

Sample Question Paper
CLASS: XII
Session: 2022-23
Applied Mathematics (Code-241)

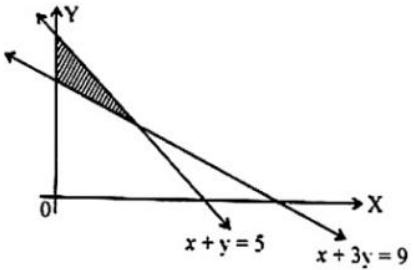
Time Allowed: 3 hrs

Maximum Marks: 80

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
- Section – A:**
3. It comprises of **20 MCQs of 1 mark** each.
- Section – B:**
4. It comprises of **5 VSA type questions of 2 marks** each.
- Section – C:**
5. It comprises of **6 SA type of questions of 3 marks** each.
- Section – D:**
6. It comprises of **4 LA type of questions of 5 marks** each.
- Section – E:**
7. It has **3 case studies**. Each case study comprises of 3 case-based questions, where **2 VSA type questions are of 1 mark** each and **1 SA type question is of 2 marks**. Internal choice is provided in **2 marks** question in each case-study.
8. Internal choice is provided in **2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D**. You have to attempt only one of the alternatives in all such questions.

<u>SECTION – A</u>		<u>marks</u>
(All questions are compulsory. No internal choice is provided in this section)		
1.	What is the least value of 'x' that satisfies $x \equiv 27 \pmod{4}$, when $27 < x \leq 36$? a) 27 b) 30 c) 31 d) 35	1
2.	Let $p > 0$ and $q < 0$ and $p, q \in Z$, then choose the correct inequality from the given below options to complete the statement: $p + q \quad \square \quad p - q$ a) $>$ b) \leq c) \geq d) $<$	1
3.	A machine makes car wheels and in a random sample of 26 wheels, the test statistic is found to be 3.07. As per the t-distribution test (of 5% level of significance), what can you say about the quality of wheels produced by the machine? (Use $t_{25}(0.05) = 2.06$) a) Superior quality b) Inferior quality c) Same quality d) Cannot say	1

12.	<p>In the given figure (I), what is the LPP shaded region known as?</p>	 <p style="text-align: center;">Figure (I)</p>	1
<p>a) Feasible region b) Feasible solution c) Optimal region d) Objective region</p>			
13.	<p>General solution of differential equation: $y \log y \, dx - x \, dy = 0$ is –</p>		1
<p>a) $y = \log Cx$ b) $y = e^{ Cx }$ c) $y = e^{-Cx}$ d) $\log y = C + x$</p>			
14.	<p>An investment of ₹ 10,000 becomes ₹ 60,000 in 4 years, then the CAGR (compound annual growth rate) is given by –</p>		1
<p>a) $\frac{\sqrt[4]{6}-1}{100}$ b) $\frac{\sqrt[4]{6}+1}{100}$ c) $[\sqrt[4]{6} - 1] \times 100$ d) $[\sqrt[4]{6} + 1] \times 100$</p>			
15.	<p>In what ratio shall I add water to the liquid detergent costing ₹ 480 per litre to get resulting mixture worth ₹ 300 per litre?</p>		1
<p>a) 5:3 b) 3:8 c) 3:5 d) 5:8</p>			
16.	<p>A grain whole-seller visits the granary market. While going around to make a good purchase, he takes a handful of rice from random sacks of rice, in order to inspect the quality of farmers produce. The handful of rice taken from a sack of rice for quality inspection is a:</p>		1
<p>a) statistic b) population c) parameter d) sample</p>			
17.	<p>For predicting the straight-line trend in the sales of scooters (in thousands) on the basis of 6 consecutive years data, the company makes use of 4-year moving averages method. If the sales of scooters for respective years are a, b, c, d, e and f respectively, then which of the following average will <u>not</u> be computed?</p>		1
<p>a) $\frac{a+b+c+d}{4}$ b) $\frac{b+c+d+e}{4}$ c) $\frac{a+c+d+e}{4}$ d) $\frac{c+d+e+f}{4}$</p>			
18.	<p>In a school, a random sample of 145 students is taken to check whether a student's average calory intake is 1500 or not. The collected data of average calories intake of sample students is presented in a frequency distribution, which is called a:</p>		1
<p>a) Statistics b) Sampling distribution c) Parameter d) Population sampling</p>			

