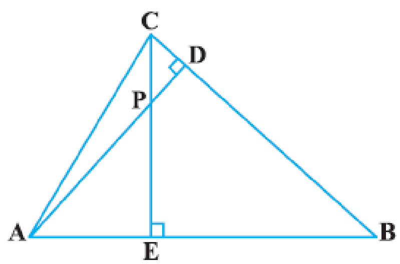
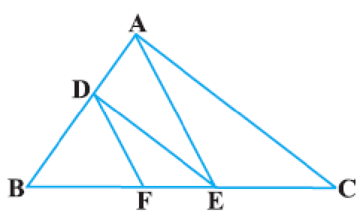
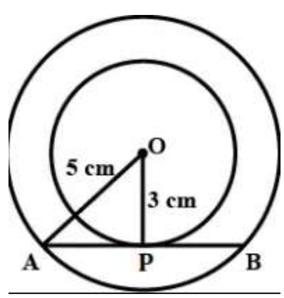
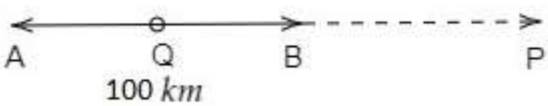
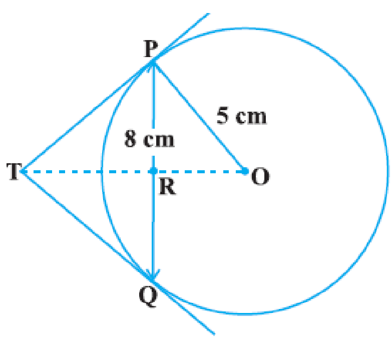
Class- X
Mathematics Basic (241)
Marking Scheme SQP-2022-23

Time Allowed: 3 Hours **Maximum Marks: 80**

	Section A	
1	(c) a^3b^2	1
2	(c) 13 km/hours	1
3	(b) -10	1
4	(b) Parallel.	1
5	(c) k = 4	1
6	(b) 12	1
7	(c) $\angle B = \angle D$	1
8	(b) 5 : 1	1
9	(a) 25°	1
10	(a) $\frac{2}{\sqrt{3}}$	1
11	(c) $\sqrt{3}$	1
12	(b) 0	1
13	(b) 14 : 11	1
14	(c) 16 : 9	1
15	(d) 147π cm^2	1
16	(c) 20	1
17	(b) 8	1
18	(a) $\frac{3}{26}$	1
19	(d) Assertion (A) is false but Reason (R) is true.	1

20	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	1
	Section B	
21	For a pair of linear equations to have infinitely many solutions : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$ $\frac{k}{12} = \frac{3}{k} \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$ Also, $\frac{3}{k} = \frac{k-3}{k} \Rightarrow k^2 - 6k = 0 \Rightarrow k = 0, 6.$ Therefore, the value of *k*, that satisfies both the conditions, is k = 6.	½ ½ ½ ½
22	 (i) In ΔABD and ΔCBE ∠ADB = ∠CEB = 90º ∠ABD = ∠CBE (Common angle) ⇒ ΔABD ~ ΔCBE (AA criterion) (ii) In ΔPDC and ΔBEC ∠PDC = ∠BEC = 90º ∠PCD = ∠BCE (Common angle) ⇒ ΔPDC ~ ΔBEC (AA criterion) [OR]  In ΔABC, DE || AC BD/AD = BE/EC ....(i) (Using BPT) In ΔABE, DF || AE BD/AD = BF/FE ....(ii) (Using BPT) From (i) and (ii) BD/AD = BE/EC = BF/FE Thus, $\frac{BF}{FE} = \frac{BE}{EC}$	½ ½ ½ ½ ½ ½ ½ ½
23	 Let O be the centre of the concentric circle of radii 5 cm and 3 cm respectively. Let AB be a chord of the larger circle touching the smaller circle at P Then AP = PB and OP⊥AB Applying Pythagoras theorem in ΔOPA, we have $OA^2=OP^2+AP^2 \Rightarrow 25 = 9 + AP^2$ $\Rightarrow AP^2 = 16 \Rightarrow AP = 4$ cm ∴ AB = 2AP = 8 cm	½ ½ ½ ½
24	Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$ $= \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2$ $= \cot^2\theta$ $= \left(\frac{7}{8}\right)^2 = \frac{49}{64}$	½ ½ ½ ½

