

Marking Scheme

Mathematics Class X (2017-18)

Section A

S.No.	Answer	Marks
1.	Non terminating repeating decimal expansion.	[1]
2.	$k = \pm 4$	[1]
3.	$a_{11} = -25$	[1]
4.	(0, 5)	[1]
5.	9 : 49	[1]
6.	25	[1]

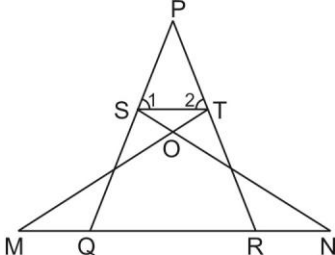
Section B

7.	$\text{LCM}(p, q) = a^3b^3$ $\text{HCF}(p, q) = a^2b$ $\text{LCM}(p, q) \times \text{HCF}(p, q) = a^5b^4 = (a^2b^3)(a^3b) = pq$	[1/2] [1/2] [1]
8.	$S_n = 2n^2 + 3n$ $S_1 = 5 = a_1$ $S_2 = a_1 + a_2 = 14 \Rightarrow a_2 = 9$ $d = a_2 - a_1 = 4$ $a_{16} = a_1 + 15d = 5 + 15(4) = 65$	[1/2] [1/2] [1/2] [1/2]
9.	For pair of equations $kx + 1y = k^2$ and $1x + ky = 1$ We have: $\frac{a_1}{a_2} = \frac{k}{1}, \frac{b_1}{b_2} = \frac{1}{k}, \frac{c_1}{c_2} = \frac{k^2}{1}$ For infinitely many solutions, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ $\therefore \frac{k}{1} = \frac{1}{k} \Rightarrow k^2 = 1 \Rightarrow k = 1, -1 \quad \dots(i)$ and $\frac{1}{k} = \frac{k^2}{1} \Rightarrow k^3 = 1 \Rightarrow k = 1 \quad \dots(ii)$ From (i) and (ii), $k = 1$	[1/2] [1/2] [1/2] [1/2]
10.	Since $\left(1, \frac{p}{3}\right)$ is the mid-point of the line segment joining the points (2, 0) and $\left(0, \frac{2}{9}\right)$ therefore, $\frac{p}{3} = \frac{0 + \frac{2}{9}}{2} \Rightarrow p = \frac{1}{3}$ The line $5x + 3y + 2 = 0$ passes through the point (-1, 1) as $5(-1) + 3(1) + 2 = 0$	[1] [1]
11.	(i) $P(\text{square number}) = \frac{8}{113}$ (ii) $P(\text{multiple of 7}) = \frac{16}{113}$	[1] [1]

12.	Let number of red balls be = x $\therefore P(\text{red ball}) = \frac{x}{12}$ If 6 more red balls are added: The number of red balls = x + 6 $P(\text{red ball}) = \frac{x+6}{18}$ Since, $\frac{x+6}{18} = 2\left(\frac{x}{12}\right) \Rightarrow x = 3$ $\therefore$ There are 3 red balls in the bag.	[1/2] [1] [1/2]
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Section C

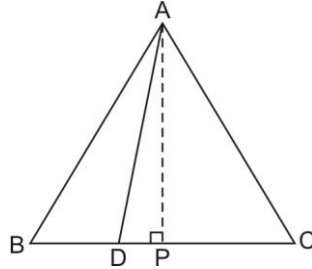
13.	Let $n = 3k, 3k + 1$ or $3k + 2$. (i) When $n = 3k$: n is divisible by 3. $n + 2 = 3k + 2 \Rightarrow n + 2$ is not divisible by 3. $n + 4 = 3k + 4 = 3(k + 1) + 1 \Rightarrow n + 4$ is not divisible by 3. (ii) When $n = 3k + 1$: n is not divisible by 3. $n + 2 = (3k + 1) + 2 = 3k + 3 = 3(k + 1) \Rightarrow n + 2$ is divisible by 3. $n + 4 = (3k + 1) + 4 = 3k + 5 = 3(k + 1) + 2 \Rightarrow n + 4$ is not divisible by 3. (iii) When $n = 3k + 2$: n is not divisible by 3. $n + 2 = (3k + 2) + 2 = 3k + 4 = 3(k + 1) + 1 \Rightarrow n + 2$ is not divisible by 3. $n + 4 = (3k + 2) + 4 = 3k + 6 = 3(k + 2) \Rightarrow n + 4$ is divisible by 3. Hence exactly one of the numbers $n, n + 2$ or $n + 4$ is divisible by 3.	[1] [1] [1]
14.	Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are the two zeroes therefore, $\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2 - 5)$ is a factor of given polynomial. We divide the given polynomial by $3x^2 - 5$. $ \begin{array}{r} \overline{) \begin{array}{l} 3x^4 + 6x^3 - 2x^2 - 10x - 5 \\ \underline{+ 3x^4} \\ 6x^3 + 3x^2 - 10x - 5 \\ \underline{+ 6x^3} \\ 3x^2 - 5 \\ \underline{+ 3x^2} \\ 0 \end{array} \\ \end{array} $ For other zeroes, $x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0, x = -1, -1$ $\therefore$ Zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -1$ and -1.	[1] [1] [1]

