CLASS – 11

WORKSHEET- WAVES

A. INTRODUCTION TO WAVE

(1 Mark Questions)

1. With propagation of longitudinal waves through a medium, the quantity transmitted is
(a) matter (b) energy
(c) matter and energy (d) energy, matter and momentum

Sol. (b)
Only energy is transmitted from one point to another and during propagation of any longitudinal waves n a medium transmission of energy through the medium without matter being transmitted.

2. Why are the longitudinal waves also called pressure waves?

Sol. Since propagation of longitudinal waves through a medium creates pressure disturbances in the medium, hence these waves are called pressure waves.

3. What is the direction of oscillations of the particles of the medium through which (i) a transverse and (ii) a longitudinal wave is propagating?

Sol. In transverse waves, particles of the medium oscillate in a direction perpendicular to the direction of propagation of waves. In longitudinal waves particles of the medium oscillates in the direction of propagation of waves.

4. Ocean waves hitting a beach are always found to be nearly normal to the shore. Why?

Sol. Ocean waves are transverse waves travelling in concentric circles of the ever-increasing radius. When they hit the shore, their radius of curvature is so large that they can be treated as plane waves. Hence they hit the shore nearly normal to the shore.

5. Which of the following wave functions does not represent a travelling wave?
(a) $y = (x - vt)^2$ (b) $y = \log(x + vt)$ (c) $y = 1/x + vt$ (d) all of these

Sol. (d) The basic requirement for a wave function to represent a travelling wave is that for all values of x and t wave function must have a finite value. Out of the given functions for y no one satisfied the given condition. Therefore none can represent a travelling wave.

(2 Marks Questions)

6. Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases. Give reason.

Sol. Solids possess both the volume, elasticity and the shear elasticity. Therefore they can support both longitudinal and transverse waves. On the other hand, gases have only the volume elasticity and no shear elasticity, so only longitudinal waves can propagate in gases.

(3 Marks Questions)

7. You have learnt that travelling wave in one dimension is represented by a fraction y = f(x, t) where x and t must appear in the combination x – v t or x +vt, i.e. y = f(x±vt). Is the converse true? Examine if the following functions for y can possibly represent a travelling wave: (a) $(x - vt)^2$ (b) $\log[(x+vt)x_0]$ (c) $\exp[-(x+vt)/x_0]$ (d) $1/(x+vt)$.

Sol. If y = f(x±vt) represents a travelling wave, then the converse may not be true i.e. every function of x – vt or x +vt may not always a travelling wave. The basic requirement for a function to represent travelling wave is that it must be finite for all value of x and t.
The functions (i), (ii) and (iv) are not finite for all values of x and t, hence they cannot represent a travelling wave. Only fraction (iii) satisfies the condition to represent a travelling wave.

B. PLANE PROGRESSIVE WAVE OR HARMONIC WAVE

(1 Mark Questions)

1. Two astronauts on the surface of the moon cannot talk to each other, why?

Sol. Two astronauts cannot talk on Moon like they do on Earth because sound needs medium to travel and there is no air on Moon.

2. What is the evidence that (i) sound is a wave, (ii) sound is a mechanical wave and (iii) sound waves are longitudinal waves?

Sol. (i) Sound waves show the phenomenon of diffraction. (ii) Sound waves require material for propagation. (iii) Sound waves cannot be polarized.

3. Do displacement, particle velocity and pressure variation in a longitudinal wave vary with the same phase?

Sol. No particle velocity is out of phase by $\pi/2$ with the displacement and the pressure variation is out by phase by it with the displacement.

4. A progressive wave is represented by $y = 5\sin(100\pi t - 2\pi x)$ where x and y are in m and t is in s. The maximum particle velocity is

(a) 100 π m/s (b) 200 π m/s (c) 400 π m/s (d) 500 π m/s

Sol. (d)

The given progressive wave is
$y=5\sin(100\pi t-2\pi x)$
particle velocity, $v_p=dy/dt = 500\pi\sin(100\pi t-2x)$
$v(p_{max})=500ms^{-1}$

5. The propagation constant of a wave is also called its
(a) wavelength (b) frequency (c) wave number
(d) angular wave number

Ans. (d)

6. A wave of wavelength 2m propagate through a medium. What is the phase difference between two particles on the line of propagation? Given that the distance between the particles is 75m.

Sol. The phase difference at any instant of time t, between two particles separated by distance Δx is given by $\Delta\phi = \frac{2\pi}{\lambda}\Delta x$ where Δx is path difference and λ is wavelength.
$\Delta\phi = \frac{2\pi}{2}\times 75$; $\Delta\phi = 75\pi$.

7. Newton assumed that sound propagation in a gas takes under
(a) isothermal condition (b) adiabatic condition
(c) isobaric condition (d) isentropic condition

Sol. (a)
Newton assumed that sound propagation in a gas takes under isothermal condition.

8. For v_{rms} is the rms speed of molecules in a gas and v is the speed of sound waves in the gas, then the ratio v_{rms}/v is
(a) $\sqrt{\frac{3}{\gamma}}$ (b) $\sqrt{\frac{\gamma}{3}}$ (c) $\sqrt{3\gamma}$ (d) $\frac{\sqrt{3}}{\gamma}$

Sol. (a)
rms speed of gas molecule is $v_{rms} = \sqrt{\frac{3P}{\rho}}$ …(i)
Speed of sound in the gas is $v = \sqrt{\frac{\gamma P}{\rho}}$ …(ii)
Divide (i) by (ii) we get, $\frac{v_{rms}}{v} = \sqrt{\frac{3}{\gamma}}$

9. What kind of thermodynamical process occur in air, when a sound wave propagates through it?

Sol. When the sound wave travel through air adiabatic changes take place in the medium.

10. State the factors on which the speed of a wave travelling along a stretched ideal string depends.

Sol. The speed of a wave travelling along a stretched ideal string depends on the tension on the string (T) and mass per unit length (m). $\upsilon = \sqrt{\frac{T}{\mu}}$

11. What is the effect of pressure on the speed of sound in air? Justify your answer.

Sol. The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma P}{\rho}}$

At constant temperature, PV = constant. Pm/ρ = constant

Since m is constant, so P/ρ = constant i.e when the pressure changes, density also changes in the same ratio so that the factor P/ρ remains unchanged. Hence, the pressure has no effect on the speed of sound in a gas for a given temperature.

(2 Marks Questions)

12. If the phase difference between two sound waves of wavelength λ is 60°, what is the corresponding path difference?

Sol. Path difference of a given phase difference δ is given by, $\Delta x = \frac{\lambda}{2\pi}\delta$

Given $\delta = 60° = \pi/3$

Therefore $\Delta x = \frac{\lambda}{2\pi} \times \frac{\pi}{3} = \frac{\lambda}{6}$

13. Define wave number and angular wave number and give their SI units.

Sol. Wave number is the number of waves present in a unit distance of medium ($\bar{\upsilon} = 1/\lambda$). SI unit of $\bar{\upsilon}$ is m^{-1}. Angular wave number of propagation constant difference is $2\pi/\lambda$. It represents phase change per unit path difference and denoted by $k = 2\pi/\lambda$. SI unit of k is rad m^{-1}.

14. A steel wire has a length of 12.0m and a mass of 2.10kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals to the speed of sound in dry air at 20°C = 343m/s.

Sol. Here l = 12.0m, M = 2.10kg, T = ?, v = 34.3 ms^{-1}

Mass per unit length, $\mu = M/l = 2.10/12.0 = 0.175\ kgm^{-1}$

As $v = \sqrt{\frac{T}{\mu}}$.