25	Perimeter of quadrant = 2r + $\frac{1}{4}$ × 2 π r ⇒ Perimeter = 2 × 14 + $\frac{1}{2}$ × $\frac{22}{7}$ × 14 ⇒ Perimeter = 28 + 22 =28+22 = 50 cm [OR] Area of the circle = Area of first circle + Area of second circle ⇒ $\pi R^2 = \pi (r_1)^2 + \pi (r_1)^2$ ⇒ $\pi R^2 = \pi (24)^2 + \pi (7)^2$ ⇒ $\pi R^2 = 576\pi + 49\pi$ ⇒ $\pi R^2 = 625\pi$ ⇒ $R^2 = 625$ ⇒ R = 25 Thus, diameter of the circle = 2R = 50 cm.	½ ½ 1 ½ ½ 1
	Section C	
26	Let us assume to the contrary, that $\sqrt{5}$ is rational. Then we can find a and b ($\neq 0$) such that $\sqrt{5} = \frac{a}{b}$ (assuming that a and b are co-primes). So, a = $\sqrt{5}$ b ⇒ $a^2 = 5b^2$ Here 5 is a prime number that divides a^2 then 5 divides a also (Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer) Thus 5 is a factor of a Since 5 is a factor of a, we can write a = 5c (where c is a constant). Substituting a = 5c We get $(5c)^2 = 5b^2$ ⇒ $5c^2 = b^2$ This means 5 divides b^2 so 5 divides b also (Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer). Hence a and b have at least 5 as a common factor. But this contradicts the fact that a and b are coprime. This is the contradiction to our assumption that p and q are co-primes. So, √5 is not a rational number. Therefore, the √5 is irrational.	1 ½ ½ ½ ½
27	$6x^2 - 7x - 3 = 0 \Rightarrow 6x^2 - 9x + 2x - 3 = 0$ ⇒ 3x(2x – 3) + 1(2x – 3) = 0 ⇒ (2x – 3)(3x + 1) = 0 ⇒ 2x – 3 = 0 & 3x + 1 = 0 x = 3/2 & x = -1/3 Hence, the zeros of the quadratic polynomials are 3/2 and -1/3. For verification Sum of zeros = $\frac{-\text{coefficient of x}}{\text{coefficient of } x^2}$ ⇒ 3/2 + (-1/3) = – (-7) / 6 ⇒ 7/6 = 7/6 Product of roots = $\frac{\text{constant}}{\text{coefficient of } x^2}$ ⇒ 3/2 x (-1/3) = (-3) / 6 ⇒ -1/2 = -1/2 Therefore, the relationship between zeros and their coefficients is verified.	½ ½ 1 1
28	Let the fixed charge by Rs x and additional charge by Rs y per day Number of days for Latika = 6 = 2 + 4 Hence, Charge x + 4y = 22 x = 22 – 4y ………(1) Number of days for Anand = 4 = 2 + 2 Hence, Charge x + 2y = 16 x = 16 – 2y ……. (2) On comparing equation (1) and (2), we get,	½ ½

