqrt{k}$. As hard spring has greater value of spring constant k therefore hard spring have more frequency than delicated spring.

13. What will be the time period of a second pendulum inside an artificial satellite?

Sol. Time period of a second pendulum inside an artificial satellite will be infinity due to the absence of gravitation force.

(2 Marks Questions)

14. The displacement of particle in SHM may be given by a $y = a \sin(\omega t + \phi)$. Show that if the time t is increased by $2\pi/\omega$, the value of y remains the same.

Sol. The displacement at any time t is $y = a \sin(\omega t + \phi)$.

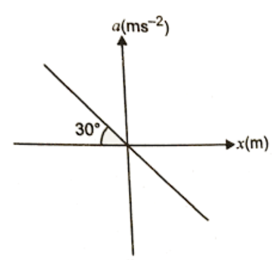
Therefore displacement at any time $(t + 2\pi/\omega)$ will be

$y = a \sin[\omega(t + 2\pi/\omega) + \phi] = a[\sin(\omega t + \phi) + 2\pi]$

$y = a \sin(\omega t + \phi)$ [since $\sin(2\pi + \phi) = \sin\phi$]

Hence the displacement at time t and $(t + 2\pi/\omega)$ are same.

15. Figure shows the acceleration displacement graph of a particle in SHM. Find the time period (in second).



Sol. The slope of given graph is $-\tan 30° = -1/\sqrt{3}$

$\therefore \frac{x}{a} = -\frac{1}{\sqrt{3}}$

We know in SHM, $a = -\omega^2 x \Rightarrow \frac{x}{a} = -\frac{1}{\omega^2}$

$\frac{1}{\omega^2} = \frac{1}{\sqrt{3}} \Rightarrow \omega = 3^{1/4} \quad \therefore T = \frac{2\pi}{3^{1/4}}$

16. Find the period of vibrating particle (SHM) which ahs acceleration of 45 cm s^{-1}, when displacement from mean position is 5cm.

Sol. Here y = 5cm and acceleration a = $45 cms^{-2}$.

We know that $a = \omega^2 y$

Therefore $45 = \omega^2 \times 5$ or $\omega = 3$ rad s^{-1}

And $T = \frac{2\pi}{\omega} = \frac{2\pi}{3} = 2.0935$ s

17. Show that the acceleration of a particle in SHM is proportional to its displacement from the mean position.

Sol. A particular type of periodic motion in which a particle moves to and fro repeatedly about a mean position under the influence of a restoring force is termed as simple harmonic motion (SHM). A body is undergoing simple harmonic motion if it has an acceleration which is (i) directed towards a fixed point, and (ii) proportional to the displacement of the body form that point.

Acceleration $a \propto -x$ or $a = -kx$ (where k is any constant) or $\frac{d^2x}{dt^2} = -kx$, where x = displacement at any instant t.

Simple harmonic motion can also be represented as the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

18. A body is executing a simple harmonic motion such that its potential energy is U_1 at x and U_2 at y. When the displacement is x + y, calculate the potential energy.

Sol. At displacement x, $U_1 = \frac{1}{2} m\omega^2 x^2 \Rightarrow x = \sqrt{\frac{2U_1}{m\omega^2}}$

Similarly $y = \sqrt{\frac{2U_2}{m\omega^2}}$

When displacement is (x + y), potential energy is

$U = \frac{1}{2} m\omega^2 (x + y)^2 - \frac{1}{2} m\omega^2 (x^2 + y^2 + 2xy)$

$= \frac{1}{2} m\omega^2 x^2 + \frac{1}{2} m\omega^2 y^2 + m\omega^2 xy = U_1 + U_2 + \sqrt{U_1 U_2}$

19. A point particle of mass 0.1kg is executing SHM of amplitude 0.1m. When the particle passes through the mean position, its kinetic energy is 0.008J. If each is 45°, then what is the equation of motion of this particle?

Sol. At mean position, KE = 0.008J

$\therefore \frac{1}{2} m v_{max}^2 = 0.008$ or $v_{max}^2 = \frac{0.008 \times 2}{m} = \frac{0.008 \times 2}{0.1} = 0.16$

$V_{rms} = 0.4\ ms^{-1}$.

Given A = 0.1m, $v_{max} = A\omega \Rightarrow \omega = \frac{v_{max}}{A} = \frac{0.4}{0.1} = 4$ rad s^{-1}

$\delta = 45° = \pi/4$ (given)

The equation of motion is x = $0.0\sin\left[(4)t + \frac{\pi}{4}\right]$ $[\because x = A\sin(\omega t + \delta)]$

20. A particle is moving on x-axis and has potential energy $U = 2 - 20x + 5x^2$ joule where x is position. The particle is released at x = - 3. If the mass of the particle is 0.1kg, then the maximum velocity (in m/s) of the particle is 25β.If the amplitude is 5m, find the value of β.

Sol. $U = 2 - 20x + 5x^2$

$\frac{dU}{dx} = -20 + 10x$

$F = -\frac{dU}{dx} = 20 - 10x = -10(x - 2)$

Therefore the particle is executing SHM about mean position x – 2 = 0 or x = 2

Therefore k = 10, $m\omega^2 = 10$

$\omega^2 = \frac{10}{m} = \frac{10}{0.1} = 100 \Rightarrow \omega = 10\text{rad s}^{-1}$

$V_{max} = 25\beta - A\omega = 5(10) = 50ms^{-1}$

$\Rightarrow \beta = 2$ (since $v_{max} = 25\beta$)

21. A 0.2 kg of mass hangs at the end of a spring. When 0.02 kg more mass is added to the end of the spring, it stretches 7cm more. If the 0.02kg mass is removed, what will be the period of vibration of the system?