Therefore $T = v^2\mu = (343)^2 \times 0.175 = 2.06\times10^4 N$

15. Discuss the effect of the following factors on the speed of sound: (a) pressure (b) density (c) humidity (d) temperature.

Sol. Effect on speed of sound due to:

(a) Pressure: There is no effect of pressure change on speed of sound as long as temperature remain constant.

(b) Density: The speed of sound is inversely proportional to the square root of density of medium.

(c) Humidity: Speed of sound increases with increase in humidity.

(d) Temperature: Speed of sound in a gas is directly proportional to square root of its temperrture.

16. A String of mass 2.50kg is under a tension of 200N. The length of the stretched string is 20.0m. If a transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

Sol. Given m = $\frac{2.50}{20.0}$ kgm^{-1}; T = 200N

Speed of transverse jerk is v = $\sqrt{\frac{T}{m}} = \sqrt{\frac{200 \times 20.0}{250}} = \sqrt{1600} = 40\ ms^{-1}$

Therefore time taken by the jerk to reach the other end= $\frac{\text{Distance}}{\text{speed}} = \frac{20}{40}$ = 0.5s.

(3 Marks Questions)

17. The equations of displacements of two waves are $y_1 = 10\sin\left[3\pi t + \frac{\pi}{3}\right]$ and $y_2 = 5\left[\sin 3\pi t + \sqrt{3}\cos 3\pi t\right]$. Find the ratio of their amplitudes.

Sol. Here y_1 = $10\sin\left[3\pi t + \frac{\pi}{3}\right]$

Therefore amplitude of this wave is A_1 = 10 and y_2 – 5[sin 3πt + $\sqrt{3}$cos πt]

$= 10\left[\cos\frac{\pi}{3}\sin 3\pi t + \frac{\sqrt{3}}{w}\cos 3\pi t\right] = 10\left[\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos 3\pi t\right] = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

Therefore amplitude of this wave is A_2 = 10

Their corresponding ratio = A_1/A_2 = 10/10 = 1

18. A mechanical wave travels along a string is described by y(x, t) = 0.005 sin (3.0t – 80x) in which numerical constants are in SI units. Calculate (a) amplitude of displacement (b) amplitude of velocity (c) wavelength (d) amplitude of acceleration (e) the time period (f) frequency of oscillation.

Sol. (a) y(x, t) = 0.005sin(3.0t – 80x)

Also, y(x, t) = A $\sin\left(\frac{2\pi t}{T} - \frac{2\pi t}{\lambda}\right)$

A = 0.005m

(b) Amplitude of velocity $\frac{2\pi}{T}$A = 3×0.005 = 0.015 ms^{-1}

(c) Wavelength, k = $\frac{2\pi}{\lambda}$ where k = 80

80 = $\frac{2\pi}{\lambda}$; $\lambda = \frac{\pi}{40}$

(d) Amplitude of acceleration = $\left(\frac{2\pi}{T}\right)^2$ A = $(3)^2$ ×0.005 = 0.045 ms^{-2}

(e) Time period; ω = $\frac{2\pi}{T}$ or T = $\frac{2\pi}{3}$ = 120sec

(f) Frequency, υ = 1/T = 0.47Hz.

19. What is the nature of sound waves in air? How is the speed of sound waves in atmosphere affected by the (i) humidity (ii) temperature?

Sol. The nature of sound wave is longitudinal in air.

(i) As v = $\sqrt{\frac{\gamma P}{\rho}}$, i.e. v $\propto \frac{1}{\sqrt{\rho}}$

The density of water vapours is less than that of dry air. Since the speed of sound is inversely proportional to the square root of density, so speed of sound increases with increase in temperature.

(ii) We know that, PV = nRT pr P = $\frac{nRT}{V}$

Also v = $\sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma nRT}{\rho V}} = \sqrt{\frac{\gamma RT}{M}}$

Where M = molecular weight of the gas. As γ, R and M are constants, so v ∝ $\sqrt{T}$, i.e. velocity of sound in a gas is directly proportional to the square root of its temperature hence we conclude that the velocity of sound in air increases with increase in temperature.

20. A stone dropped from the top of the tower 300 m high splashes into water of a pond near the base of the tower. When is the splash heard at the top? Speed of sound in air = 340m/s, g = 9.8 m/s^2.

Sol. Let t be the time taken by the stone to reach the water surface.

Here s = 300m, u = 0, a = g = 9.8ms^{-2}

As s = ut + ½ at^2

Therefore 300 = ½ × 9.8 × t^2 or t^2 = $\frac{2\times 300}{9.8}$ = 61.2

Therefore t = $\sqrt{61.2}$ = 7.82s

Time taken by the splash to reach from water surface to the top, t' = $\frac{\text{distance}}{\text{speed}} = \frac{300}{340}$ = 0.88s

Therefore time taken by the splash to be heard on the top = t + t' = 7.82+0.88 = 8.7s.

21. A steel wire has a length of 12.0 ms and a mass of 2.10kg. What should be the tension in the wire equals the speed of sound in dry air at 20°C is 343 ms^{-1}?

Sol. Speed of transverse wave in the steel wire is given by v = $\sqrt{\frac{T}{m}}$ or $v^2 = \frac{T}{m}$ or $T = v^2 m$

Given v = 343 ms^{-1}. M = 2.10/12.0 kg m^{-1}
Therefore T = $(343)^2 \times (2.10/12.0) = 2.06\times10^4$N

22. Use the formula v = √λP/r to explain why the speed of sound in air (a) is independent of pressure, (b) increases with temperature (c) increases with humidity.

Sol. (a) Take the relation v = $\sqrt{\frac{\gamma P}{\rho}}$ …(i)

Where, Density, ρ = Mass Volime = M V ρ=Mass/Volume=M/V
M=Molecular weight of the gas V= Volume of the gas

Hence, equation (i) reduces to : v = $\sqrt{\frac{\gamma PV}{M}}$ ... (ii)

Now from the ideal gas equation for n = 1: PV = RT
For constant T, PV = Constant
Since both M and γ are constants, v = Constant
Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas.

(b) Take the relation: v = $\sqrt{\frac{\gamma P}{\rho}}$ ... (i)

For one mole of an ideal gas, the gas equation can be written as: PV = RT
p = RT/V ... (ii)
Substituting equation (ii) in equation (i), we get

v = $\sqrt{\frac{\gamma RT}{V_\rho}} = \sqrt{\frac{\gamma RT}{M}}$ ... (iv)

Where, Mass , M= ρV is a constant, Y and R are also constants
We conclude from equation (iv) that v ∝ √ T.
Hence, the speed of sound in a gas is directly proportional to the square root of the temperature of the gaseous medium, i.e., the speed of the sound increases with an increase in the temperature of the gaseous medium and vice versa.
(c) Let v_m and v_d be the speeds of sound in moist air and dry air respectively
Let ρ_m and ρ_d be the densities of moist air and dry air respectively

Take the relation v = $\sqrt{\frac{\gamma P}{\rho}}$ ... (i)

And the speed of sound in dry air is: v_d = $\sqrt{\frac{\gamma PV}{M}}$ (ii)

On dividing equations (i) and (ii), we get:

$v_m/v_d = \sqrt{\frac{\gamma P}{\rho_m} \times \frac{\rho_d}{\gamma P}} = \sqrt{\frac{\rho_d}{\rho_m}}$

However, the presence of water vapour reduces the density of air, i.e.,
$\rho_d < \rho_m$
$\therefore v_m > v_d$

Hence, the speed of sound in moist air is greater than it is in dry air. Thus, in a gaseous medium, the speed of sound increases with humidity.