15.	Let the ten's and the units digit be y and x respectively. So, the number is $10y + x$. The number when digits are reversed is $10x + y$. Now, $7(10y + x) = 4(10x + y) \Rightarrow 2y = x \dots(i)$ Also $x - y = 3 \dots(ii)$ Solving (1) and (2), we get $y = 3$ and $x = 6$. Hence the number is 36.	[1/2] [1/2] [1] [1/2] [1/2]
16.	Let x-axis divides the line segment joining $(-4, -6)$ and $(-1, 7)$ at the point P in the ratio $1 : k$. Now, coordinates of point of division $P\left(\frac{-1-4k}{k+1}, \frac{7-6k}{k+1}\right)$ Since P lies on x-axis, therefore $\frac{7-6k}{k+1} = 0$ $\Rightarrow 7 - 6k = 0$ $\Rightarrow k = \frac{7}{6}$ Hence the ratio is $1 : \frac{7}{6} = 6 : 7$ Now, the coordinates of P are $\left(\frac{-34}{13}, 0\right)$. OR Let the height of parallelogram taking AB as base be h. Now $AB = \sqrt{(7-4)^2 + (2+2)^2} = \sqrt{3^2 + 4^2} = 5$ units. Area (ΔABC) = $\frac{1}{2}[4(2-9) + 7(9+2) + 0(-2-2)] = \frac{49}{2}$ sq units . Now, $\frac{1}{2} \times AB \times h = \frac{49}{2}$ $\Rightarrow \frac{1}{2} \times 5 \times h = \frac{49}{2}$ $\Rightarrow h = \frac{49}{5} = 9.8$ units .	[1/2] [1] [1/2] [1] [1] [1] [1]
17.	$\angle SQN = \angle TRM$ (CPCT as $\Delta NSQ \cong \Delta MTR$)  Since, $\angle P + \angle 1 + \angle 2 = \angle P + \angle PQR + \angle PRQ$ (Angle sum property) $\Rightarrow \angle 1 + \angle 2 = \angle PQR + \angle PRQ$ $\Rightarrow 2\angle 1 = 2\angle PQR$ (as $\angle 1 = \angle 2$ and $\angle PQR = \angle PRQ$) $\angle 1 = \angle PQR$	[1] [1]

Also $\angle 2 = \angle PRQ$
 And $\angle SPT = \angle QPR$ (common)
 $\Delta PTS \sim \Delta PRQ$ (By AAA similarity criterion)

[1]

OR



Construction: Draw $AP \perp BC$

[1/2]

In ΔADP , $AD^2 = AP^2 + DP^2$

[1/2]

$AD^2 = AP^2 + (BP - BD)^2$

[1/2]

$AD^2 = AP^2 + BP^2 + BD^2 - 2(BP)(BD)$

$AD^2 = AB^2 + \left(\frac{1}{3}BC\right)^2 - 2\left(\frac{BC}{2}\right)\left(\frac{BC}{3}\right)$

[1]

$AD^2 = \frac{7}{9}AB^2$ ($\because BC = AB$)

$9AD^2 = 7AB^2$

[1/2]

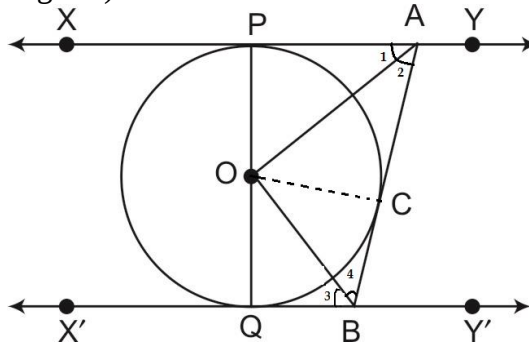
18.