	<p>For questions 19 and 20, two statements are given – one labelled Assertion(A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:</p> <p>(i) Both A and R are true and R is the correct explanation of the assertion (ii) Both A and R are true but R is not the correct explanation of the assertion (iii) A is true, but R is false (iv) A is false, but R is true</p> <p style="text-align: center;">1</p>	
19.	<p><u>Assertion (A)</u> : Kuhu and Beena are two equally capable badminton players. Probability that Beena will beat Kuhu in 3 games out of 4 is 25%</p> <p><u>Reason (R)</u> : The probability of r successes in n trials, denoted by $P(X = r)$ is given by $P(X = r) = {}^n C_r p^r q^{n-r}$, $r = 0, 1, \dots, n$ where p denotes success and q denotes failure in each trial.</p> <p>a) (i) b) (ii) c) (iii) d) (iv)</p>	1
20.	<p><u>Assertion (A)</u> : If the nominal rate of interest is 12.5% and the inflation is 2%, then the effective rate of interest is 10.5%</p> <p><u>Reason (R)</u> : If the interest is calculated only at the end of an year, then the effective rate of interest is same as the nominal rate of interest.</p> <p>a) (i) b) (ii) c) (iii) d) (iv)</p>	1
<p>SECTION – B</p> <p>(All questions are compulsory. In case of internal choice, attempt any one question only)</p>		
21.	<p>₹ 2,50,000 cash is equivalent to a perpetuity of ₹ 7,500 payable at the end of each quarter. What is the rate of interest convertible quarterly?</p>	2
22.	<p>Find value of $2a + 3b - c$, if $A = \begin{bmatrix} 0 & -1 & 28 \\ a - 8 & 0 & 3b \\ -c + 2 & 2 & 0 \end{bmatrix}$ is a skew-symmetric matrix</p> <p style="text-align: center;">OR</p> <p>There are two real value(s) of x, for which the value of the determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & x & -1 \\ 0 & 4 & 2x \end{vmatrix}$ is 86.</p> <p>Find the value(s) of x</p>	2
23.	<p>A book publisher sells a hard cover edition of a book for ₹ 72 and a paperback edition for ₹ 40. In addition to a fixed weekly cost of ₹ 9,600, the cost of printing hardcover and paperback editions are ₹ 56 and ₹ 28 per book respectively. Each edition requires 5 minutes on the printing machine whereas hardcover binding takes 10 minutes and paperback takes 2 minutes on the binding machine. The printing machine and the binding machine are available for 80 hours each week. Formulate the linear programming problem to maximise the publisher's profit.</p>	2
24.	<p>A boatman takes half as much time in rowing his boat for a certain distance downstream than upstream. What is the ratio between his speed of rowing the boat in still water and speed of current?</p>	2

OR

In a 200-metre race, Anuj can beat Param by 5 metre or 3 seconds. How much time did Anuj take to complete the race?

25. Mitul invested ₹ 3,50,000 in a fund. At the end of the year the value of the fund is ₹ 4,37,500. What is the nominal rate of interest, if the market price is same at the end of the year?

2

SECTION – C

(All questions are compulsory. In case of internal choice, attempt any one question only)

26. Find the interval(s) in which the function $f(x) = \frac{x^4}{4} - 2x^3 + \frac{11x^2}{2} - 6x$, is strictly increasing and strictly decreasing.