23. A bat emits ultrasound frequency 100 kHz in air. If this sound meets a water surface, what is the wavelength of (i) the reflected sound (ii) the transmitted sound? Speed of sound in air = 340 ms^{-1} and in water = 1486 ms^{-1}.

Sol. Here v = 100kHz = 10^5Hz, υ_d = 340 ms^{-1}, υ_w = 1486 ms^{-1}

Frequency of both the reflected and transmitted sound remains unchanged.

(i) Wavelength of reflected sound, $\lambda_a = \frac{\upsilon_a}{v} = \frac{340}{10^5} = 3.4\times10^{-3}$m

(ii) Wavelength of transmitted sound, $\lambda_w = \frac{\upsilon_w}{v} = \frac{1486}{10^5} = 1.49\times10^{-2}$m

24. A hospital uses and ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in a tissue in which the speed of sound is 1.7 kms^{-1}? The operating frequency of the scanner is 4.2 MHz.

Sol. Here υ = 1.7 kms^{-1} = 1.7×10^3 ms^{-1}, v = 4.2 MHz = 4.2×10^6Hz

Wavelength, $\lambda = \frac{\upsilon}{v} = \frac{1.7\times10^3}{4.2\times10^6} = 4.047\times10^{-4}$m

25. A transverse harmonic wave on a strong is described by:

$$Y(x, t) = 3.0 \sin(36t + 0.081x + \pi/4)$$

Where x, y are in cm and t in s. The positive direction of x is from left to right.

(i) Is this a travelling or a stationary wave? If it is travelling, what are the speed and direction of its propagation

(ii) What are its amplitude and frequency?

(iii) What is the initial phase at the origin?

(iv) Hat is the least distance between two successive crests in the wave?

Sol. Given y(x, t) = 3.0sin(36t + 0.018x + π/4) …(i)

The standard equation for a harmonic wave is y(x, t) = A $\sin\left(\frac{2\pi}{T}t + \frac{2\pi}{\lambda}x + \phi_0\right)$ …(ii)

Comparing (i) and (ii) we get A = 3.0, $2\pi/T = 36$, $2\pi/\lambda = 0.018$, $\phi_0 = \pi/4$

(i) The given equation represents travelling wave propagating from right to left (as x term is +ve)

$\upsilon = \frac{\lambda}{T} = \frac{\lambda/2\pi}{T/2\pi} = \frac{1/0.018}{1/36} = \frac{36}{0.018}$ = 2000 cms^{-1} = 20ms^{-1}

(ii) Amplitude, A = 3.0cm

Frequency, $v = \frac{1}{T} = \frac{36}{2\pi} = \frac{18}{3.14} = 5.73s^{-1}$

(iii) Initial phase at the origin, $\phi = \pi/4$ rad

(iv) Least distance between two successive crests is equal to wavelength,

$\lambda = 2\pi/0.018$ = 349.0cm = 3.49m.

26. Given below are some functions of x and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all

(a) $y = 2\cos(3x)\sin(10t)$ (b) $y = 2\sqrt{x - vt}$

(c) $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$ (d) $y = \cos x \sin t + 2x \sin 2t$

Sol. (a) As the function is the product of two separate harmonic functions of x and t, so it represents a stationary wave.

(b) It cannot represent any type of wave.

(c) Here $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t) = 3\sin\theta + 4\cos\theta$ [$\theta = 5x - 0.5t$]

If we put $A\cos\alpha = 3$ and $A\sin\alpha = t$ then $y = a\sin(\theta + \alpha)$

It represents a simple harmonic travelling wave of amplitude, $A = \sqrt{3^2 + 4^2} = 5$ and $\alpha =$ $= \tan^{-1}(4/3)$.

(5 marks Questions)

27. For the wave described by $y(x, t) = 3.0\sin(36t + 0.018x + \pi/4)$. Plot the displacement (u) versus (t) graphs for x = 0, 2 and 4cm. What are the shaped of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

Sol. All the waves have different phases.

The given transverse harmonic wave is:

$y(x,t) = 3.0\sin(36t + 0.018x + \pi/4)$ ...(i)

For x = 0, the equation reduces to:

$y(0,t) = 3.0\sin(36t + \pi/4)$

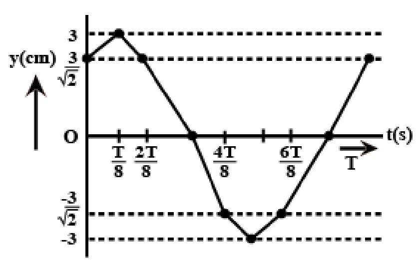
Also, $\omega = 2\pi/t = 36 \text{rad/s}^{-1}$

$\therefore t = \pi/18 s$

Now, plotting y vs. t graphs using the different values of t, as listed in the given table

t	0	T/8	2T/8	3T/8	4T/8	5T/8	6T/8	7T/8	T
y	$\frac{\sqrt{3}}{2}$	3	$\frac{\sqrt{3}}{2}$	0	$\frac{-\sqrt{3}}{2}$	- 3	$\frac{-\sqrt{3}}{2}$	0	$\frac{\sqrt{3}}{2}$

For x=0,x=2, and x=4, the phases of the three waves will get changed. This is because amplitude and frequency are invariant for any change in x. The y-t plots of the three waves are shown in the given figure.



28. For the travelling harmonic wave y(x, t) = 2.0 cos 2π(10t – 0.0080x + 0.35s), where x and y are in cm and t is in s. Calculate the phase difference between oscillatory motion of two points by a distance of (a) 4m (b) 0.5m (c) λ/2 (d) 3λ/4.

Sol. Given equation of a travelling harmonic wave is

y(x, t) = 2.0 cos 2p(10t – 0.0080x + 0.35) …(i)

The standard equation of a travelling harmonic wave is

y(x, t) = $A\cos\left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \phi_0\right]$ …(ii)

Comparing eqn (i) and (ii) we get $\frac{2\pi}{\lambda} = 2\pi \times 0.0080\text{cm}^{-1}$ …(iii)

$= \frac{2\pi}{\lambda} = 2\pi \times 10 \text{ and } \phi_0 = 0.35$

We know that phase difference = $\frac{2\pi}{\lambda}$ × path difference …(iv)

(a) When path difference = 4m = 400cm, then from (iv)

Phase difference = $\frac{2\pi}{\lambda}$×400 = 2π × 0.0080 × 400 [by using (iii)]

= 6.4π rad.

(b) When path difference = 0.5m = 50cm then phase difference = 2π ×0.0080×50 = 0.8π rad

(c) When path difference λ/2, then phase difference = $\frac{2\pi}{\lambda} \times \frac{\lambda}{2}$ = π rad

(d) When path difference = $\frac{2\pi}{\lambda}$

Phase difference = $\frac{2\pi}{\lambda} \times \frac{2\pi}{\lambda} = \frac{3\pi}{2}$rad = $\left(\pi + \frac{\pi}{2}\right)$

∵ cos (π +θ) = - cos θ

∴ Effective phase difference = $\frac{\pi}{2}$ rad

29. The equation of a plane progressive wave is given by equation: y = 10 sin 2π(t – 0.005x), where x and y are in cm and t in seconds. Calculate (i) amplitude (ii) frequency (iii) wavelength (iv) velocity of wave.

Sol. Here y = 10 sin 2π(t – 0.005x)

y = 10sin$\frac{2\pi}{200}(200t - x)$ …(i)

The equation of a travelling wave is given by

y = a sin$\frac{2\pi}{\lambda}$(vt –x) …(ii)

Comparing the equations (i) and (ii) we have a = 10cm, A = 200cm and v = 200 cms^{-1}

Now frequency, $\upsilon = \frac{v}{\lambda} = \frac{200}{200}$ = 1 Hz.