Join OC

In ΔOPA and ΔOCA

$OP = OC$ (radii of same circle)

$PA = CA$ (length of two tangents)



[1]

$AO = AO$ (Common)

$\therefore \Delta OPA \cong \Delta OCA$ (By SSS congruency criterion)

Hence, $\angle 1 = \angle 2$ (CPCT)

[1]

Similarly $\angle 3 = \angle 4$

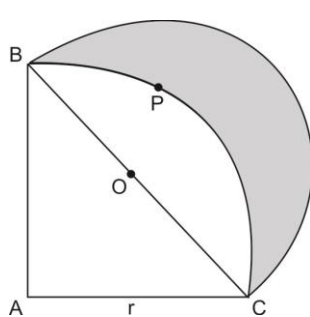
Now, $\angle PAB + \angle QBA = 180^\circ$

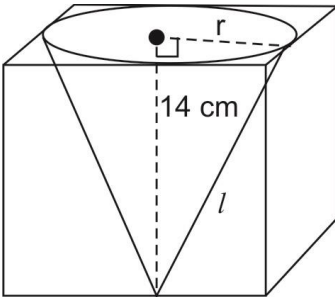
$\Rightarrow 2\angle 2 + 2\angle 4 = 180^\circ$

[1]

$\Rightarrow \angle 2 + \angle 4 = 90^\circ$

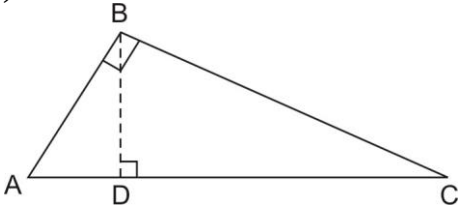
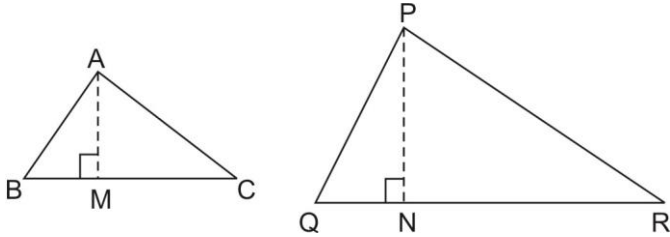
$\Rightarrow \angle AOB = 90^\circ$ (Angle sum property)

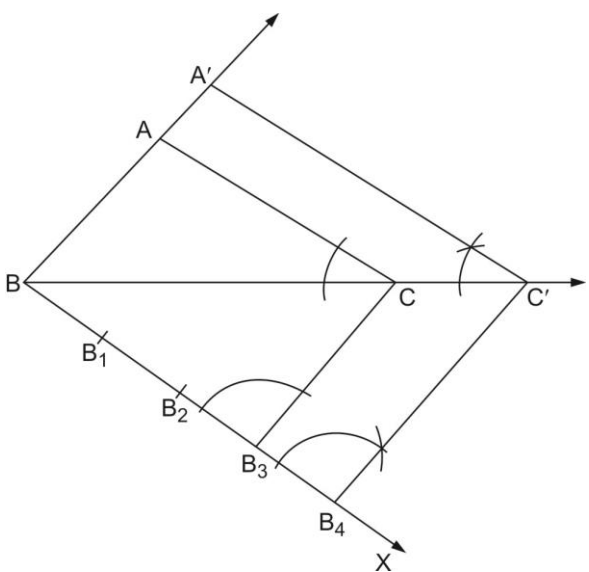
19.	$\frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\cot^2 66^\circ + \sec^2 27^\circ} + \frac{\sin^2 63^\circ + \cos 63^\circ \sin 27^\circ + \sin 27^\circ \sec 63^\circ}{2(\operatorname{cosec}^2 65^\circ - \tan^2 25^\circ)}$ $= \frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\tan^2(90^\circ - 66^\circ) + \operatorname{cosec}^2(90^\circ - 27^\circ)} + \frac{\sin^2 63^\circ + \cos 63^\circ \cos(90^\circ - 27^\circ) + \sin 27^\circ \operatorname{cosec}(90^\circ - 63^\circ)}{2[\operatorname{cosec}^2 65^\circ - \cot^2(90^\circ - 25^\circ)]}$ $= \frac{\operatorname{cosec}^2 63^\circ + \tan^2 24^\circ}{\tan^2 24^\circ + \operatorname{cosec}^2 63^\circ} + \frac{\sin^2 63^\circ + \cos^2 63^\circ + \sin 27^\circ \operatorname{cosec} 27^\circ}{2(\operatorname{cosec}^2 65^\circ - \cot^2 65^\circ)}$ $= 1 + \frac{1+1}{2(1)} = 2$ OR $\sin \theta + \cos \theta = \sqrt{2}$ $\Rightarrow (\sin \theta + \cos \theta)^2 = (\sqrt{2})^2$ $\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta = 2$ $\Rightarrow 1 + 2 \sin \theta \cos \theta = 2$ $\Rightarrow \sin \theta \cos \theta = \frac{1}{2} \quad \dots(i)$ we know, $\sin^2 \theta + \cos^2 \theta = 1 \quad \dots(ii)$ Dividing (ii) by (i) we get $\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{1}{1/2}$ $\Rightarrow \tan \theta + \cot \theta = 2$	[1] [1] [1] [1/2] [1] [1/2] [1]
20.	We know, $AC = r$ In ΔACB, $BC^2 = AC^2 + AB^2$ $\Rightarrow BC = AC\sqrt{2} \quad (\because AB = AC)$ $\Rightarrow BC = r\sqrt{2}$  Required area = ar(ΔACB) + ar(semicircle on BC as diameter) – ar(quadrant ABPC) $= \frac{1}{2} \times r \times r + \frac{1}{2} \times \pi \times \left(\frac{r\sqrt{2}}{2}\right)^2 - \frac{1}{4} \pi r^2$ $= \frac{r^2}{2} + \frac{\pi r^2}{4} - \frac{\pi r^2}{4}$ $= \frac{r^2}{2} = \frac{196}{2} \text{ cm}^2 = 98 \text{ cm}^2$	[1] [1] [1]