Sol. Mass added m = 0.02

Length stretched x = 7 cm = 0.07 m

| Force due to weight| = | restoring force of spring|

⇒ mg = kx

$\Rightarrow k = \frac{0.02 \times 9.8}{0.07} = 2.86$ N/m

Time period, $T = 2\pi\sqrt{\frac{L}{G}}$

$\Rightarrow T = 2\pi\sqrt{\frac{0.2}{2.86}} = 1.6s.$

(3 Marks Questions)

22. A particle executes simple harmonic oscillation with an amplitude a. The period of oscillation is T. What will be the minimum time taken by the particle to travel half of the amplitude from the equilibrium position?

Sol. In simple harmonic motion, the displacement x(t) of a particle from equilibrium position at any time t is given by $x(t) = a\sin\omega t$ where a is the amplitude. At x(t) = a/2

$a/2 = a\sin\omega t$ or $½ = \sin\omega t$ or $\sin 30° = \sin\omega t$

or $\sin\left(\frac{\pi}{6}\right) = \sin\omega t \Rightarrow \frac{\pi}{6} = \frac{2\pi}{T} t$ ($\because \omega = \frac{2\pi}{T}$, where T is the time period of oscillation)
or t = T/12.

23. A block is resting on a piston which is moving vertically with a SHM of period 1.0s. At what amplitude of vibration will the block and the piston separate? What is the maximum velocity of the piston at this amplitude?

Sol. We are given that T = 1.0 s
Further, the maximum acceleration in SHM, i.e.,

$$a_{max} = \omega^2 A$$

For the block and the piston to separate,

$$a_{max} \geq g \text{ or } \omega^2 A \geq g$$

or $(2\pi/T)^2 A \geq g$ or $A \geq \frac{gT^2}{4\pi^2}$

or $A \geq \frac{(9.8\,m/s^2)(1.0\,s)^2}{39.48}$ (as $4\pi^2 = 39.48$)

or $A \geq 0.248\,m$

Thus, the block and the piston separate, when A = 0.248 m
Clearly.

$$v_{max} = \omega A = \left(\frac{2\pi}{T}\right) A = \left(\frac{2\times 3.14}{1.0\,s}\right)(0.248\,m)$$

$$= 1.56\,m/s$$

24. A body is describing SHM has a maximum acceleration of 8π m/s^2 and maximum speed of 1.6m/s. Find the time period and the amplitude.

Sol. $\omega^2 A = 8p$ …(i)
$\omega^2 A = 1.6$ …(ii)
Solving these two equations, we get $\omega = 5\pi$ rad/s
Therefore $T = \frac{2\pi}{\omega} = \frac{2\pi}{5\pi} = 0.4s$ and $A = \frac{1.6}{\omega} = \frac{1.6}{5\pi} = 0.102m$

25. If the displacement x and velocity v of a particle executing SHM are related through the experiment $4v^2 = 25 - x^2$, then determine its time period.

Sol. Velocity in SHM is given as $V^2 = \omega^2(A - x^2)$
Thus $V^2 = ¼ (5^2 - x^2)$, so, $\omega = ½$
Thus we get the time period, $T = 2\pi/\omega = 4\pi$ sec.

26. What is Simple Harmonic Motion? What is phase difference between displacement and acceleration in SHM. A simple harmonic motion is described by a = -25x where a is acceleration (m/s) and x is displacement (m). What is the time period?

Sol. Simple Harmonic Motion or SHM is defined as a motion in which the restoring force is directly proportional to the displacement of the body from its mean position. The direction of this restoring force is always towards the mean position. The acceleration of a particle executing simple harmonic motion is given by $a(t) = -\omega^2 x(t)$. Here, ω is the angular velocity of the particle.

Displacement of the particle executing SHM x=Asinwt

Acceleration of the particle a=dt2d2x

⟹ a=−Aw2sinwt=Aw2sin(π+wt)

Thus phase difference between displacement and acceleration of the particle is π radian.

Simple harmonic motion is defined by the equation: $a = (-x)\omega^2$ …(1)

Where a and x have usual meanings as in question and ω = angular velocity.

The relation between time period (t) ad angular frequency is: t = 2π/ω ,,,(2)

By comparing a = - 25x in equation (1) we get ω = 5

From equation (2) time period, t = (2 × 3.14) /5 = 3.14/2.5 = 1.25 sec.

27. Show that the total energy of a particle executing SHM is directly proportional to the square of its amplitude and frequency.

Sol. Let the energy be E, amplitude = A, frequency = ω

The total energy is given as $E = ½ m\omega^2A^2$

Thus $E \propto A^2$ and $E \propto \omega^2$.

28. A simple harmonic motion is described by y = A sin ωt. Find the time at which kinetic energy and potential energy of the simple harmonically oscillating particle are equal to each other.

Sol. ____

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29. Find the displacement of a simple harmonic oscillator at which its PE is half of the maximum energy of the oscillator.

Sol. PE of oscillator, $U = ½ m\omega^2y^2$

[y = displacement, maximum energy of oscillator, $E = ½ m\omega^2A^2$]

U = ½ E or $½ m\omega^2y^2 = ½ m\omega^2A^2$

Or $y^2 = A/2$ or $y = \pm A/\sqrt{2}$

30. A man of mass 60kg is standing on a platform executing SHM in vertical direction. The displacement from mean position of platform varies as $y = 0.5 \sin(2\pi\upsilon t)$. What will be the minimum value of υ, for which the man will feel weightlessness at the highest point?

Sol. $\omega = 2\pi\upsilon$, $A = 0.5$, $a = -\omega^2 A$

For weightlessness we have $a = g$

Thus $g = -\omega^2 A$

$\omega^2 = g/0.5 = 2g$ Or $4\pi^2\upsilon^2 = 2g$

$\upsilon^2 = g/2\pi^2$ or $\upsilon = \frac{\sqrt{2g}}{2\pi}$

31. Show that for small oscillations the motion of a simple pendulum is simple harmonic. Derive an expression for its time period. What would be the time period of simple pendulum at the centre of the earth. Justify your answer.