30. A transverse harmonic wave on a string is described by y(x, t) = 3.0sibn (36t + 0.018x + π/4) where x and y are in cm and t in s. The positive direction of x is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?
(b) What are its amplitude and frequency?
(c) What is the initial phase at the origin?
(d) What is the least distance between two successive crests in the wave?

Sol. The equation of the form y(x,t) = A $\sin\left(\frac{2\pi}{\lambda}(\upsilon t + x) + \phi\right)$ …(i)
represents a harmonic wave of amplitude A, wavelength l and traveling from right to left with a velocity v.
Now the give equation for the transverse harmonic wave is
y(x, t) = 3.0sin(36t + 0.018x + π/4) = 3.0sin $\left[0.018\left(\frac{36}{0.018}t + x\right) + \frac{\pi}{4}\right]$
= 3.0sin [0.018(2000t +x) + π/4) …(ii)
(a) Since the equation (i) and (ii) are of the same form, the gien equation also represents a travelling wave propagating from right to left. Further the coefficient of t gives the speed of the wave. Therefore v = 2000cms^{1} = 20ms^{-1}
(b) Obviously amplitude, A = 3.0cm
Further $\frac{2\pi}{\lambda}$ = 0.018 or λ = $\frac{2\pi}{0.018}$cm
Therefore $\upsilon = \frac{v}{\lambda} = \frac{2000}{2\pi} \times 0.018$ = 5.73 s^{-1}
(c) Initial phase at origin, ϕ = π/4 rad
(d)Least distance between two successive crests in the wave is equal to wavelength. Therefore λ = $\frac{2\pi}{0.018}$ = 349.0cm = 3.49m

31. A standing wave set up in a medium is given by $y = 4\cos\left(\frac{\pi x}{3}\right)$ where x and y are in cm and t is in seconds. (i) Write the equation of the two component waves and give amplitude and velocity of each wave. (ii) What is the distance between the adjacent nodes? (iii)What is the velocity of the particle of the medium at x = 3cm and time t = 1/8s?

Sol. (i) The standing wave is formed by superposition of the waves y_1 = 2sin(40πt = πx/3) and y_2 = 2sin(40πt + πx/3) as y = y_1 + y_2 and their amplitude = 2cm, ω = 40π rads^{-1} and k = π/3 rad cm^{-1}
Velocity, v = $\frac{\omega}{k} = \frac{40\pi}{\pi/3} = 120\ \text{cm/s}$
(ii) $\because \frac{2\pi}{\lambda} = k$ or $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/3} = 6\text{cm}$
Distance between adjacent nodes = λ/2 = 3cm
(iii) Particle velocity at v = 3cm at time t = 1/8s
v = $\frac{dy_1}{dt} = 80\pi\cos\left(40\pi t - \frac{\pi x}{3}\right) = 80\pi\cos\left(\frac{40\pi}{8} - \frac{\pi\times 3}{3}\right) = 80\pi\ \text{m/s}$

32. The transverse displacement of a string (clamped at its two ends) is given by y(x, t) = 0.06 sin ($2\pi/3$) × cos $120\pi t$ where x, y are in m and t in s. The length of the string is 1.5m and its mass is 3.0×10^{-2}kg. Answer the following:

(a) Does the function represent a travelling or a stationary wave?

(b) Interpret the wave as a superposition of two waves travelling in opposite directions. What are the wavelength frequency and speed of propagation of each wave?

(c) Determine the tension in the string.

Sol. $y(x, t) = 0.06 \sin\frac{2\pi}{3} \times \cos 120\,\pi t$ …(i)

(a) The displacement which involves harmonic functions of x and t separately represents a stationary wave and the displacement, which is harmonic functuion of the form (ct +x) represents a travelling wave. Hence the equation given above represents a stationary wave.

(b) When a wave pulse $y_1 = a \sin\frac{2\pi}{\lambda}(ct - x\_$ travelling along x axis is superimposed by the reflected pulse.

$y_2 = -a \sin\frac{2\pi}{\lambda}(ct + x)$ from the other end, a stationary wave is formed and is given by

$y = y_1 + y_2 = -a\sin\frac{2\pi}{\lambda} \times \cos\frac{2\pi}{\lambda}vt$ …(ii)

On comparing (i) and (ii) we have

$\frac{2\pi}{\lambda} = \frac{2\pi}{3}$ or $\lambda = 3\text{m}$

$\frac{2\pi}{\lambda}v = 120\pi$ or $v = 60\lambda = 60 \times 3 = 180\text{ms}^{-1}$

Now frequency $\upsilon = \frac{v}{\lambda} = \frac{180}{3} = 60$ Hz

(c) Velocity of transverse wave in a string is given by $v = \sqrt{\frac{T}{\mu}}$

Here $\mu = \frac{3\times10^{-2}}{1.5} = 2 \times 10^{-2}\text{kgm}^{-1}$

Also v = 180 ms^{-1}

Therefore $T = v^2\mu = (180)^2 \times 2\times10^{-2} = 648\text{N}$.

C. REFLECTION AND REFRACTION OF WAVES

(1 Mark Questions)

1. The phenomenon of echo is an example of

(a) reflection (b) refraction (c) beat (d) resonance

Sol. (a)

Like all waves, sound waves can be reflected. Sound waves suffer reflection from the large obstacles. As a result of reflection of sound wave from a large obstacle, the sound is heard which is named as an echo. Ordinarily echo is not heard as the reflected sound gets merged with the original sound. Certain conditions have to be satisfied to hear an echo distinctly (as a separate

sound).
Hence, the phenomenon of echo of sound waves is due to reflection.

2. When you shout in front of a hill, your own shout is repeated. Explain.

Sol. The sound is heard more than once because of the time difference between the initial production of the sound waves and their return from the reflecting surface. For example, when we shout or clap near a suitable reflecting object such as a tall building or a mountain, we will hear the same sound again a little later.

(2 Marks Questions)

3. Explain why we cannot hear an echo in a small room?

Sol. For an echo of a simple sound to be heard, the minimum distance between the speaker and the walls should be 17m. so in any room having length less than 17m, our ears cannot distinguish between sound received directly and sound received after reflection due to persistence of hearing. The sensation of hearing of any sound persists in our brain for 0.1s.

4. What do you mean by reverberation? What is reverberation time?

Sol. The phenomenon of persistence or prolongation of sound after the source has stoped emitting sound is called reverberation. The time for which the sound persists until it becomes inaudible is called the reverberation time.

D. SUPERPOSITION OF WAVE

E. STANDING WAVES

(1 Mark Questions)

1. Why do stationary waves not transport energy?

Sol. There is no energy transport in a standing wave because the two waves that make them up carry equal energy in opposite directions.

2. When you shout in front of an open organ pipe, what happens to the wavelength of the fundamental note?

Sol. In an open organ pipe, the distance between any two consecutive nodes or antinodes is equal to the half of the wavelength. Thus, for an organ pipe, the wavelength of the fundamental note is twice the length of the organ pipe.

3. When are stationary waves produced?

Sol. Stationary waves are produced when two exactly identical progressive waves (having the same amplitude, wavelength & same speed) traveling through a medium along the same path in exactly opposite directions, interfere with each other.

(2 Marks Questions)

4. What are the differences between stationary waves and progressive waves?

Sol.

	Stationary waves	**Progressive waves**
(i)	The disturbance remains confined to a particular region, and there is no onward motion.	The disturbance travels forward, being handed over from one particle to the neighbouring particles.
(ii)	There is no transfer of energy in the medium.	Energy is transferred in the medium along the waves.
(iii)	The amplitude of vibration of particles varies from zero at nodes to maximum at antinodes.	The amplitude of vibration of each particle is same.