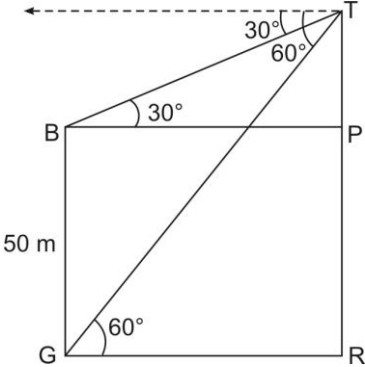
21.	Let the area that can be irrigated in 30 minute be $A \text{ m}^2$. Water flowing in canal in 30 minutes = $\left(10,000 \times \frac{1}{2}\right) \text{ m} = 5000 \text{ m}$ Volume of water flowing out in 30 minutes = $(5000 \times 6 \times 1.5) \text{ m}^3 = 45000 \text{ m}^3 \dots(i)$ Volume of water required to irrigate the field = $A \times \frac{8}{100} \text{ m}^3$...(ii) Equating (i) and (ii), we get $A \times \frac{8}{100} = 45000$ $A = 562500 \text{ m}^2.$ OR $l = \sqrt{7^2 + 14^2} = 7\sqrt{5}$ Surface area of remaining solid = $6l^2 - \pi r^2 + \pi rl$, where r and l are the radius and slant height of the cone.  $= 6 \times 14 \times 14 - \frac{22}{7} \times 7 \times 7 + \frac{22}{7} \times 7 \times 7\sqrt{5}$ $= 1176 - 154 + 154\sqrt{5}$ $= (1022 + 154\sqrt{5}) \text{ cm}^2$	[1/2] [1] [1/2] [1/2] [1] [1/2] [1] [1]
22.	$\text{Mode} = \ell + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ $= 60 + \left(\frac{29 - 21}{58 - 21 - 17}\right) \times 20$ $= 68$ So, the mode marks is 68. Empirical relationship between the three measures of central tendencies is: 3 Median = Mode + 2 Mean $3 \text{ Median} = 68 + 2 \times 53$ Median = 58 marks	[1] [1] [1]