Sol. A motion be Simple harmonic motion only when ,

1. Acceleration of particle is just opposite to motion of body
2. Acceleration is directly proportional to displacement e.g., $a = -\omega^2 x$

A pendulum moves in such a way that angle is formed by it is θ , as you know , along tangent , motion of pendulum is just opposite to force. As shown in figure.

now, restoring force , $F = -mg\sin\theta$,

When displacement of pendulum is very small , then $\sin\theta \approx \theta$ so, $F = -mg\theta$, also here it is clear , $\theta = x/L$

$\therefore F = -mgx/L$

Now use F = ma { Newton's second law }

$ma = -mgx/L \Rightarrow a = -gx/L$

Here what you see both the above conditions are satisfied when displacement of pendulum is small.

Now, comapre both the expressions ,

$\therefore \omega^2 = g/L$

we know, $\omega = 2\pi/T$, here T is time period .

so, $\{2\pi/T\}^2 = g/L$

$\Rightarrow T = 2\pi\sqrt{L/g}$

Hence, for pendulum time period is $T = 2\pi\sqrt{L/g}$

The time period at of simple pendulum at the centre of the earth = $T = 2\pi\sqrt{(lg)}$ so at the centre of the Earth 'g' is zero. The time period would be infinite.

32. The vertical motion of a huge piston in a machine is approximately SHM with a frequency of 0.5/s. A block of 10kg is placed on piston, what is the maximum amplitude for the block and piston to remain together?

Sol. Given, $v = 0.5\ s^{-1}$, $g = 9.8\ ms^{-1}$

$$a = \omega^2 y = (2\pi\nu)^2 y = 4\pi^2\nu^2 y$$

a_{max} at the extreme position *i.e.*, $r = y$

$a_{max} = 4\pi^2\nu^2 r$ and $a_{max} = g$ to remain in contact.

$$\text{or } r = \frac{g}{4\pi^2\nu^2}$$

$$= \frac{9.8}{4\pi^2 \times (0.5)^2} = 0.993\text{m}.$$

(5 Marks Questions)

33. A body oscillates with SHM along the x-axis. Its displacement varies with time according to the equation x = (4.00m) cos(πt + π/4). Calculate (a) displacement (b) velocity (c) acceleration at t = 1.00s (d) the maximum speed and maximum acceleration and (e) phase at t = 2.00s.

Sol. By comparing the given equation with the general equation for SHM along X axis i.e. $x = A\cos(\omega t + \phi_0)$ we get A = 4.00m, ω = π rad/s, $\phi_0 = \pi/4$

(a) Displacement at t = 1.00s, i.e x = (4.00m) cos (π × 1 × π/4)

= (4.00)(- cos π/4) = (4.00)(- 0.707) = - 2.8m

(b) Velocity at t = 1.00s i.e. $v = -\omega A\sin(\omega t + \phi_0)$

Or $v = -(\pi)(4.00)\sin[\pi \times 1 + \pi/4]\text{m/s} = -(\pi)(4.00)\sin\left(\frac{5\pi}{4}\right)\text{m/s}$

$= (4\pi) \times 1/\sqrt{2} = 8.87$ m/s

(c) $a = -\omega^2 A\cos(\omega t + \phi_0) = -\pi^2 \times 4.00\cos(\pi\times1+\pi/4)$

$= -(4.00\,\pi^2)(-\cos\pi/4)\text{ m/s}^2 = 4.00 \times (3.14)^2 \times 0.707\text{ m/s}^2 = 27.9\text{ m/s}^2$

(d) Maximum velocity, $v_{max} = \omega A = \pi \times 4.00 = 12.56$m/s.

Maximum acceleration, $a_{max} = \omega^2 A = \pi^2 \times 4.00 = 39.4\text{ m/s}^2$

(e) Phase, $(\omega t + \phi_0) = (\pi/\text{s}) \times 2\text{s} + \frac{\pi}{4} = 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$

34. Deduce an expression for the (a) displacement (b) velocity (c) acceleration of a particle executing SHM.

Sol. Displacement of a body in SHM = A cos (ωt +ϕ)

(a) Displacement (x): At t = 0, displacement x = A i.e extreme position when ωt = ϕ = 90°, displacement x = 0 at mean position at any point x = A cos (ωt +ϕ)

(b) Velocity (V): - Velocity of body in SHM = $V = \frac{dx}{dt} = \frac{d}{dt}\{A\omega\cos(\omega t + \phi)\}$

Therefore $V = A\omega\sin(\omega t + \phi)$ or $V = -\omega\sqrt{A^2 - x^2}$

When (ωt = ϕ) = 0 then velocity v = 0. For points (ωt = ϕ) = 90°,

Velocity V = - Aω i.e. velocity is maximum

(c) Acceleration (a): Acceleration of a body in SHM = $\frac{dy}{dx}$

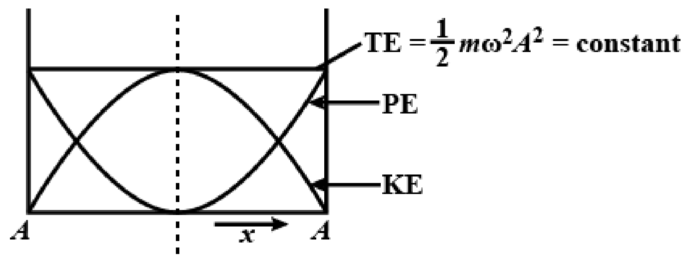
$= \frac{d}{dt}(-A\omega\sin(\omega t + \phi) = -A\omega^2\cos(\omega t + \phi) = -\omega^2 x)$

$a_{max} = -\omega^2 A$.

35. (a) Draw a graph showing the variation of kinetic energy and potential energy of a particle executing SHM with its displacement from mean position.

(b) Show that total mechanical energy of a particle executing simple harmonic motion remains conserved with time, when dissipative forces are neglected.

Sol. (a) The variations of kinetic energy K, potential energy U and total energy W with displacement x. The graphs for K and U are parabolic while that for E is a straight line parallel to the displacement axis. At x = 0 the energy is all kinetic and for $x = \pm A$, the energy is all potential.