5. Differentiate between harmonics and overtones.

Sol. A harmonic is made of vibration that is a whole number multiple of the fundamental mode. The first harmonic is the fundamental frequency. The second harmonic is twice its frequency etc. Many instruments, especially bells, oscillate in modes that are not whole number multiples of the fundamental frequency. These higher modes are called overtones. Overtones include harmonic, but harmonic do not include overtones. The first overtone is not the fundamental. The second harmonic is the first overtone.

(3 Marks Questions)

6. Give any three differences between progressive wave and stationary wave. A stationary wave is y = 12 sin 300t cos 2x. What is the distance between two nearest modes?

Sol. Same as Q 4

Node: Point of wave where velocity of particle is maximum e.g., mean position of a wave is known as node.

$y = 12 \sin 300t.\cos 2x$ [use trigonometric formula, $2\sin A.\cos B = \sin(A+B\_) + \sin(A - B)$]

Now $y = 6[\sin(300t + 2x) + \sin(300t - 2x)]$

$= 6\sin(300t + 2x) + 6\sin(300t - 2x)$

At mean postionnodwe will be formed so y = 0

$\sin(300t + 2x) = -\sin(300t - 2x) = \sin(300t + 2x) = \sin(\pi + 300t - 2x)$

$4x = -\pi$ or $x = -\pi/4$

Hence node formed at $x = \pi/4$ and $-\pi/4$. So distance between node = $\pi/4 + \pi/4 = \pi/2$.

7. An open pipe has a fundamental frequency of 240 Hz. The first overtone of a closed organ pipe has the same frequency as the first overtone of the open pipe. How long is each pipe? Velocity of sound at room temperature is 350m/s.

Sol. The fundamental frequency of the open organ pipe is 240Hz.

The first overtone of an open organ pipe is also the second harmonic.

So its frequency is 240×2 = 480Hz.

If υ_c be the fundamental frequency of a closed organ pipe, the frequency of its first overtone which is also the third harmonic is $3\upsilon_c$.

$3\upsilon_c = 480$ or $\upsilon_c = 160$Hz

The length of closed organ pipe is given by $\upsilon_C = \frac{v}{4L} \Rightarrow L = \frac{v}{4\upsilon_C} = \frac{350}{4\times160} = 54.7$cm

The length of organ open pipe is given by $\upsilon_0 = \frac{v}{2L}$

$L = \frac{v}{2\upsilon_0} = \frac{350}{2\times240} = 72.9$cm.

8. The length of a wire between the two ends of a sonometer is 105cm. Where should the two bridges be placed so that the fundamental frequencies of the three segments are in the ratio of 1:5:15?

Sol. Total length of the wire, L = 105cm

$\upsilon_1:\upsilon_2:\upsilon_3 = 1:5:15$

Let L_1, L_2 and L_3 be the length of the three parts.

As $\upsilon \propto 1/L$

Therefore $L_1:L_2:L_3 = 15:5:1$

Sum of the ratios = 15+5+1=21

Therefore $L_1 = 15/21 \times 105 = 75$cm, $L_2 = 5/21 \times 105 = 25$cm, $L_3 = 1/20 \times 105 = 5$cm

Hence the bridges should be placed at 75cm and (75+25) = 100cm from one end.

9. A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a turning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effect may be neglected.

Sol. The frequency of nth mode of vibration for a closed end pipe is given by

$v_n = \frac{(2n-1)\upsilon}{4L}$ where n = 1, 2, 3, …

Suppose the lengths $L_1 = 25.5$cm and $L_2 = 79.3$cm of the resonance columns correspond to $n = n_1$ and $n = n_2$ respectively. Then

$v = \frac{(2n_1-1)\upsilon}{4L_1} = \frac{(2n_2-\upsilon)}{4L_2}$ …(1)

$\textbf{or } 340 = \frac{(2n_1-1)\upsilon}{4\times 25.5} = \frac{(2n_2-1)\upsilon}{4\times 79.3}$ or $\frac{2n_1-1}{2n_2-1} = \frac{25.5}{79.3} = \frac{1}{3}$

This is possible if $n_1 = 1$ and $n_2 = 2$. Thus the resonance length 25.5cm corresponds to the fundamental note and 79.3cm corresponds to first overtone or third harmonic. From equation (1) we get

$\upsilon = \frac{4L_1 v}{2n_1 - 1} = \frac{4\times 25.5\times 340}{2\times 1 - 1} = 34680 \text{ cms}^{-1} = 346.8 \text{ ms}^{-1}$.

(5 Marks Questions)

10. What are stationary waves? Explain the formation of stationary waves graphically.

Sol. Stationary Waves are defined as a combination of two waves having equal amplitude and frequency but moving in opposite directions. A standing wave is formed due to interference. Specifically, a standing wave is a wave that oscillates in time but its peak altitude profile does not move in space.

Explanation: Consider two harmonic progressive waves (formed by strings) that have the same amplitude and same velocity but move in opposite directions. Then the displacement of the first wave (incident wave) is

$y_1 = A \sin(kx - \omega t)$ (waves move toward right) …(1)

and the displacement of the second wave (reflected wave) is

$y_2 = A \sin(kx + \omega t)$ (waves move toward left) …(2)

both will interfere with each other by the principle of superposition, the net displacement is $y = y + y_2$ …… (3)

Substituting equation (1) and equation (2) in equation (3), we get

$y = A \sin(kx - \omega t) + A \sin(kx + \omega t)$ …(4)

Using trigonometric identity, we rewrite equation (4) as

$y(x, t) = 2A \cos(\omega t) \sin(kx)$ …(5)

This represents a stationary wave or standing wave, which means that this wave does not move either forward or backward, whereas progressive or travelling waves will move forward or backward. Further, the displacement of the particle in equation (5) can be written in more compact form,

$y(x, t) = A' \cos(\omega t)$ where, $A' = 2A \sin$ (foe), implying that the particular element of the string executes simple harmonic motion with amplitude equals to A'. The maximum of this amplitude occurs at positions for which

$$\sin(kx) = 1 \Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots.. = m\pi$$

where m takes half integer or half integral values. The position of maximum amplitude is known as antinode.

Expressing wave number in terms of wavelength, we can represent the anti-nodal positions as

$$x_m = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2}, \text{ where, } m = 0,1,2... \qquad ...(6)$$

For $m = 0$ we have maximum at $x_0 = \frac{\lambda}{2}$

For $m = 1$ we have maximum at $x_1 = \frac{3\lambda}{4}$

For $m = 2$ we have maximum at $x_2 = \frac{5\lambda}{4}$ and so on.

The distance between two successive antinodes can be computed by

$$x_m - x_{m-1} = \left(\frac{2m+1}{2}\right)\frac{\lambda}{2} - \left(\frac{(2m+1)+1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{2}$$

Similarly, the minimum of the amplitude A' also occurs at some points in the space, and these points can be determined by setting sin (kx) = 0 ⇒ kx = 0, π, 2π, 3π, … = nπ where n takes integer or integral values. Note that the elements at these points do not vibrate (not move), and the points are called nodes. The nth nodal positions is given by,

$$x_n = n\frac{\lambda}{2} \text{ where, } n = 0,1,2,... \qquad ...(7)$$

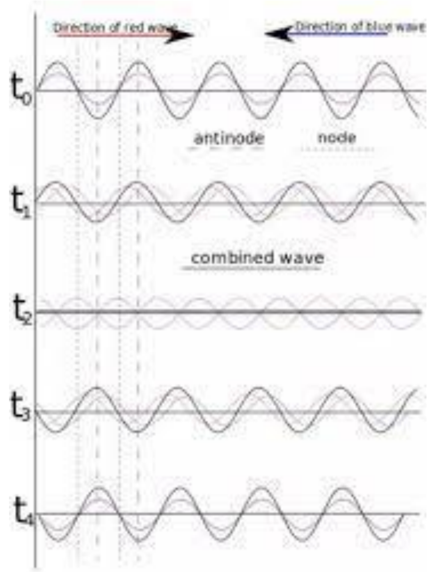
For $n = 0$ we have minimum at $x_0 = 0$

For $n = 1$ we have minimum at $x_1 = \frac{\lambda}{2}$

For n = 2 we have maximum at $x_2 = \lambda$ and so on.