Section D

23.	Let original speed of the train be x km/h. Time taken at original speed = $\frac{360}{x}$ hours Time taken at increased speed = $\frac{360}{x+5}$ hours Now, $\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$ $\Rightarrow 360 \left[\frac{1}{x} - \frac{1}{x+5} \right] = \frac{4}{5}$ $\Rightarrow x^2 + 5x - 2250 = 0$ $\Rightarrow x = 45$ or -50 (as speed cannot be negative) $\Rightarrow x = 45$ km/h OR Discriminant = $b^2 - 4ac = 36 - 4 \times 5 \times (-2) = 76 > 0$ So, the given equation has two distinct real roots $5x^2 - 6x - 2 = 0$ Multiplying both sides by 5. $(5x)^2 - 2 \times (5x) \times 3 = 10$ $\Rightarrow (5x)^2 - 2 \times (5x) \times 3 + 3^2 = 10 + 3^2$ $\Rightarrow (5x - 3)^2 = 19$ $\Rightarrow 5x - 3 = \pm\sqrt{19}$ $\Rightarrow x = \frac{3 \pm \sqrt{19}}{5}$ Verification: $5 \left(\frac{3 + \sqrt{19}}{5} \right)^2 - 6 \left(\frac{3 + \sqrt{19}}{5} \right) - 2 = \frac{9 + 6\sqrt{19} + 19}{5} - \frac{18 + 6\sqrt{19}}{5} - \frac{10}{5} = 0$ Similarly, $5 \left(\frac{3 - \sqrt{19}}{5} \right)^2 - 6 \left(\frac{3 - \sqrt{19}}{5} \right) - 2 = 0$	[1] [1/2] [1½] [1] [1] [1] [1/2] [1/2]
24.	Let the three middle most terms of the AP be $a - d$, a, $a + d$. We have, $(a - d) + a + (a + d) = 225$ $\Rightarrow 3a = 225 \Rightarrow a = 75$ Now, the AP is $a - 18d, \dots, a - 2d, a - d, a, a + d, a + 2d, \dots, a + 18d$ Sum of last three terms: $(a + 18d) + (a + 17d) + (a + 16d) = 429$ $\Rightarrow 3a + 51d = 429 \Rightarrow a + 17d = 143$ $\Rightarrow 75 + 17d = 143$ $\Rightarrow d = 4$ Now, first term = $a - 18d = 75 - 18(4) = 3$ $\therefore$ The AP is 3, 7, 11, ..., 147.	[1] [1/2] [1] [1/2] [1]

25.	Given: A right triangle ABC right angled at B. To prove: $AC^2 = AB^2 + BC^2$ Construction: Draw $BD \perp AC$ Proof: In $\triangle ADB$ and $\triangle ABC$ $\angle ADB = \angle ABC$ (each 90°) $\angle BAD = \angle CAB$ (common)  $\triangle ADB \sim \triangle ABC$ (By AA similarity criterion) Now, $\frac{AD}{AB} = \frac{AB}{AC}$ (corresponding sides are proportional) $\Rightarrow AB^2 = AD \times AC$... (i) Similarly $\triangle BDC \sim \triangle ABC$ $\Rightarrow BC^2 = CD \times AC$... (ii) Adding (1) and (2) $AB^2 + BC^2 = AD \times AC + CD \times AC$ $\Rightarrow AB^2 + BC^2 = AC \times (AD + CD)$ $\Rightarrow AB^2 + BC^2 = AC^2$, Hence Proved. OR Given: $\triangle ABC \sim \triangle PQR$ To prove: $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$ Construction: Draw $AM \perp BC$, $PN \perp QR$  $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{\frac{1}{2} \times BC \times AM}{\frac{1}{2} \times QR \times PN} = \frac{BC}{QR} \times \frac{AM}{PN}$... (i) In $\triangle ABM$ and $\triangle PQN$ $\angle B = \angle Q$ ($\because \triangle ABC \sim \triangle PQR$) $\angle M = \angle N$ (each 90°) $\triangle ABM \sim \triangle PQN$ (AA similarity criterion) Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$... (ii) But $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ ($\triangle ABC \sim \triangle PQR$) ... (iii)	[1/2] [1] [1] [1/2] [1] [1/2] [1] [1/2]
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	Hence, $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \frac{BC}{QR} \times \frac{AM}{PN}$ from (i) $= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [from (ii) and (iii)] $= \left(\frac{AB}{PQ}\right)^2$ $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$ Using (iii)	[1/2] [1/2]
26.	Draw $\triangle ABC$ in which $BC = 7$ cm, $\angle B = 45^\circ$, $\angle A = 105^\circ$ and hence $\angle C = 30^\circ$. Construction of similar triangle $A'BC'$ as shown below: 	[1] [3]
27.	$\begin{aligned} \text{LHS} &= \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} \\ &= \frac{\cos \theta - \sin \theta + 1}{\cos \theta + \sin \theta - 1} \times \frac{\cos \theta + \sin \theta + 1}{\cos \theta + \sin \theta + 1} \\ &= \frac{(\cos \theta + 1)^2 - \sin^2 \theta}{(\cos \theta + \sin \theta)^2 - 1^2} \\ &= \frac{\cos^2 \theta + 1 + 2 \cos \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta - 1} \\ &= \frac{2 \cos^2 \theta + 2 \cos \theta}{2 \sin \theta \cos \theta} \\ &= \frac{2 \cos \theta (\cos \theta + 1)}{2 \sin \theta \cos \theta} \\ &= \frac{\cos \theta + 1}{\sin \theta} = \operatorname{cosec} \theta + \cot \theta = \text{RHS} \end{aligned}$	[1] [1] [1] [1]