(b) The energy of a harmonic oscillator is partly kinetic and partly potential. When a body is displaced from the equilibrium position by doing work upon it, it acquires potential energy. When the body is released, it begins to move back to equilibrium position, the acquires kinetic energy. At any instant, the displacement of a particle executing SHM is given by $x = A\cos(\omega t + \phi_0)$

So velocity, $v = dx/dt = -\omega A\sin(\omega t + \phi_0)$

Hence kinetic energy of the particle at any time t is given by $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi_0)$

But $A^2\sin^2(\omega t + \phi_0) = A^2[1 = \cos^2(\omega t + \phi_0)] = A^2 - A^2\cos^2(\omega t + \phi_0) = A^2 - x^2$

Or $K = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$

When the displacement of a particle from its equilibrium position is x, the restoring force is given by $dW = -Fdx = +kxdx$

The total work done in moving the particle from mean position (x = 0) to displacement x is given by $W = \int dW = \int_0^x kxdx = k\left[\frac{x^2}{2}\right]_0^x = \frac{1}{2}kx^2$

The work done against the restoring force is stored as the potential energy of the particle. Hence potential energy of a particle at displacement x is given by

$U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2A^2\cos(\omega t + \phi_0)$

At any displacement x, the total energy of a harmonic oscillator is given by

$E = K + U = \frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2$ or $E = \frac{1}{2}KA^2 = \frac{1}{2}m\omega^2A^2 = 2\pi^2 m\upsilon^2A^2$ ($\because \omega = 2\pi\upsilon$)

Thus the total mechanical energy of a harmonic oscillator is independent of time or displacement. Hence in the absence of any frictional force, the total energy fo harmonic oscillator is conserved.

C. COMBINATION OF SHM

(2 Marks Questions)

1. Prove that the equation x = a sin ωt + b cos ωt shows SHM.

Sol. SHM = F = - kx [F ∝ - x]

ma = - kx [a ∝ - x]

x = a sin ωt + b cos ωt

$v = \frac{dx}{dt} = \frac{d}{dt}(a\sin\omega t) + \frac{d}{dt}(b\cos\omega t) = a\,\omega\cos\omega t - b\,\omega\sin\omega t$

$a = \frac{d\theta}{dt} = \frac{d}{dt}(a\omega\cos\omega t) - \frac{d}{dt}(b\omega\sin\omega t)$

$= -a\omega^2\sin\omega t - b\omega^2\cos\omega t$

$= -\omega^2[a\sin\omega t + b\cos\omega t] = -\omega^2 x$

Or a ∝ - x

This represents SHM.

2. Show that motion of a particle represented by y = sin ωt – cos ωt is a simple harmonic motion with time period 2π/ω.

Sol. y = sin ωt – cos ωt

$= \sqrt{2}\left(\cos\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right) = \sqrt{2}\left(\cos\frac{\pi}{4}\sin\omega t - \sin\frac{\pi}{4}\cos\omega t\right)$

Therefore $y = \sqrt{2}\sin(\omega t - \frac{\pi}{4})$

Therefore (sin ωt – cos ωt) represents SHM.

$y = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right) = \sqrt{2\sin}\left(\omega t - \frac{\pi}{4} + 2\pi\right) = \sqrt{2}\sin(\omega\left(t + \frac{2\pi}{\omega}\right) - \frac{\pi}{4}$

Therefore time period = $\frac{2\pi}{\omega}$

D. MISCELLANEOUS SHM

(1 Marks Questions)

1. The length of the simple pendulum which ticks seconds is

(a) 0.5m (b) 1m (c) 1.5m (d) 2m

Sol. (b)

The time period of a simple harmonic pendulum is $T = 2\pi\sqrt{\frac{L}{G}}$ where L is the length fo pendulum or $L = \frac{gT^2}{4\pi^2}$ ….(i)

The time period of the simple pendulum which ticks seconds is 2s.
Therefore T = 2s

Substituting in (i) we get, L = $\frac{(0.98\text{ms}^{-2})(2\text{s})^2}{4\times(3.14)^2}$ = 1m

2. What is the effect on the time period of a simple pendulum if the mass of the bob is doubled?

(a) Halved (b) Doubled (c) becomes 8 times (d) no effect

Sol. (d)

Time period of a simple pendulum, $T = 2\pi\sqrt{\frac{L}{G}}$

It means T is independent of mass of the bob m. Hence if mass doubled, T will not change.

3. There are two springs, one delicate and another hard or stout one. For which spring, the frequency of the oscillator will be more?

Sol. We have $v = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

So keeping mass constant $v \propto \sqrt{k}$. As hard spring has greater value of spring constant k therefore hard spring have more frequency than delicated spring.

4. Write the expression for time period of a simple pendulum.

Sol. $T = 2\pi\sqrt{\frac{L}{G}}$

5. Define force constant of a spring.

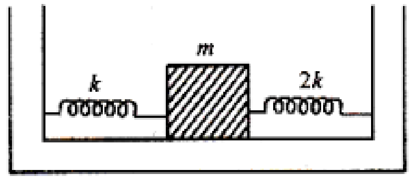
Sol. The force constant of a spring is the change in the force it exerts, divided by the change in deflection of the spring. (k = F/x).

6. Does the direction of acceleration at various points during the oscillation of a simple pendulum remain towards mean position?

Sol. No the resultant of tension in the string and weight of bob is not always towards the mean position.

(2 Marks Questions)

7. Two springs of force constants K and 2K are connected in a block of m as shown in the figure. What is the frequency of oscillation of this block?



Sol. For the given condition, $K_{eq} = \frac{K(2K)}{(2K)+K} = \frac{2K^2}{3K} = \frac{2}{3}K$

Now $T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{3m}{2K}} = 2\pi\sqrt{\frac{3}{2}}\sqrt{\frac{m}{K}}$

Hence $\omega = \frac{1}{2\pi}\sqrt{\frac{2}{3}}\sqrt{\frac{K}{m}}$

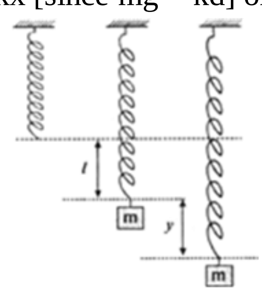
8. The formula for time period T for a loaded spring, $T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$. Foes the time period depend on the length of the spring?