The distance between any two successive nodes can be calculated as

$$x_n - x_{n-1} = n\frac{\lambda}{2} - (n-1)\frac{\lambda}{2} = \frac{\lambda}{2}$$



F. BEATS

(1 Mark Questions)

1. When two waves of almost equal frequencies v_1 and v_2 reach at a point simultaneously, the time interval between successive maxima is

   (a) $v_1 + v_2$ (b) $v_1 – v_2$ (c) $1/ v_1 + v_2$ (d) $1/ v_1 – v_2$

Sol. (d)

When two waves of almost equal frequencies v_1 and v_2 reach at a point simultaneously, beats are produced.

Beat frequency, $v_{beat} = v_1 – v_2$

Time interval between successive maxima = $1/v_{beat} = 1/v_1 – v_2$.

2. Which of the following phenomenon is used by the musicians to tune their musical instruments?

   (a) interference (b) diffraction (c) beats (d) polarization

Ans. (c)

3. Why do we not hear beats due to sound waves emitted by the violin section of an orchestra?

Sol. All the violin section of an orchestra are tuned to the same frequency. Since there is no difference in the frequencies of these violins, no bears are heard. (For beats to be heard, the frequencies should be slightly different).

4. Two sound source produce 12 beats in 4 seconds. By how much do their frequencies differ?

Sol. Beat frequency is equal to the beats produced per second. Therefore $v = 12/4 = 3$Hz.

(2 Marks Questions)

5. If two sound waves of frequencies 480Hz and 536Hz superpose, will they produce beats? Would you hear the beats?

Sol. Yes, the sound waves will produce 56 beats every second. But due to persistence of hearing, we would not be able to hear these beats.

6. Why is the sonometer box hollow and provided with holes?

Sol. When the stem f the tuning fork gently pressed against the top of sonometer box, the air enclosed in box also vibrates and increases the intensity of sound. The holes bring the inside air in contact with the outside air and check the effect of elastic fatigue.

7. How does the frequency of a tuning fork change, when the temperature is increased?

Sol. As the temperature increases, the length of the prong of the tuning fork increases and Young's modulus changes. So frequency of the tuning fork decreases.

(3 Marks Questions)

8. Calculate the speed of sound in a gas in which two waves of lengths 100cm and 101cm produce 24 beats in 6 seconds.

Sol. Here λ_1 = 100cm = 1m, λ_2 = 101cm = 1.01m

Let v be the velocity of the sound in the gas. Then $\upsilon_1 = \frac{v}{\lambda_1}$ or $\frac{v}{1}$ and $\upsilon_2 = \frac{v}{\lambda_2} = \frac{v}{1.01}$

Beat frequency $\upsilon_1 \square \upsilon_2$ = 24/6 = 4Hz

v = v/1.01 = 4 of v = 404m/s.

9. Two sitar strings A and B playing the note 'Ga' are slight out of tune and produce beats of frequency 6Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3Hz. If the original frequency of A I s324Hz, what is the frequency of B?

Sol. We know that $\upsilon \propto \sqrt{T}$ where υ = frequency, T = tension

The decrease in the tension of a string decreases its frequency. So let us assume that original frequency υ_A of A is more than υ_B of B.

Thus $\upsilon_A - \upsilon_B = \pm$ 6Hz (given) and υ_A = 324Hz.

Therefore $324 - \upsilon_B = \pm 6$ or $\upsilon_B = 324 \pm 6$ = 318Hz or 330Hz

On reducing tension of A, $\Delta\upsilon$ = 3Hz.

If υ_B = 330Hz and on decreasing tension in A, υ_A will be reduced i.e. no. of beats will increase, but this is not so because no. of beats becomes 3. Therefore υ_B must be 318Hz because on reducing the tension in string A, its frequency may be reduced to 321Hz, thus giving 3 beats with υ_B = 318Hz.

10. What is beat phenomenon?

Sol. When two or more waves superimpose each other with slightly different frequencies, then a sound of periodically varying amplitude at a point is observed. This phenomenon is known as beats. The number of amplitude maxima per second is called the frequency.

If we have two sources, then their difference in frequency gives the beat frequency

Number of beats per second, n = $|f_1 - f_2|$ per second.

(5 Marks Questions)

11. What are beats? Explain their formation analytically? Prove that the beat frequency is equal to the difference in frequencies of the two superposing waves.

Sol. The periodic variation in the intensity of the sound is called the beats.

Explanation:

When two waves of nearly equal frequencies, traveling in the same direction superimpose, the intensity of sound at a point changes with time periodically. The periodic variation in the intensity of sound is called beats.

One loud sound followed by a faint sound or one faint sound followed by one loud sound constitute a beat.

The time-lapse between two successive loud sounds or two faint sounds is called the beat period and the reciprocal of the beat period is called beat frequency.

Analytical treatment of beats:

Let the two waves be represented by

$y_1 = rsin\omega_1 t = rsin2\pi\nu_1 t$ (1)

$y_2 = rsin\omega_2 t = rsin2\pi\nu_2 t$ (2)

If y is the resultant displacement due to superimposition of the waves , then

$$y = y_1 + y_2 = r[sin2\pi\nu_1 t + sin2\pi\nu_2 t]$$
$$= 2rsin\pi(\nu_1 + \nu_2)tcos\pi(\nu_1 - \nu_2)t$$
$$= [2rcos\pi(\nu_1 - \nu_2)t]sin\pi(\nu_1 + \nu_2)t$$
$$= Asin\pi(\nu_1 + \nu_2)t$$

Where $A=2rcos\pi(\nu_1 - \nu_2)t$ is the amplitude of the resultant wave. As intensity is directly proportional to the square of the amplitude, therefore, intensity is maximum when A is maximum. i.e.

$cos\pi(\nu_1 - \nu_2)t = \pm 1$

or $\pi(\nu_1 - \nu_2)t = n\pi$

or $t = \frac{n}{\nu_1 - \nu_2}$

i.e. At times $t = 0, \frac{1}{\nu_1 - \nu_2}, \frac{2}{\nu_1 - \nu_2}$ .... intensity will be maximum.

Therefore, the interval between two successive loud sounds is,

$T = \frac{1}{\nu_1 - \nu_2}$

Therefore, beat frequency, $b = \nu_1 - \nu_2$

i.e. The beat frequency is equal to the difference of frequencies of two superimposing harmonic waves.

Hence, proved.

G. INTERFERENCE

(1 Mark Questions)

1. Intensities of two waves, which produce interference are 9:4. The ratio of maximum and minimum intensity is

   (a)9:4 (b) 3:2 (c) 25:1 (d) 5:1

Ans. (c)

(3 Marks Questions)

2. Two periodic waves of intensities I_1 and I_2 pas through a region at the same time in the same direction. What is the sum of the maximum and minimum intensities?