	22 – 4y = 16 – 2y ⇒ 2y = 6 ⇒ y = 3 Substituting y = 3 in equation (1), we get, x = 22 – 4 (3) ⇒ x = 22 – 12 ⇒ x = 10 Therefore, fixed charge = Rs 10 and additional charge = Rs 3 per day [OR]  AB = 100 km. We know that, Distance = Speed × Time. AP – BP = 100 ⇒ 5x − 5y = 100 ⇒ x−y=20.....(i) AQ + BQ = 100 ⇒ x + y = 100….(ii) Adding equations (i) and (ii), we get, x − y + x + y = 20 +100 ⇒ 2x = 120 ⇒ x = 60 Substituting x = 60 in equation (ii), we get, 60 + y = 100 ⇒ y = 40 Therefore, the speed of the first car is 60 km/hr and the speed of the second car is 40 km/hr.	1 1 ½ ½ 1 1
29	 Since OT is perpendicular bisector of PQ. Therefore, PR=RQ=4 cm Now, OR = $\sqrt{\mathbf{OP^2} - \mathbf{PR^2}} = \sqrt{\mathbf{5^2} - \mathbf{4^2}}$ =3cm Now, ∠TPR + ∠RPO = 90° (∵TPO=90°) & ∠TPR + ∠PTR = 90° (∵TRP=90◦) So, ∠RPO = ∠PTR So, ΔTRP ~ ΔPRO [By A-A Rule of similar triangles] So, $\frac{TP}{PO} = \frac{RP}{RG}$ $\Rightarrow \frac{TP}{5} = \frac{4}{3} \Rightarrow$ TP = $\frac{20}{3}$ cm	½ ½ ½ ½ ½ ½
30	$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$ $= \frac{\tan^2\theta}{\tan\theta-1} + \frac{1}{\tan\theta\,(1-\tan\theta)}$ $= \frac{\tan^3\theta-1}{\tan\theta\,(\tan\theta-1)}$ $= \frac{(\tan\theta-1)\,(\tan^3\theta+\tan\theta+1)}{\tan\theta\,(\tan\theta-1)}$ $= \frac{(\tan^3\theta+\tan\theta+1)}{\tan\theta}$ $= \tan\theta + 1 + \sec = 1 + \tan\theta + \sec\theta$ $= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ $= 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\,\cos\theta}$	½ ½ ½ ½ ½

	$= 1 + \frac{1}{\sin\theta\ \cos\theta} = 1 + \sec\theta \operatorname{cosec}\theta$ [OR] sin θ + cos θ = $\sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$ $\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$ $\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \Rightarrow 1 \sin\theta\cos\theta = 1$ Now tanθ + cotθ = $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\text{isn}\ \theta}$ $= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ $= \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1$	½ ½ ½ ½ ½ ½ ½
31	(i) P(8) = $\frac{5}{36}$ (ii) P(13) = $\frac{0}{36} = 0$ (iii) P(less than or equal to 12) = 1	1 1 1
	Section D	
32	Let the average speed of passenger train = *x* km/h. and the average speed of express train = (*x* + 11) km/h As per given data, time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance. Therefore, $\frac{132}{x} - \frac{132}{x+11} = 1$ $\Rightarrow \frac{132\ (x+11-x)}{x\ (x+11)} = 1 \Rightarrow \frac{132\ x\ 11}{x\ (x+11)} = 1$ ⇒ 132 × 11 = $x(x + 11) \Rightarrow x^2 + 11x - 1452 = 0$ ⇒ $x^2 + 44x - 33x - 1452 = 0$ ⇒ $x(x + 44) - 33(x + 44) = 0 \Rightarrow (x + 44)(x - 33) = 0$ ⇒ $x = -44, 33$ As the speed cannot be negative, the speed of the passenger train will be 33 km/h and the speed of the express train will be 33 + 11 = 44 km/h. [OR] Let the speed of the stream be x km/hr So, the speed of the boat in upstream = (18 - x) km/hr & the speed of the boat in downstream = (18 + x) km/hr ATQ, $\frac{\text{distance}}{\text{upstream speed}} - \frac{\text{distance}}{\text{downstream speed}} = 1$ $\Rightarrow \frac{24}{18-x} - \frac{24}{18+x} = 1$	½ 1 ½ 1 1 ½ ½ ½ ½ 1