Sol. Although length of the spring does not appear in the expression for the time period, yet the time period depends on the length of the spring. It is because force constant fo the spring depends on the length of the spring.

9. A massless spring of spring constant k is attached with a mass m and is made to oscillate vertically. Deduce the expression for its time period.

Sol. If a spring is suspended vertically from a rigid support and a body of mass m is attached to its lower end, the spring gets stretched to a distance of due to the weight mg of the body. Because of the elasticity of the spring, a restoring force equals to kd begins to act in the upward direction. Here k is the spring factor of the spring. In the position of equilibrium, mg = kd.

If the body be pulled vertically downwards, through a small distance x from its equilibrium, position and then released, it begins to oscillate up and down about this position. The weight mg of the body acts vertically downwards while the restoring force k(d +x) due to elasticity acts vertically upwards. Therefore the resultant force on the body is F = mg – k(+x) = kd – kd – kx [since mg = kd] or F = - kx



If d^2x/dt^2 is the acceleration of the body, then

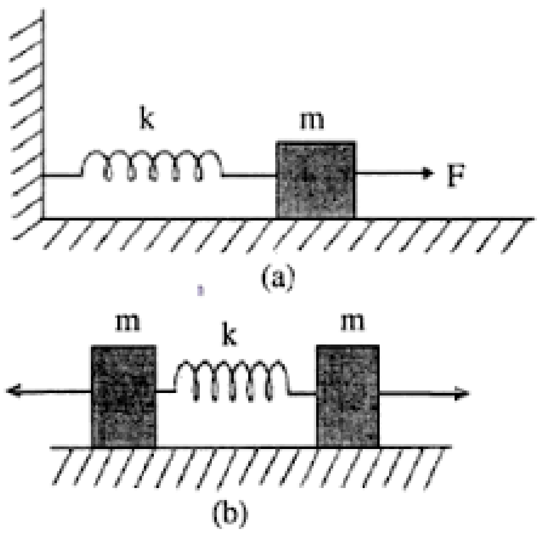
$m\frac{dx^2}{dt^2} = kx$ or $\frac{dx^2}{dt^2} = -\frac{k}{m}x = -\omega^2 x$

Thus acceleration is proportional to displacement x and is directly opposite to it. Hence the body executes SHM and its time period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \text{ or } T = 2\pi\sqrt{\frac{m}{k}}$$

(3 Marks Questions)

10. Figures (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F is applied to the free end stretches the spring. Figure (b) shows the same spring with both ends attached to mass m at either end. Each end of the spring in figure (b) is stretched by the same force F. What will be the maximum extension of the spring in both cases? Also, find out the time period for each case.



Sol. (i) Maximum extension of the spring:

(a) Suppose the maximum extension produced in the spring is y. Then, F = ky (in magnitude) or y = F/k

(b) In this case, force F on each mass acts as the force of reaction developed due to force F on the other mass. Therefore in this case also, maximum extension is given by y = F/k.

(ii) Period of oscillation:

If T_1 is the time period in case (a), then $T_1 = 2\pi\sqrt{\frac{m}{k}}$

In case (b), the time period of oscillation of a two body oscillator (two bodies of mass m_1 and m_2 connected at the ends of a spring of spring constant (k) is given by $T_2 = 2\pi\sqrt{\frac{\mu}{k}}$

Where μ is called the reduced mass of the system defined as $\mu = \frac{m_1 m_2}{m_1 + m_2}$

In the present case, $m_1 = m_2 = m$. So, $\mu = \frac{m \times m}{m + m} = \frac{m}{2}$

Thus $T_2 = 2\pi\sqrt{\frac{m/2}{k}} = 2\pi\sqrt{\frac{m}{2k}}$

11. An infinite number of springs with spring constant k, 2k, 4k, 8k, 16k, …∞. Respectively are connected in series. What is the equivalent spring constant?

Sol. $\frac{1}{k_s} = \frac{1}{k} + \frac{1}{2k} + \frac{1}{4k} + \frac{1}{8k} + \frac{1}{16k} + \cdots \text{ to } \infty$

$= \frac{1}{k}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots \text{ to } \infty\right]$

$= \frac{1}{k}\left[\frac{1}{1-1/2}\right] = \frac{2}{k} \text{ or } k_s = \frac{k}{2}$

12. Determine the period of small oscillations of a pendulum that is bob suspended by a thread L = 20cm in length, if it is located in a liquid whose density is 3 times less than that of bob.

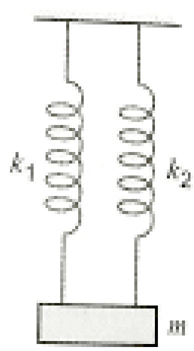
Sol. $g_{eff} = \frac{\text{Weight}-\text{Upthrust}}{\text{Mass}} = \frac{V\rho g - V\frac{\rho}{3}g}{V\rho} = \frac{2}{3g}$

$T = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{2l}{2g}} = 2\pi\sqrt{\frac{3\times0.2}{2\times9.8}} = 1.1\text{s}.$

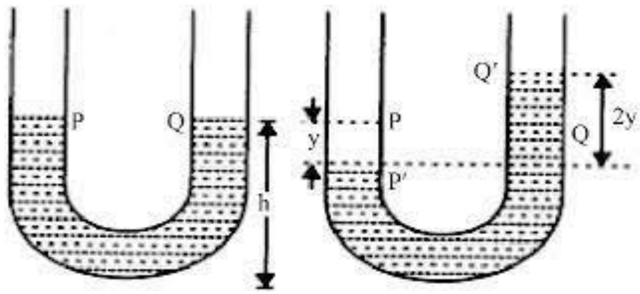
(5 Marks Questions)

13. (a) One end of a U tube containing mercury is conned to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that when the suction pump is removed, the liquid column of mercury in the U-tube executes SHM.