Sol. Other factors such as ω and v remaining the same, $I = A^2 \times$ constant (K) or $A = \sqrt{\frac{I}{K}}$

On superposition $A_{max} = A_1 + A_2$ and $A_{min} = A_1 - A_2$

Therefore $A_{max}^2 = A_1^2 + A_2^2 + 2\,A_1A_2$

$\Rightarrow \frac{I_{max}}{K} = \frac{I_1}{K} + \frac{I_2}{K} + \frac{2\sqrt{I_1I_2}}{K}$

$A_{min}^2 = A_1^2 - A_2^2 - 2A_1A_2$

$\Rightarrow \frac{I_{min}}{K} = \frac{I_1}{K} - \frac{I_2}{K} - \frac{2\sqrt{I_1I_2}}{K}$

Therefore $I_{max} + I_{min} = 2I_1 + 2I_2$

H. DOPPLER'S EFFECT

(1 Mark Questions)

1. Doppler effect is applicable for

   (a) sound waves only (b) light waves only

   (c) both sound and light waves (d) none of these

Ans. (c)

2. A train approaching a railway platform with a speed of 20m/s starts blowing the whistle. Speed of sound in air is 340 m/s. If the frequency of the emitted sound from the whistle is 640Hz, the frequency of sound as heard by the person standing on the platform is

   (a) 600 Hz (b) 640 Hz (c) 680Hz (d) 720 Hz

Sol. (c)

The apparent frequency of sound, observed by a stationary listener , when a source of sound is approaching to listener , is given by

$v' = \upsilon\left(\frac{v}{v - v_s}\right)$

Given υ =640Hz, v = 340m/s, v_s = 20m/s

Hence v' = $\mathbf{640}\left(\frac{340}{340-20}\right) = \frac{640\times340}{320}$=680Hz.

3. What is Doppler effect?

Sol. An increase (or decrease) in the frequency of sound, light, or other waves as the source and observer move towards (or away from) each other. The effect causes the sudden change in pitch noticeable in a passing siren, as well as the red shift seen by astronomers.

4. An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. The percentage change in the apparent frequency is

(a) zero (b) 5% (c) 10% (d) 20%

Ans. (d)

(2 Marks Questions)

5. What is the speed of the observer for whom a note is 10 percent lower than the emitted frequency?

Sol. As the apparent frequency (υ) is less than the emitted frequency (υ), the observer must move away from the source. If v is the speed of sound and v_0 that of the observer, then v' = $\left(\frac{v-v_0}{v}\right)\upsilon$

As apparent frequency is 10% lower than the emitted frequency,

$\therefore \upsilon' = \upsilon - \frac{10}{100}\upsilon = \frac{90}{100}\upsilon = 0.9\upsilon$

Or $0.9\upsilon = \left(\frac{v-v_0}{v}\right)\upsilon$ or $0.9 = \left(\frac{v-v_0}{v}\right)$

Or $0.9v = v - v_0$ or $v_0 - v - 0.9v = 0.1v$

Thus the speed of the observer is $(1/10)^{th}$ of the speed of sound.

6. A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the Sonar with a speed of 360km/h. What is the frequency of sound reflected by the submarine. Take the speed of sound in water to be 1450m/s.

Sol. Frequency, υ = 45 kHz, speed of sound = 1450 ms^{-1}, speed of enemy submarine = 360 km h^{-1} = 100 ms^{-1}.

Firstly sound is observed by enemy submarine. Here observer (enemy submarine) is moving towards the source (SONAR), so v_a = 100 ms^{-1}, v_s = 0. Frequency of sonar waves received by the enemy submarine

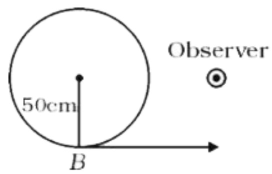
$\upsilon' = \frac{v-v_0}{v-v_s}\times\upsilon = \frac{1450+100}{1450-0}\times 40 = \frac{1550}{1450}\times 40$ = 42.76 kHz

After the sound is reflected, enemy submarine acts as a source of sound of frequency υ'. This source moves with a speed of 100 ms^{-1} towards the observer (SONAR) so v_s = +100 ms^{-1}, v_0 = 0. Frequency of sound reflected by the enemy submarine,

$\upsilon'' = \frac{v-v_0}{v-v_s}\times\upsilon' = \frac{1450-0}{1450-100}\times 42.76$ = 45.93 kHz.

(3 Marks Questions)

7. A whistle revolve in a circle with angular velocity $\omega = 20$ rad/s. If the frequency of the sound is 385Hz and speed is 340 m/s, then find the frequency heard by the observer when the whistle is at B.



Sol. Speed of source (whistle), $v_s = r\omega = 0.5\times20 = 10$m/s

Actual frequency, $\upsilon = 385$ Hz, speed of sound, v = 340 m/s

When the whistle (at point B) is moving towards the observer, then frequency

$\upsilon' = \frac{v}{v-v_s} \times \upsilon = \frac{340}{340-10} \times 385 = 396.7$Hz

8. A railway engine and a car are moving parallel but in opposite direction with velocities 144 km/h and 72 km/h respectively. The frequency of engine's whistle is 500Hz and the velocity of sound in 340 m/s. Calculate the frequency of sound heard in the car when (i) the car and engine are approaching each other (ii) both are moving away from each other.

Sol. Apparent frequency, $f' = f_0\left(\frac{v+v_0}{v-v_0}\right)$

(i)$f' = 500\left(\frac{340+144\times5/18}{340-72\times5/18}\right) = 500 \times \frac{380}{320} = 593.75$Hz

(ii) $f' = f_0\left(\frac{v-v_0}{v+v_s}\right) = 500 \times \left(\frac{340-144\times5/8}{340+72\times5/8}\right) = 500 \times \frac{300}{360} = 416.67$ Hz

(5 marks Questions)

9. A train standing in a station yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10m/s. What are frequency, wavelength and speed of sound for an observer standing on the station platform? Is the situation exactly identical to the case when the air is still and the observer runs toward the yard at a speed of 10m/s? The speed of sound in still air can be taken as 340m/s.

Sol. Here v = 340m/s. $\upsilon = 400$Hz

(a) Speed of wind, $v_w = 10$m/s

As the direction of blow of wind (yard to station) is the same as the direction of sound, therefore, for the observer standing on the platform;

Velocity of sound, $V' = v + v_w = 340+10 = 250$m/s

As there is no relative motion between source of the sound and the observer; the frequency of sound will remain unchanged.
Thus frequency of sound = 400Hz
Wavelength of sound, $\lambda' = \frac{v'}{\upsilon} = \frac{350}{400}$ = 0.875 m
(b) Speed of the observer, v_0 = 10m/s (towards yard). When the observer moves towards the source of sound, the apparent frequency,
$\upsilon' = \frac{v+v_0}{v}\upsilon = \frac{340+10}{340} \times 400$ = 411.5 Hz
The wavelength of sound waves is not affected due to the motion of the observer and hence the wavelength of the sound waves will remain unchanged.
Speed of sound relative to the observer = 340+10 = 350m/s.
Therefore, situations (a) and (b) are not equivalent.

10. A train standing at the outer signal of a railway station blows a whistle of frequency 400Hz in still air.
(i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10m/s. (b) recedes from the platform with a speed of 10m/s.
(ii) What is the speed of sound in each case if the speed of sound in still air is 340 m/s.

Sol. (i) Here υ = 400Hz, v = 340m/s
(a) Train approaches the platform, v_s = 10m/s
$\upsilon' = \frac{v}{v-v_s} \times \upsilon = \frac{340\times400}{340-10}$ = 412.12 Hz
(b) Train recedes from the platform, v_s =10m/s
$\upsilon' = \frac{v}{v+v_s} \times \upsilon = \frac{340\times400}{340+10}$ = 388.6Hz
(ii) The speed of speed in each case remains same i.e. 340m/s.