$\Rightarrow 24\left[\frac{1}{18-x} - \frac{1}{18+x}\right] = 1 \Rightarrow 24\left[\frac{18+x-(18-x)}{(18-x).(18+x)}\right] = 1$ 1

$\Rightarrow 24\left[\frac{2x}{(18-x).(18+x)}\right] = 1 \Rightarrow 24\left[\frac{2x}{(18-x).(18+x)}\right] = 1$

$\Rightarrow 48x = 324 - x^2 \Rightarrow x^2 + 48x - 324 = 0$ 1

$\Rightarrow (x + 54)(x - 6) = 0 \Rightarrow x = -54$ or 6 ½

As speed to stream can never be negative, the speed of the stream is 6 km/hr. ½

33

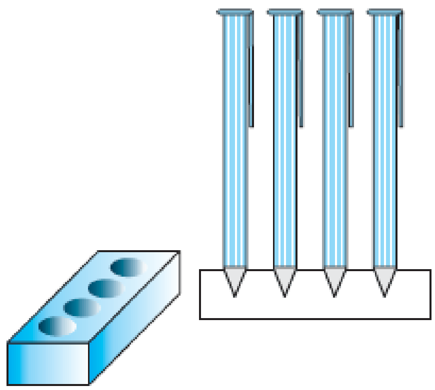
Figure ½

Given, To prove, constructions 1½

Proof 2

Application ---- 1

34



Volume of one conical depression = $\frac{1}{3} \times \pi r^2 h$ ½

$= \frac{1}{3} \times \frac{22}{7} \times 0.5^2 \times 1.4 \text{ cm}^3 = 0.366 \text{ cm}^3$ 1½

Volume of 4 conical depression = 4 x 0.366 cm^3

$= 1.464 \text{ cm}^3$ ½

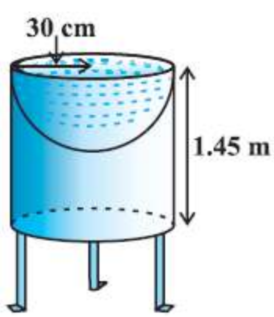
Volume of cuboidal box = L x B x H ½

$= 15 \times 10 \times 3.5 \text{ cm}^3 = 525 \text{ cm}^3$ 1½

Remaining volume of box = Volume of cuboidal box – Volume of 4 conical depressions ½

$= 525 \text{ cm}^3 - 1.464 \text{ cm}^3 = 523.5 \text{ cm}^3$ 1

[OR]



Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere.

