

Physics Master Academy - Only Teaching Noting Else.

Class – XII

MATHEMATICS (041)

SQP Marking Scheme (2019-20)

TIME: 3 Hrs.

Maximum Marks: 80

SECTION A		
1	(c) 9	1
2	(a) $3 \times p$	1
3	(b) $p=3, q=\frac{27}{2}$	1
4	(b) 0.25	1
5	(c) (2,3)	1
6	(b) $\frac{\pi}{3}$	1
7	(c) $\frac{8}{15}$	1
8	(b) $\frac{1}{5} \sin^{-1} \left(\frac{5x}{3} \right) + c$	1
9	(a) 0	1
10	(b) $\vec{r} = (-\hat{i} + 3\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j})$	1
11	$g \left(\left[-\frac{5}{4} \right] \right) = g(-2) = 2$	1
12	2	1
13	$y = 2$	1
14	$\frac{-3}{2}$ OR decreasing at rate of 72 units/sec.	1
15	2 units OR $\frac{5}{7}(-2\hat{i} - 3\hat{j} + 6\hat{k})$	1
16	Apply $R_1 \rightarrow R_1 + R_2$ $\begin{vmatrix} l+m+n & m+n+l & n+l+m \\ n & l & m \\ 1 & 2 & 2 \end{vmatrix}$ $= 2(l+m+n) \begin{vmatrix} 1 & 1 & 1 \\ n & l & m \\ 1 & 1 & 1 \end{vmatrix}$; yes $(l+m+n)$ is a factor	1
17	$\int_{-2}^2 (x^3 + 1) dx = \int_{-2}^2 (x^3) dx + \int_{-2}^2 1 dx = I_1 + I_2$ $= 0 + [x]_{-2}^2$ (As I_1 is odd function) $= 2+2$ $= 4$	1

18	<p>Let $x + \sin x = t$ So $(1 + \cos x)dx = dt$ $I = 3 \int \frac{dt}{t} = 3 \log t + c = 3 \log (x + \sin x) + c$ or directly by writing formula</p> $\int \frac{f'(x)}{f(x)} dx = \log f(x) + c$ <p style="text-align: center;">OR</p> $\int \cos 4x dx = \frac{\sin 4x}{4} + c$	1
19	<p>let $(1 + x^2) = t$ so $2x dx = dt$ $\Rightarrow I = \frac{1}{2} \int e^t dt = \frac{1}{2} e^t + c = \frac{1}{2} e^{(1+x^2)} + c$</p>	1
20	<p>$\frac{dy}{dx} = e^x e^y$ $\Rightarrow \frac{dy}{e^y} = e^x dx$ integrating both sides $\Rightarrow -e^{-y} + c = e^x$ $\Rightarrow e^x + e^{-y} = c$</p>	1
SECTION B		
21	<p>$= \sin^{-1} \left(\frac{\sin x}{\sqrt{2}} + \frac{\cos x}{\sqrt{2}} \right)$ if $-\frac{\pi}{4} < x < \frac{\pi}{4}$ $= \sin^{-1} \left(\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} \right)$ if $-\frac{\pi}{4} + \frac{\pi}{4} < x + \frac{\pi}{4} < \frac{\pi}{4} + \frac{\pi}{4}$ $= \sin^{-1} \left(\sin \left(x + \frac{\pi}{4} \right) \right)$ if $0 < \left(x + \frac{\pi}{4} \right) < \frac{\pi}{2}$ i.e. principal values $= \left(x + \frac{\pi}{4} \right)$</p> <p style="text-align: center;">OR</p> <p>Let 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$ So 2 divides $[(a - b) + (b - c)]$ 2 divides $(a - c)$: Yes relation R is transitive $[0] = \{0, \pm 2, \pm 4, \pm 6, \dots\}$</p>	1
	OR	1
	<p>Let 2 divides $(a - b)$ and 2 divides $(b - c)$: where $a, b, c \in Z$ So 2 divides $[(a - b) + (b - c)]$ 2 divides $(a - c)$: Yes relation R is transitive $[0] = \{0, \pm 2, \pm 4, \pm 6, \dots\}$</p>	1
22	<p>$y = ae^{2x} + be^{-x} \dots \dots \dots (1)$ $\frac{dy}{dx} = 2ae^{2x} - be^{-x} \dots \dots \dots (2)$ $\frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x} \dots \dots \dots (3)$ putting values on LHS $= \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$ $= (4ae^{2x} + be^{-x}) - (2ae^{2x} - be^{-x}) - 2(ae^{2x} + be^{-x})$ $= 4ae^{2x} + be^{-x} - 2ae^{2x} + be^{-x} - 2ae^{2x} - 2be^{-x}$ $= 0$</p>	1