11. (a) What is Doppler effect?
(b) Derive an expression for the apparent frequency when a source moves towards a stationary observer.
(c) A policeman on duty detects a drop of 15% in the pitch of the horn of a motor car as it crosses him. Calculate the speed of car, if the velocity of sound is 330 m/s.

Sol. (a) A Doppler effect: The Doppler effect or Doppler shift (or simply Doppler, when in context) is the change in frequency of a wave in relation to an observer who is moving relative to the wave source.
(b) Let n = actual frequency of the source, n_0 = apparent frequency of the source, v = velocity of sound in air, v_s = velocity of the source, v_1 = velocity of the observer.
If the source is moving towards observer then apparent frequency is given by
n = $n_0 \left(\frac{v}{v-v_s}\right)$ i.e. apparent frequency increases.

(c) $\frac{f_1}{f_2} = \left[\frac{c}{c-v} / \frac{c}{c+v}\right]$

$= \frac{100}{90} = \frac{c+v}{c-v}$

100c – 100v = 90c – 90v

10c = 190v

v = 10/190 c = 1/19 ×330m/s

v = 17.3 m/s.

12. Explain why (or how):

(a) in a sound wave, a displacement node is a pressure antinode and vice versa.

(b) bats can ascertain distances, directions, nature and sizes of the obstacles without any "eyes".

(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes.

(d) solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and

(e) the shape of a pulse gets distorted during propagation in a dispersive medium.

Sol. (a) At the displacement node (the point of zero displacement), the variation of pressure is maximum. Hence the displacement node is the pressure antinode and vice-versa.

(b) Bats can produce and detect ultrasonic waves (sound waves of frequencies above 20KHz). (i) From the interval of time between their producing the waves and receiving the echo after the reflection from an object, they can estimate the distance of the object from them. (ii) From the intensity of the echo, they can estimate the nature and size of the object. (iii) Also, from the small time interval between the reception of the echo by their two ears, they can determine the direction of the object.

(c) The instruments produce different overtones (integral multiples of fundamental frequency). Hence the quality of sound produced by the two instruments of even same fundamental frequency is different.

(d) Solids have both volume and shear elasticity. So both longitudinal and transverse waves can propagate through them. On the other hand, gases have only volume elasticity and not shear elasticity. So only longitudinal waves can propagate through them.

(e) When the pulse passes through a dispersive medium, the wavelength of the wave changes. Consequently, the shape of the pulse changes i.e. it gets distorted.

13. A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 ms~1. (b) recedes from the platform with a speed of 10 ms^{-1}(ii) What is the speed of sound in each case? The speed of sound in still air can be taken as 340 ms^{-1} .

Sol. Given, f = 400 Hz, v = 340 ms^{-1},v_s = 10 ms^{-1}

(i) speed of train .

(a) train approaches the platform

$$f' = \frac{v}{v - v_s} \times f$$

$$= \frac{340}{340 - 10} \times 400$$

$$= 412 \text{ Hz}$$

(b) train recedes from the platform

$$f' = \frac{v}{v + v_s} \times f$$

$$= \frac{340}{340 + 10} \times 400$$

$$= 389 \text{ Hz}$$

(ii). The speed of sound does not change, i.e., it is 340 ms^{-1} for both cases.

I. ASSERTION REASON TYPE QUESTIONS:

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false **(d) If both assertion and reason are false**

(e) If assertion is false but reason is true

1. Assertion: Transverse waves are not produced in liquids and gases.
Reason: Light waves are transverse waves.

Ans. (e) Assertion is false, but reason is true.

2. Assertion: Sound passes through air in the form of longitudinal waves.
Reason: Longitudinal waves are easier to propagate.

Ans. (c) Assertion is true but reason is false
A longitudinal wave motion travel in the form of compressions and rarefaction which involve changes in volume and density of the medium. As air possesses volume elasticity, therefore sound comes to us from the source in the form of longitudinal waves only.

3. Assertion: Particle velocity and wave velocity both are independent of time.
Reason: For the propagation of wave motion, the medium must have the properties of elasticity inertia.

Ans. (e) Assertion is false but reason is true
The velocity of every oscillating particle of the medium is different of its different positions in one oscillation but the velocity of wave motion is always constant i.e. particle velocity vary with respect to time, while the wave velocity is independent of time.

4. Assertion: Wave produced by a motor boat sailing in water are both longitudinal and transverse waves.
Reason: The longitudinal and transverse waves cannot be produced simultaneously.

Ans. (c) Assertion is true but reason is false

The propeller of a motor boat cuts the water surface laterally and also pushes it in backward direction. Hence it will result in both longitudinal and transverse waves.

5. Assertion: Violet shift indicates decrease in apparent wavelength of light.

Reason: Violet shift indicates decrease in apparent wavelength of light.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.

As $\lambda_c < \lambda_r$. therefore violet shift means apparent wavelength of light from a star decreases. Obviously apparent frequency increases. This would happen when the star is approaching the earth.

J. CHALLENGING PROBLEMS:

1. One end of a long string of linear mass density 8.0 x 10^{-3} kg m^{-1} is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At t = 0, the left end (fork end) of the string x = 0 has zero transverse displacement (y = 0) and is moving along positive y-direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement y as function of x and t that describes the wave on the string.

Sol. Tension in the string, T = 90×9.8 = 882N; Mass per unit length of the string, m= 8.0×10^{-3} kg m^{-1}; Frequency of the wave, v = 250Hz; Amplitude of the wave, A = 5.0cm = 0.05m

The velocity of the transverse wave along the string is

$\upsilon = \sqrt{\frac{T}{m}} = \sqrt{\frac{882}{8\times10^{-3}}} = 3.32 \times 10^2 ms^{-1}$

Angular frequency, $\omega = 2\pi v = 2\times3.142\times256 = 16.1\times10^2$ rad s^{-1}

As $\upsilon = \frac{\omega}{k}$ $\therefore k = \frac{\omega}{\upsilon} = \frac{16.1\times10^2}{3.31\times10^2} = 4.84$ rad m^{-1}

As the wave propagates along the positive X axis, so the displacement equation is

$y = A \sin(\omega t - kx)$ or $y = 0.05\sin(16.1\times10^2 t - 4.84x)$, x and y are in m.

2. A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be 1450 ms^{-1}.

Sol. Same as Q 6 Sec H.

3. Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about 4.0 km s^{-1} . A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur?

Sol. Suppose the earthquake occurs at a distance of x km from the seismograph.

Speed of S wave = 4.0 km s^{-1}
Time taken by the S wave to reach the seismograph = x/4 s
Speed of P wave = 8.0 kms^{-1}
Time taken by the P wave to each the seismograph = x/8 s
But $\frac{x}{4} - \frac{x}{8}$ = 4×60s
Therefore x = 1920 km.

4. A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

Sol. As the bat approaches the stationary flat wall surface, the apparent frequency is

$$v' = \frac{\upsilon}{\upsilon - \upsilon_s} v$$

The stationary flat surface (source) reflects the sound of frequency v' to the bat (observer) moving towards the flat surface. So the apparent frequency is

$$v'' = \frac{\upsilon + \upsilon_0}{\upsilon} \times v' = \frac{\upsilon + \upsilon_0}{\upsilon} \times \frac{\upsilon}{\upsilon - \upsilon_s} v = \frac{\upsilon + \upsilon_0}{\upsilon - \upsilon_s} \times v$$

$$= \frac{\upsilon + 0.03\upsilon}{\upsilon - 0.03\upsilon} \times v = \frac{1.03}{0.97} \times 40\text{kHz} = 42.47 \text{ kHz.}$$