(b) An arrangement of springs for SHM is shown in the figure. If mass of block is m, then find frequency of oscillation.



Sol. (a) The suction pump creates the pressure difference, thus mercury rises in one limb of the U tube. When it is removed, a net force acts on the liquid column due to thte difference in levels of mercury in the two limbs and hence the liquid column executes SHM which can be explained as:



Consider the mercury contained n a vertical U tube the level P and Q in its two limbs. Let ρ = density of the mercury, L = total length of the mercury column in both the limbs, A = internal cross sectional area of U tube, m = mass of mercury in U tube = LAρ.

Le the mercury is depressed in left limb to P' by small distance y, then it rises by the same amount in the right limb to position Q'.

Therefore difference in levels in the two limbs = P'Q' = 2y

Therefore volume of mercury contained in the column of length 2y = A × 2y

m' = A × 2y × ρ

. If W = m'g = A × 2y × ρ × g

This weight produces the resulting force (F) which sends to bring back the mercury to its equilibrium position.

Therefore F = - 2Ayρg = - (2Aρg)y

If a = acceleration produced in the liquid column, then

$a = \frac{F}{m} = -\frac{(2A\rho g)y}{LA\rho} = -\frac{2yg}{L} = -\frac{2g}{2h}y$ (since L = 2h) …(i)

where h = height fo mercury in each limb. Now from (i) it is clear that a = y and –ve sign shows that it acts opposite to y, so the motion of mercury in U tube is SHM in natur having time period (T) given by

$T = 2\pi\sqrt{\frac{y}{a}} = 2\pi\sqrt{\frac{2h}{2g}} = 2\pi\sqrt{\frac{h}{g}}$

(b) No effect on time period if the amplitude of pendulum is increased or decreased.

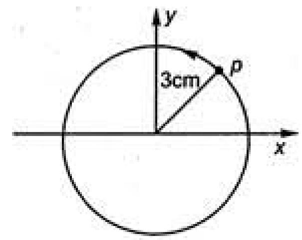
14. Show that the time period of a oscillation of a liquid column in a U tube is independent of the mass of the liquid column, the density of the liquid and the cross section area of U tube but depends only upon the length of the liquid column and on the value of the acceleration due to gravity.

Sol. Same as Q13 (a)

E. CIRCULAR MOTION AND SHM

(1 Marks Questions)

1. The figure shows circular motion of a reference particle to represent simple harmonic motion. What is the amplitude of simple harmonic motion?



Sol. The amplitude in simple harmonic motion is equal to the radius of the circle of reference. Hence the amplitude of simple harmonic motion is 3cm.

(3 Marks Questions)

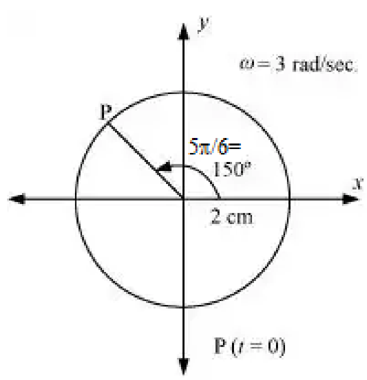
2. Plot the reference circle for each of the following simple harmonic motion. Indicate the initial (t= 0) opposition of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s)

(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right)$ (b) $x = \cos\left(\frac{\pi}{6} - t\right)$

Sol. (a) Here x = - 2 sin $\left(3t + \frac{\pi}{3}\right) = 2\cos\left[\frac{\pi}{2} + \left(3t + \frac{\pi}{3}\right)\right] = 2\cos\left(3t + \frac{5\pi}{6}\right)$

Therefore A = 2cm, $\phi = 5\pi/6$ and $\omega = 3$ rad s^{-1}

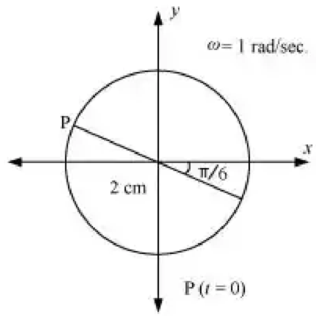
Therefore at t = 0, the particle is at point P such that $\phi = \angle POX = \frac{5\pi}{6}$ as shown in figure.



(b) Here x = cos (π/ 6 – t) = cos (t - $\frac{\pi}{6}$) [since cos (-θ) = cos θ]

Therefore A = 1cm, $\phi = -\frac{\pi}{6}$ and $\omega = 1$ rad s^{-1}

Therefore at t = 0, the particle is at point P such that $\phi = \angle POX = -\frac{\pi}{6}$ as shown in figure.



F. DAMPING OSCILLATION AND RESONANCE

(1 Marks Questions)

1. A particle oscillating under a force is a $\vec{F} = -k\vec{x} - b\vec{v}$ is a(k and b are constants).
   (a) simple harmonic motion (b) linear oscillator
   (c) damped oscillator (d) forced oscillator

Ans. (c)

2. At resonance, the amplitude of forced oscillations is
   (a) minimum (b) maximum (c) zero (d) none of these

Sol. (b)
At resonance the amplitude of forced oscillations is maximum.

3. Marching troops are asked to break their steps while crossing the bridge. Why?

Sol. If soldiers while crossing a bridge march in steps, the frequency of the matching steps of soldiers may match with natural frequency of oscillations of the suspended bridge. Resonance will take place in this situation, then amplitude of the oscillations of the bridge will increase enormously which may cause the collapsing of bridge. So to avoid this situation, marching troops are asked to break their steps while crossing the bridge.

4. Why is loud sound heard at resonance?

Sol. When resonance happens the body starts vibrating with a greater amplitude when it experiences an external source of frequency very similar to its own natural frequency. This produces a loud sound.

5. At a certain speed of a bus, its whole body starts vibrating strongly. Explain.

Sol. The vibration can be defined as an action under which particles of any object oscillate at a similar frequency. The vibrations in any object depend on the magnitude of the external load that applied to the object.