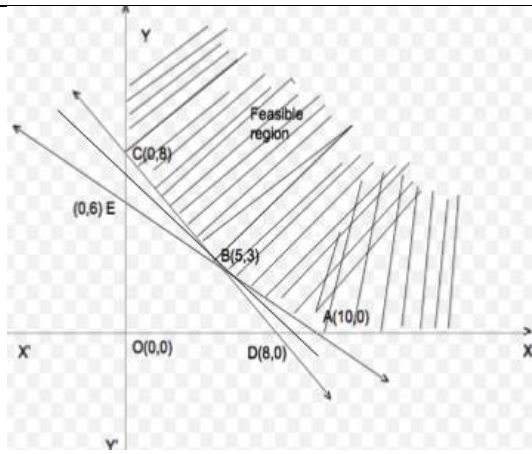
23	$x^2 = 2y \dots\dots\dots(1)$ $\Rightarrow 2x \frac{dx}{dt} = 2 \frac{dy}{dt} \quad (\text{given } \frac{dy}{dt} = \frac{dx}{dt})$ $\Rightarrow 2x \frac{dx}{dt} = 2 \frac{dx}{dt}$ $\Rightarrow x = 1$ <p>from (1) $y = \frac{1}{2}$ so point is $(1, \frac{1}{2})$</p>	1
24	$= (\vec{a} - \vec{b}) \cdot \{(\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})\}$ $= (\vec{a} - \vec{b}) \cdot \{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a}\}$ $= (\vec{a} - \vec{b}) \cdot \{\vec{b} \times \vec{c} - \vec{b} \times \vec{a} + \vec{c} \times \vec{a}\} \dots\dots(\vec{c} \times \vec{c} = 0)$ $= (\vec{a} - \vec{b}) \cdot \{\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\}$ $= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{a} \times \vec{b}) + \vec{a} \cdot (\vec{c} \times \vec{a}) - \vec{b} \cdot (\vec{b} \times \vec{c}) - \vec{b} \cdot (\vec{a} \times \vec{b}) - \vec{b} \cdot (\vec{c} \times \vec{a})$ $= \vec{a} \cdot (\vec{b} \times \vec{c}) + 0 + 0 - 0 - 0 - \vec{b} \cdot (\vec{c} \times \vec{a})$ $= \vec{a} \cdot (\vec{b} \times \vec{c}) - \vec{b} \cdot (\vec{c} \times \vec{a})$ $= 0$ <p>(STP remains same if vectors $\vec{a}, \vec{b}, \vec{c}$ are changed in cyclic order)</p> <p style="text-align: center;">OR</p>	1
	$(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0$ $\Rightarrow \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} + \vec{c} \cdot \vec{b} + \vec{c} \cdot \vec{c} = 0$ $\Rightarrow \vec{a} ^2 + \vec{b} ^2 + \vec{c} ^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\Rightarrow 3^2 + 5^2 + 7^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\Rightarrow 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -(9 + 25 + 49)$ $\Rightarrow (\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -\frac{83}{2}$	1
25	<p>Vector in the direction of first line $\vec{b} = (3\hat{i} + 4\hat{j} + 5\hat{k})$ Vector in the direction of second line $\vec{d} = (4\hat{i} - 3\hat{j} + 5\hat{k})$ Angle θ between two lines is given by $\cos \theta = \frac{\vec{b} \cdot \vec{d}}{ \vec{b} \vec{d} }$</p> $\cos \theta = \frac{(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (4\hat{i} - 3\hat{j} + 5\hat{k})}{ (3\hat{i} + 4\hat{j} + 5\hat{k}) (4\hat{i} - 3\hat{j} + 5\hat{k}) }$ $\Rightarrow \cos \theta = \frac{12 - 12 + 25}{\sqrt{9 + 16 + 25} \sqrt{9 + 16 + 25}}$ $\Rightarrow \cos \theta = \frac{25}{\sqrt{50} \sqrt{50}}$ $\Rightarrow \cos \theta = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{3}$	1
26	<p>$P(A) = \frac{80}{100} = \frac{4}{5}$, $P(B) = \frac{90}{100} = \frac{9}{10}$ $P(\text{Agree}) = P(\text{Both speaking truth or both telling lie})$ $= P(AB \text{ or } \bar{A}\bar{B})$</p>	1