28.	In $\triangle BTP \Rightarrow \tan 30^\circ = \frac{TP}{BP}$  Correct Figure $\Rightarrow \frac{1}{\sqrt{3}} = \frac{TP}{BP}$ $BP = TP\sqrt{3} \quad \dots(i)$ In $\triangle GTR$, $\tan 60^\circ = \frac{TR}{GR} \Rightarrow \sqrt{3} = \frac{TR}{GR} \Rightarrow GR = \frac{TR}{\sqrt{3}} \quad \dots(ii)$ Now, $TP\sqrt{3} = \frac{TR}{\sqrt{3}}$ (as $BP = GR$) $\Rightarrow 3TP = TP + PR$ $\Rightarrow 2TP = BG \Rightarrow TP = \frac{50}{2} \text{ m} = 25 \text{ m}$ Now, $TR = TP + PR = (25 + 50) \text{ m}$. Height of tower $= TR = 75 \text{ m}$. Distance between building and tower $= GR = \frac{TR}{\sqrt{3}}$ $\Rightarrow GR = \frac{75}{\sqrt{3}} \text{ m} = 25\sqrt{3} \text{ m}$	[1] [1/2] [1/2] [1] [1/2] [1/2]
29.	Capacity of mug (actual quantity of milk) $= \pi r^2 h - \frac{2}{3} \pi r^3$ $= \pi r^2 \left(h - \frac{2}{3} r \right)$ $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(14 - \frac{2}{3} \times \frac{7}{2} \right)$ $= \frac{2695}{6} \text{ cm}^3$ Amount dairy owner B should charge for one mug of milk $= \frac{2695}{6} \times \frac{80}{1000} = ₹ 35.93$ Value exhibited by dairy owner B: honesty (or any similar value)	[1] [1] [1] [1]

30.

Daily pocket allowance (in ₹)	Number of children (f_i)	Mid-point (x_i)	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11–13	3	12	–3	–9
13–15	6	14	–2	–12
15–17	9	16	–1	–9
17–19	13	18	0	0
19–21	k	20	1	k
21–23	5	22	2	10
23–25	4	24	3	12

$$\Sigma f_i = 40 + k$$

$$\Sigma f_i u_i = k - 8$$

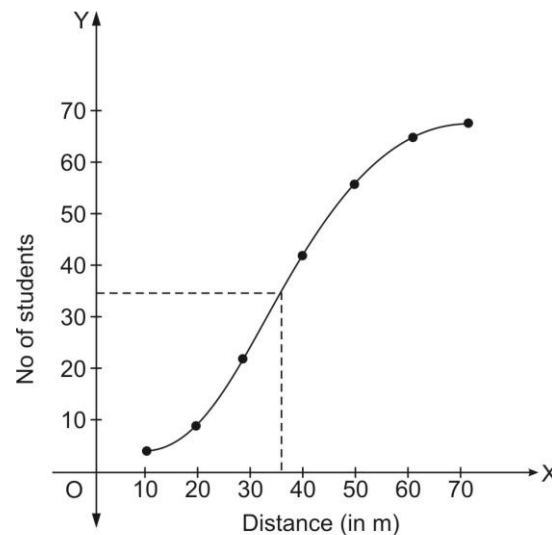
$$\text{Mean} = \bar{x} = a + h \left(\frac{\Sigma f_i u_i}{\Sigma f_i} \right)$$

$$18 = 18 + 2 \left(\frac{k - 8}{40 + k} \right)$$

$$\Rightarrow k = 8$$

OR

Less than	Number of Students
10	4
20	9
30	22
40	42
50	56
60	64
70	68



Less than Ogive

Median distance is value of x that corresponds to

$$\text{Cumulative frequency } \frac{N}{2} = \frac{68}{2} = 34$$

Therefore, Median distance = 36 m

[2]

[1]

[1]

[1]

[2]

[1]