Then, the total surface area = CSA of cylinder + CSA of hemisphere ½

$= 2\pi rh + 2\pi r^2 = 2\pi\, r\,(h + r)$ 2

$= 2 \times \frac{22}{7} \times 30\,(145 + 30) \text{ cm}^2$ 1

$= 2 \times \frac{22}{7} \times 30 \times 175 \text{ cm}^2$ ½

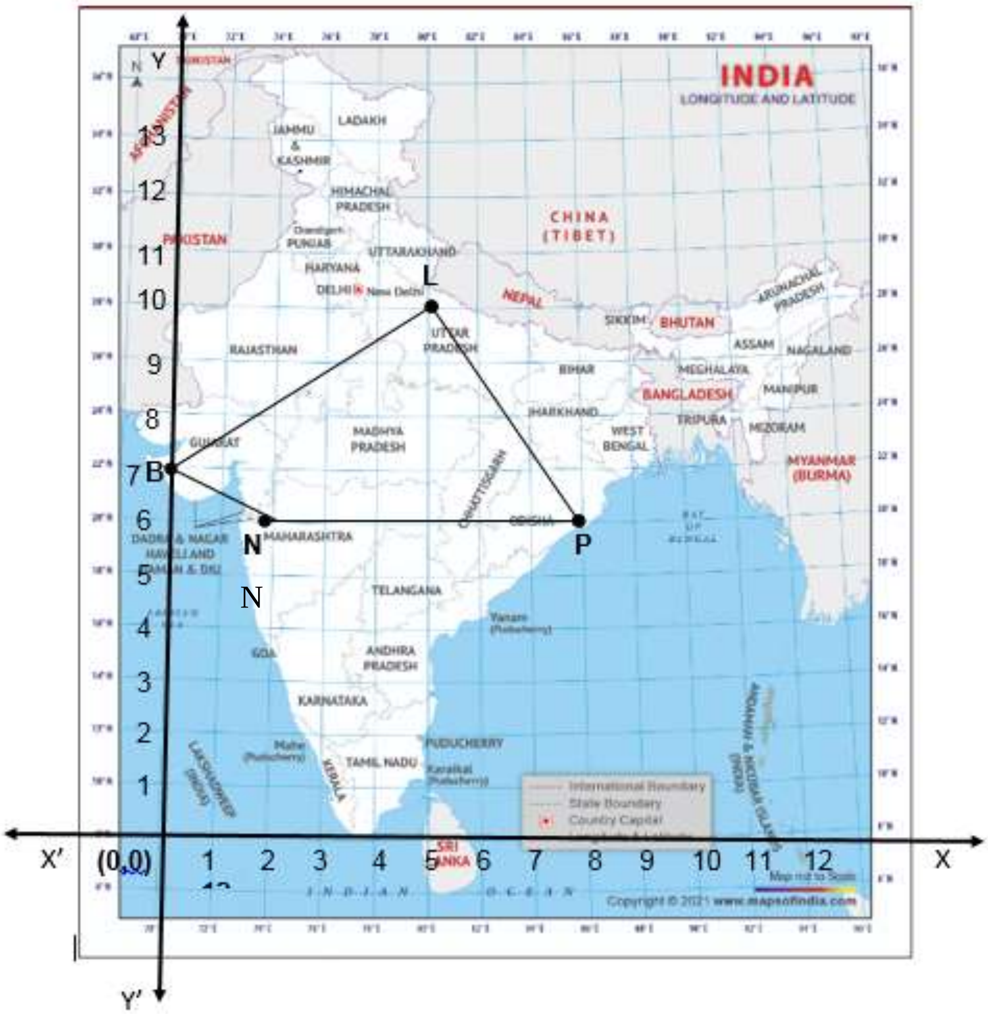
$= 33000 \text{ cm}^2 = 3.3 \text{ m}^2$ 1

35

Class Interval	Number of policy holders (f)	Cumulative Frequency (cf)
Below 20	2	2
20-25	4	6
25-30	18	24
30-35	21	45
35-40	33	78
40-45	11	89
45-50	3	92
50-55	6	98
55-60	2	100

1

	n = 100 ⇒ n/2 = 50, Therefore, median class = 35 – 40, Class size, h = 5, Lower limit of median class, l = 35, frequency f = 33, cumulative frequency cf = 45 $\Rightarrow$Median = l + $\left[\frac{\frac{n}{2}-cf}{f}\right] \times h$ $\Rightarrow$Median = 35 + $\left[\frac{50-45}{33}\right] \times 5$ = 35 + $\frac{25}{33}$ = 35 + 0.76 = 35.76 Therefore, median age is 35.76 years	½ 1½ 1 1
	Section E	
36	1	Since the production increases uniformly by a fixed number every year, the number of Cars manufactured in 1st, 2nd, 3rd, . . .,years will form an AP. So, a + 3d = 1800 & a + 7d = 2600 So d = 200 & a = 1200	 ½ ½
	2	t_{12} = a + 11d ⇒ t_{30} = 1200 + 11 x 200 ⇒ t_{12} = 3400	½ ½
	3	$S_n = \frac{n}{2}[2a+(n-1)d]$ ⇒ $S_{10} = \frac{10}{2}[2\ x\ 1200+(10-1)\ 200]$ ⇒ $S_{10} = \frac{13}{2}[2\ x\ 1200 + 9 \text{ x } 200]$ ⇒ S_{10} = 5 x [2400 + 1800] ⇒ S_{10} = 5 x 4200= 21000 [OR] Let in n years the production will reach to 31200 $S_n = \frac{n}{2}[2a+(n-1)d]$ = 31200 ⇒ $\frac{n}{2}[2\ x\ 1200+(n-1)200]$ = 31200 ⇒ $\frac{n}{2}[2 \text{ x } 1200+(n-1)200]$ = 31200 ⇒ n [$12+(n-1)$] = 312 ⇒ n^2 + 11n -312 = 0 ⇒ n^2 + 24n - 13n -312 = 0 ⇒ (n +24)(n -13) = 0 ⇒ n = 13 or – 24. As n can't be negative. So n = 13	½ ½ ½ ½ ½ ½ ½ ½
37	**Case Study – 2**	