	$= P(A)P(B) \text{ or } P(\bar{A})P(\bar{B})$ $= \left(\frac{4}{5}\right)\left(\frac{9}{10}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{10}\right)$ $= \frac{36+1}{50} = \frac{37}{50}$ $= \frac{74}{100} = 74\%$	1
	SECTION C	
27	<p>Let $y = f(x) = \frac{2x+3}{x-3}$(1)</p> <p>Let $x_1, x_2 \in A = \mathbb{R} - \{3\}$</p> <p>Let $f(x_1) = f(x_2)$</p> $\Rightarrow \frac{2x_1 + 3}{x_1 - 3} = \frac{2x_2 + 3}{x_2 - 3}$ $\Rightarrow (2x_1 + 3)(x_2 - 3) = (2x_2 + 3)(x_1 - 3)$ $\Rightarrow (2x_1x_2 - 6x_1 + 3x_2 - 9) = (2x_1x_2 - 6x_2 + 3x_1 - 9)$ $\Rightarrow -6x_1 + 3x_2 = -6x_2 + 3x_1$ $\Rightarrow 9x_1 = 9x_2$ $\Rightarrow x_1 = x_2$ <p>Now $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$</p> <p>so $f(x)$ is one-one</p> <p>For onto</p> $y = \frac{2x + 3}{x - 3}$ $\Rightarrow xy - 3y = 2x + 3$ $\Rightarrow xy - 2x = 3y + 3$ $\Rightarrow x(y - 2) = 3(y + 1)$ $\Rightarrow x = \frac{3(y+1)}{(y-2)} \text{(2)}$ <p>equation (2) is defined for all real values of y except 2 i.e $y \in \mathbb{R} - \{2\}$ which is same as given set $B = \mathbb{R} - \{2\}$ (co-domain=range)</p> <p>Also $y = f(x)$</p> $f(x) = f\left(\frac{3(y+1)}{(y-2)}\right)$ $= \frac{2\left[\frac{3(y+1)}{(y-2)}\right] + 3}{\frac{3(y+1)}{(y-2)} - 3} \left(\text{since } f(x) = \frac{2x + 3}{x - 3}\right)$ $\frac{2(3y + 3) + 3(y - 2)}{3y + 3 - 3y + 6} = \frac{9y}{9} = y$ <p>Thus for every $y \in B$, there exists $x \in A$ such that $f(x) = y$</p> <p>Thus function is onto.</p> <p>Since $f(x)$ is one-one and onto so $f(x)$ is invertible.</p> <p>Inverse is given by $x = f^{-1}(y) = \frac{3(y+1)}{(y-2)}$</p>	<p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p>
28	$\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$ <p>Let $x = \sin A$, $y = \sin B$</p> $\sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$ $\cos A + \cos B = a(\sin A - \sin B)$ $\Rightarrow 2 \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) = 2a \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right)$ $\Rightarrow \cos\left(\frac{A-B}{2}\right) = a \sin\left(\frac{A-B}{2}\right)$	<p>$\frac{1}{2}$</p> <p>1</p>