3

27. Two badminton teams A and B are staying in the same hotel. Team A has 2 male and 3 female players accompanied by 1 coach. Team B comprises of 1 male, 2 female players and 2 coaches. The daily diet requirement (calories and protein) for each person is as given below:

	Calories	Protein
Male player	2500	65 g
Female player	1900	50 g
Coach	2000	54 g

3

Use matrix algebra to calculate the total diet requirement of calories and protein for each team.

28. Evaluate $\int \frac{dx}{(1+e^x)(1+e^{-x})}$

OR

Evaluate $\int x \log(1 + x^2) dx$

3

29. Under the pure market competition scenario, the demand function p_d and the supply function p_s for a certain commodity are given as $p_d = \frac{8}{x+1} - 2$ and $p_s = \frac{x+3}{2}$ respectively, where p is the price and x is the quantity of the commodity. Using integrals, find the producer's surplus.

OR

The demand function p for maximising a profit monopolist is given by $p = 274 - x^2$ while the marginal cost is $4 + 3x$, for x units of the commodity. Using integrals, find the consumer surplus

3

30. Surjeet purchased a new house, costing ₹ 40,00,000 and made a certain amount of down payment so that he can pay the balance by taking a home loan from XYZ Bank. If his equated monthly instalment is ₹ 30,000, at 9% interest compounded monthly (reducing balance method) and payable for 25 years, then what is the initial down payment made by him?
[Use $(1.0075)^{-300} = 0.1062$]

3

31. 10 years ago, Mr Mehra set up a sinking fund to save for his daughter's higher studies. At the end of 10 years, he has received an amount of ₹ 10,21,760. What amount did he put in the sinking fund at the end of every 6 months for the tenure, which paid him 5% p.a. compounded semi-annually?
[Use $(1.025)^{20} = 1.6386$]

3

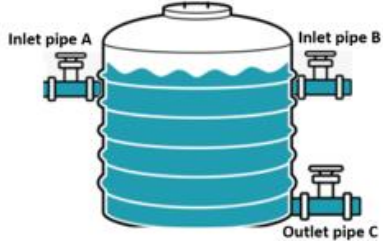
SECTION – D

(All questions are compulsory. In case of internal choice, attempt any one question only)

32.	<p>It is known that 3% of plastic buckets manufactured in a factory are defective. Using the Poisson distribution on a sample of 100 buckets, find the probability of:</p> <p>(i) Zero defective buckets (ii) At most one bucket is defective</p> <p>[Use $e^{-3} = 0.049$]</p> <p style="text-align: center;">OR</p> <p>In a math aptitude test, student scores are found to be normally distributed having mean as 45 and standard deviation 5. What percentage of students have scores -</p> <p>(i) more than the mean score? (ii) between 30 and 50?</p>	5
33.	<p>An event management company charges ₹ 4,800 per guest, for a bulk booking for 100 guests. In addition, it offers a discount of ₹ 200 for each group of 10 guests over and above 100 guest booking. What is the number of guests that will maximise the amount of money the company receives on a booking? What is the maximum profit on such booking?</p> <p style="text-align: center;">OR</p> <p>To manufacture 'x' number of dolls, a company's total cost function $C(x)$ is given by $C(x) = 100 + 0.025x^2$ and the total revenue function $R(x)$ is described as $R(x) = 5x$. Given that $C(x)$ and $R(x)$ are in thousand rupees, what number of dolls shall be manufactured to maximise the profit of the company? What is the maximum profit?</p>	5
34.	<p>Rahul is at the whole sale market to purchase folding tables and chairs, to later sell them at his furniture shop. He has only ₹ 5,760 to spend and his van has space to carry at the most 20 items. A table costs him ₹ 360 and a chair costs ₹ 240. Back at his shop, he plans to sell a table at a profit of ₹ 22 and a chair at a profit of ₹ 18. Given that he can sell all the items that he purchases, how many tables and chairs shall he purchase in order to maximise his profit?</p>	5
35.	<p>The equilibrium conditions for three competitive markets are described as given below, where p_1, p_2 and p_3 are the equilibrium price for each market respectively.</p> $p_1 + 2p_2 + 3p_3 = 85$ $3p_1 + 2p_2 + 2p_3 = 105$ $2p_1 + 3p_2 + 2p_3 = 110$ <p>Using matrix method, find the values of respective equilibrium prices.</p>	5