(2 Marks Questions)

6. In a forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. What is the relation between ω_1 and ω_2?

Sol. Both amplitude and energy of the particle can be maximum only in the case of resonance. For resonance to occur, $\omega_1 = \omega_2$.

(3 Marks Questions)

7. A 21.2 kg object oscillates at the end of a vertical spring that has a spring constant 20500 N/m. The effect of air resistances is represented by the damping coefficient b = 2kg/s. Fine the time interval that elapses while the energy of the system drops to 10% of its initial value (given ln 10 = 2.302).

Sol. $\tau = \frac{m}{b} = \frac{21.2}{2} = 10.6s$

$E = E_0 e^{-1/\tau}$ or $0.1 = e^{-t/10.6}$ or $e^{t/10.6} = 10$ or $\frac{t}{10.6} = \ln 10$

t = (10.6)(2.302) = 24.4s.

(5 Marks Questions)

8. Explain damped harmonic oscillation and the equation of such oscillations.

Sol. Let us understand the effect of a damping force on the motion of a harmonic oscillator. Suppose we have a block of mass mm connected to an elastic spring having a spring constant *k* that is oscillating vertically. Now, this block is pushed down slightly and released immediately. The spring block system will start oscillating. Let the angular frequency of the oscillation be ω.

Then, $\omega=\sqrt{(k/m)}$)

Although in reality, a damping force will be exerted by the surrounding medium, i.e. air on the block's motion. Due to this, the mechanical energy of the spring-block system will decrease. The loss in the system's energy will appear in the form of heat energy in the surroundings. The magnitude and the type of damping forces acting on the system will depend on the nature of the surrounding medium. When the spring block system is immersed in fluid instead of air, due to the larger magnitude of the damping thus, the dissipation of energy will be much faster.

Damped Harmonic Oscillator Derivation

We know from the stokes law that the damping force, in general, is directly proportional to the velocity. It acts in a direction opposite to the direction of velocity.

Let the damping force acting on the spring-block system be F_d,

$F_d = -bv$——(1)

Where v: is the velocity of the block

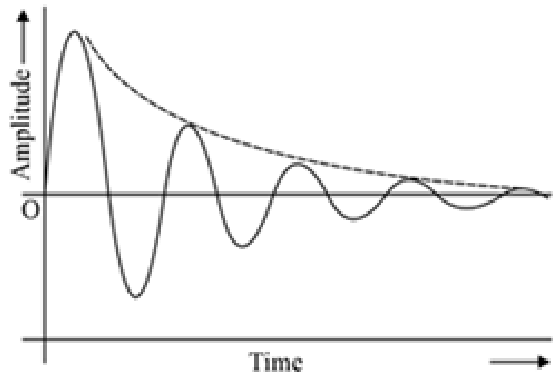
b: it is a positive constant whose value depends on the properties of the medium, for example, viscosity and the shape and size of the block.
The equation (1) is only valid for small values of the velocity. When the mass mm is hung vertically attached to the spring, as shown in the figure, when the block is released from rest, the spring will elongate a little, and the mass attached to the spring will ultimately settle at a height. The point O represents the position of equilibrium of the mass.
When the mass is pushed up or pulled down by some distance,
The restoring force acting on the block due to the spring, $F_s=-kx$
Where x is the displacement of the mass from its equilibrium position, k is the spring constant
Thus, at any time t, the total force acting on the mass, $F=-kx-bv$——(2)

Let a(t)a(t) is the acceleration of the mass in time t.
By applying Newton's laws of motion along the direction of motion,
The force due to the motion, $F=ma(t)$——(3)

Using the equations (2) and (3), we get:
$-kx(t)-bv(t)=ma(t)$——(4)

Since we are only concerned with the one-dimensional motion of the mass, we can drop the vector notation. We know,
Velocity can be determined by the first derivative of displacement, v(t)=dxdtv(t)=dxdt
Acceleration can be determined by the second derivative of displacement, $a(t)=d^2x/dt^2$

Substituting the values from above into equation (4), we get:
$m=d^2x/dt^2=-kx-b.dx/dt$
$m.d^2x/dt^2 + b.dx/dt+kx=0$
The block's motion under the effect of a damping force, which is proportional to the velocity, can be determined by the solution of the above equation. The solution of this equation is of the form:
$x(t)=Ae-^{bt/2m}\cos(\omega‘t+\phi)$——(5)
In this equation, AA is the amplitude, ‘ω‘ is the angular frequency of the damped oscillator, ϕ is the phase constant, and the cosine function has a period of ‘2πω‘
The angular frequency can be given as:
$\omega‘=\sqrt{}(k/m)-(b^2/4m^2)$
The function x(t) in equation (5) is not exactly periodic because of the presence of the factor $e^{-bt/2m}$, which as we can see, decreases continuously with time. However, if the decrease in one time period is small, the motion of the mass can be taken to be periodic.
The equation (5) can be graphically represented as:



It is a cosine function whose amplitude $A^{e^{-bt/2m}}$ is gradually decreasing with time. We know the expression for the mechanical energy of an undamped oscillator can be given as, $E=1/2\ kA^2$

When the applied damping is small, the amplitude is not constant, but it depends on the time. Thus, for the given damped oscillator, the expression for mechanical energy can be given as:

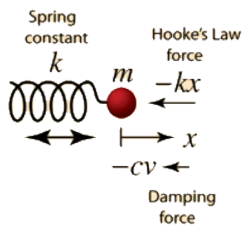
$E(t)=1/2k(A^{e-bt/2m})^2$

$E(t)=1/2kA^{e-bt/m}$——(6)

From equation (6), we can see that the total mechanical energy decreases exponentially with time.

9. Discuss driven oscillations.

Sol. If a damped oscillator is driven by an external force, the solution to the motion equation has two parts, a transient part and a steady-state part, which must be used together to fit the physical boundary conditions of the problem.