1	LB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ ⇒ LB = $\sqrt{(0 - 5)^2 + (7 - 10)^2}$ LB = $\sqrt{(5)^2 + (3)^2}$ ⇒ LB = $\sqrt{25 + 9}$ LB = $\sqrt{34}$ Hence the distance is 150 $\sqrt{34}$ km	½ ½
2	Coordinate of Kota (K) is $\left(\frac{3 \text{ x } 5 + 2 \text{ x } 0}{3+2}, \frac{3 \text{ x } 7 + 2 \text{ x } 10}{3+2}\right)$ $= \left(\frac{15+0}{5}, \frac{21+20}{5}\right) = \left(3, \frac{41}{5}\right)$	½ ½
3	L(5, 10), N(2,6), P(8,6) LN = $\sqrt{(2 - 5)^2 + (6 - 10)^2}$ = $\sqrt{(3)^2 + (4)^2}$ = $\sqrt{9 + 16}$ = $\sqrt{25} = 5$ NP = $\sqrt{(8 - 2)^2 + (6 - 6)^2}$ = $\sqrt{(4)^2 + (0)^2}$ = 4 PL = $\sqrt{(8 - 5)^2 + (6 - 10)^2}$ = $\sqrt{(3)^2 + (4)^2}$ ⇒ LB = $\sqrt{9 + 16}$ = $\sqrt{25} = 5$ as LN = PL ≠ NP, so Δ LNP is an isosceles triangle. **[OR]**	½ ½ ½ ½

	Let A (0, b) be a point on the y – axis then AL = AP	
	$\Rightarrow \sqrt{(5-0)^2+(10-b)^2} = \sqrt{(8-0)^2+(6-b)^2}$	½
	$\Rightarrow (5)^2+(10-b)^2 = (8)^2+(6-b)^2$	½
	$\Rightarrow 25+100-20b+b^2 = 64+36-12b+b^2 \Rightarrow 8b = 25 \Rightarrow b = \frac{25}{8}$	½
	So, the coordinate on y axis is $\left(0, \frac{25}{8}\right)$	½

38 **Case Study – 3**

1	$\sin 60° = \frac{PC}{PA}$	½
	$\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{PA} \Rightarrow PA = 12\sqrt{3}$ m	½
2	$\sin 30° = \frac{PC}{PB}$	½
	$\Rightarrow \frac{1}{2} = \frac{18}{PB} \Rightarrow PB = 36$ m	½
3	$\tan 60° = \frac{PC}{AC} \Rightarrow \sqrt{3} = \frac{18}{AC} \Rightarrow$ AC = $6\sqrt{3}$ m	1
	$\tan 30° = \frac{PC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{CB} \Rightarrow$ CB = $18\sqrt{3}$ m	½
	Width AB = AC + CB = $6\sqrt{3} + 18\sqrt{3} = 24\sqrt{3}$ m	½
	[OR]	
	RB = PC =18 m & PR = CB = $18\sqrt{3}$ m	½
	$\tan 30° = \frac{QR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{18\sqrt{3}} \Rightarrow$ QR = 18 m	1
	QB = QR + RB = 18 + 18 = 36m. Hence height BQ is 36m	½