SECTION – E


(All questions are compulsory. In case of internal choice, attempt any one question only)

36.	<p>CASE STUDY – I</p> <p>An overhead water tank has three pipes A, B and C attached to it (as shown in figure (II)). The inlet pipes A and B can fill the empty tank independently in 15 hours and 12 hours respectively. The outlet pipe C alone can empty a full tank in 20 hours.</p> <p>Based on the above information, answer the following questions. Show steps to support your answers.</p>	 <p style="text-align: center;">FIGURE (II)</p>
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a)	For a routine cleaning of the tank, the tank needs to be emptied. If pipes A and B are closed at the time when the tank is filled to two-fifth of its total capacity, how long will pipe C take to empty the tank completely?	1
b)	How long will it take for the empty tank to fill completely, if all the three pipes are opened simultaneously?	1
c)	On a given day, pipes A, B and C are opened (in order) at 5 am, 8 am and 9 am respectively, to fill the empty tank. In how many hours will the tank be filled completely? OR Given that the tank is half-full, only pipe C is opened at 6 AM, to empty the tank. After closing the pipe C and an hour's cleaning time, tank is filled completely by pipe A and B together. What is the total time taken in the whole process?	2


37. **CASE STUDY – II**

When observed over a long period of time, a time series data can predict trend that can forecast increase or decrease or stagnation of a variable under consideration. Such analytical studies can benefit a business for forecasting or prediction of future estimated sales or production. Mathematically, for finding a line of best-fit to represent a trend, many methods are available. Methods like moving-averages and least-squares squares are some of the techniques to predict such trends.



Mrs. Shamita runs a bread factory and the record of her sales of bakery items for the period of 2015 - 2019 is as follows:

Year	2015	2016	2017	2018	2019
Sales (in ₹ thousands)	35	42	46	41	48



Based on the above information, answer the following questions. Show steps to support your answers.

a)	By taking year 2017 as origin, use method of least-squares to find the best-fit trend line equation for Mrs. Shamita's business. Show the steps of your working. OR Demonstrate the technique to fit the best-suited straight-line trend by the method of 3-years moving averages. Also draw the trend line.	2
b)	What are the estimated sales for Mrs. Shamita's business for year 2022?	1
c)	Mrs Shamita wishes to grow her business to yearly sale of ₹ 67400. In which year will she be able to reach her target?	1

38. **CASE STUDY – III**

According to an educational board survey, it was observed that class XII students apply at least one to four weeks ahead of colleges application deadline. Let X represent the week when an average student applies ahead of a college's application deadline and the probability of student to get admission in the college $P(X = x)$ is given as follows:

$$P(X = x) = \begin{cases} \frac{kx}{6} & \text{when } x = 0, 1 \text{ or } 2 \\ \frac{(1-k)x}{6} & \text{when } x = 3 \\ \frac{kx}{2} & \text{when } x = 4 \\ 0 & \text{when } x > 4 \end{cases}$$

Where k is a real number.

Based on the above information, answer the following questions. Show steps to support your answers.

a)	Find the value of k .	1
b)	What is the probability that Sonali will get admission in the college, given that she applied at least 2 weeks ahead of application deadline?	1
c)	<p>Calculate the mathematical expectation of number of weeks taken by a student to apply ahead of a college's application deadline.</p> <p style="text-align: center;">OR</p> <p>To promote early admissions, the college is offering scholarships to the students for applying ahead of deadline as follows:</p> <p style="text-align: center;">₹ 50000 for applying 4 weeks early, ₹ 20000 for applying 3 weeks early, ₹ 12000 for applying 2 weeks early, and ₹ 9600 for applying 1 week early</p> <p>What is the expected scholarship offered by the college?</p>	2