The motion equation is of the form

Newton's 2nd Law terms | Sinusoidal driving force

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F_0\cos(\omega t+\varphi_d)$$

and has a general solution

$$x(t)=x_{transient}+x_{steady\ \ state}$$

In the underdamped case this solution takes the form

Transient solution | Steady-state solution

$$x(t)=A_he^{-\gamma t}\sin(\omega' t+\varphi_h)+A\cos(\omega t-\varphi)$$

Determined by initial position and velocity | Determined by driving force

The initial behavior of a damped, driven oscillator can be quite complex. The parameters in the above solution depend upon the initial conditions and the nature of the driving force, but deriving the detailed form is an involved algebra problem. The form of the parameters is shown below.

G. ASSERTION REASON TYPE QUESTIONS:

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false **(d) If both assertion and reason are false**

(e) If assertion is false but reason is true

1. Assertion: In SHM, acceleration is always directed towards the mean position.

Reason: In SHM, the body has to stop momentary at the extreme position and move back to mean position.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.

SHM is basically to and fro motion about the mean position. So when the body goes away from mean position and acceleration always try to return the body towards mean position. As the acceleration in SHM is always in opposite phase to that of displacement. The displacement of the particle in SHM at an instant is directed away from the mean position then acceleration at that instant is directed towards the mean position.

2. Assertion: The graph between velocity and displacement for a simple harmonic motion is a parabola.

Reason: Velocity does not change uniformly with displacement in simple harmonic motion.

Ans. (e) Assertion is false but Reason is true.

In SHM, $v = \omega\sqrt{a^2 - y^2}$ or $v^2 = \omega^2 a^2 - \omega^2 y^2$. Dividing both sides by $\omega^2 a^2$, $\frac{v^2}{\omega^2 a^2} + \frac{y^2}{a^2} = 1$.

This is the equation of an ellipse. Hence the graph between v and y is an ellipse not a parabola.

3. Assertion: The graph of total energy of a particle in SHM with respect to position is a straight line with zero slope.

Reason: Total energy of particle in SHM remains constant throughout its motion.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.

The total energy of SHM = kinetic energy of particle + potential energy of particle.

At mean position the total energy of the particle in SHM is in form of kinetic energy. At extreme position, the total energy of the particle in SHM is in the form of potential energy.

4. Assertion: If a hole were drilled through the centre of earth and ball dropped into the hole at one end, it will not oscillate.

Reason: The ball will comes out of other and if passed through centre hole in earth.

Ans. (d) Both Assertion and Reason are false

The ball will not get out of the other end of the hole, because it will execute SHM. The restoring force of ball at any instant is proportional to displacement. (F = mg' = mgh) which is one of condition of SHM. Also, on reaching the other end of the hole, its velocity becomes zero. Acceleration of ball is maximum and directed towards the centre of earth.

5. Assertion: The periodic time of hard spring is less as compared to that of soft spring.
Reason: The periodic time depends upon the spring constant, which is large for hard spring.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.

The time period of an oscillating spring is given by $T = 2\pi\sqrt{\frac{m}{k}}$ ι.ε $T \propto \frac{1}{\sqrt{k}}$. Since the spring constant i alrge for hard spring, therefore hard spring has a less periodic time as compared to soft spring.

H. CHALLENGING PROBLEMS

1. Two pendulums of length 100cm and 121cm start oscillating. At some instant the two are at the mean position in the same phase. After how many oscillations of the longer pendulum will the two be in the same phase the mean position again?

Sol. Two pendulums of length 121cm and 100cm.

Let, L_1=121cm=1/10021=1.21m; L_2=100cm=100/100=1m

We have to find the vibrations made by the shorter pendulum, such that both will be in same phase from the reaction,

T_1=longer pendulum, T_2=shorter pendulum

$T=2\pi\sqrt{\frac{L}{g}}$

$T \propto \sqrt{L}$

$T_1/T_2 \propto \sqrt{(L_1/L_2)}$

$T_1/T_2 \propto \sqrt{(1.21/1)}$

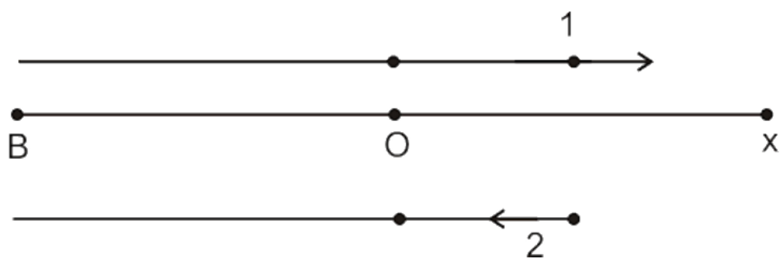
$T_1/T_2=1.1/1$

$10T_1=11T_2$

10 vibrations of longer pendulum= 11 vibrations of shorter pendulum

2. Two particles are oscillating along two close parallel straight lines side by side, with the same frequency and amplitudes. They pass each other, moving in opposite direction when their displacement is half of the amplitude. The mean positions of the two particles lie on a straight line perpendicular to the paths of the two particles. Find the phase difference.

Sol.



$x_1 = A\sin(\omega t + \phi_1)$

$x_1 = \frac{A}{2}$

$\frac{A}{2} = A\sin(\omega t + \phi_1)$

$\frac{1}{2} = \sin(\omega t + \phi_1)$

$\omega t + \phi_1 = \frac{\pi}{6}$

$x_2 = A\sin(\omega t + \phi_2)$

$x_2 = \frac{A}{2}$

$\frac{A}{2} = A\sin(\omega t + \phi_2)$

$\left(\pi - \frac{\pi}{6}\right) = \omega t + \phi_2$

(As particle 2 is moving towards mean position at A/2 displacement)

Phase difference = $\phi_2 - \phi_q$

$$= \left(\frac{\pi - \pi}{6}\right) - \omega t - \left[\frac{\pi}{6} - \omega t\right]$$

$$= \pi - 2 \times \frac{\pi}{6}$$

$$= \frac{2\pi}{3}$$