	$\Rightarrow \cot\left(\frac{A-B}{2}\right) = a$ $\Rightarrow \frac{A-B}{2} = \cot^{-1} a$ $\Rightarrow A-B = 2 \cot^{-1} a$ $\Rightarrow \sin^{-1} x - \sin^{-1} y = 2 \cot^{-1} a$ <p>differentiating w.r.t. x</p> $\Rightarrow \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$ <p style="text-align: center;">OR</p>	<p>1</p> <p>1</p> <p>$\frac{1}{2}$</p>
	$x = a(\cos 2\theta + 2\theta \sin 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(-2 \sin 2\theta + 2 \sin 2\theta + 4\theta \cos 2\theta)$ $\Rightarrow \frac{dx}{d\theta} = a(4\theta \cos 2\theta) \dots \dots \dots (1)$ $y = a(\sin 2\theta - 2\theta \cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(2 \cos 2\theta + 4\theta \sin 2\theta - 2 \cos 2\theta)$ $\Rightarrow \frac{dy}{d\theta} = a(4\theta \sin 2\theta) \dots \dots \dots (2)$ <p>using (1) and (2)</p> $\Rightarrow \frac{dy}{dx} = \frac{a(4\theta \sin 2\theta)}{a(4\theta \cos 2\theta)}$ $\Rightarrow \frac{dy}{dx} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$ <p>Differentiating again with respect to x, we get</p> $\Rightarrow \frac{d^2y}{dx^2} = 2 \sec^2 2\theta \cdot \frac{d\theta}{dx}$ $\Rightarrow \frac{d^2y}{dx^2} = 2 \sec^2 2\theta \cdot \frac{1}{a(4\theta \cos 2\theta)}$ $\left. \frac{d^2y}{dx^2} \right _{\theta=\frac{\pi}{8}} = 2 \sec^2 \frac{\pi}{4} \cdot \frac{1}{a\left(4 \frac{\pi}{8} \cos \frac{\pi}{4}\right)}$ $= \frac{8\sqrt{2}}{\pi a}$	<p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p>
29	$x \frac{dy}{dx} - y = \sqrt{x^2 + y^2}$ $\Rightarrow x \frac{dy}{dx} = y + \sqrt{x^2 + y^2}$ $\Rightarrow \frac{dy}{dx} = \frac{y + \sqrt{x^2 + y^2}}{x} \dots \dots \dots (1)$ <p style="text-align: right;">let $y = vx$</p> <p>differentiating with w.r.t. x</p> $\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ <p>put in (1)</p>	<p>1</p>

	$\Rightarrow v + x \frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2 x^2}}{x}$ $\Rightarrow v + x \frac{dv}{dx} = \frac{x(v + \sqrt{1 + v^2})}{x}$ $\Rightarrow x \frac{dv}{dx} = v + \sqrt{1 + v^2} - v$ $\Rightarrow x \frac{dv}{dx} = \sqrt{1 + v^2}$ $\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ <p>integrating both sides</p> $\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$ $\Rightarrow \log(v + \sqrt{1 + v^2}) = \log x + \log c$ $\Rightarrow \log(v + \sqrt{1 + v^2}) = \log cx$ $\Rightarrow (v + \sqrt{1 + v^2}) = cx$ $\Rightarrow \left(\frac{y}{x} + \sqrt{1 + \left(\frac{y}{x} \right)^2} \right) = cx$ $\Rightarrow y + \sqrt{x^2 + y^2} = cx^2$	<p>1</p> <p>$1\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>							
30	<p>Consider $I = \int_1^3 x^2 - 2x dx$</p> $ x^2 - 2x = \begin{cases} -(x^2 - 2x) & \text{when } 1 \leq x < 2 \\ (x^2 - 2x) & \text{when } 2 \leq x \leq 3 \end{cases}$ $I = \int_1^2 x^2 - 2x dx + \int_2^3 x^2 - 2x dx$ $I = \int_1^2 -(x^2 - 2x) dx + \int_2^3 (x^2 - 2x) dx$ $I = - \left[\frac{x^3}{3} - x^2 \right]_1^2 + \left[\frac{x^3}{3} - x^2 \right]_2^3$ $I = - \left(-\frac{4}{3} + \frac{2}{3} \right) + \left(\frac{4}{3} \right)$ $I = \frac{6}{3} = 2$	<p>1</p> <p>1</p> <p>1</p> <p>1</p>							
31	<p>Let X denotes the smaller of the two numbers obtained</p> <p>So X can take values 1,2,3,4,5,6</p> <p>P(X=1 is smaller number)</p> $P(X=1) = \frac{6}{7C_2} = \frac{6}{21} = \frac{2}{7}$ <p>(Total cases when two numbers can be selected from first 7 numbers are $7C_2$)</p> $P(X=2) = \frac{5}{7C_2} = \frac{5}{21}$ $P(X=3) = \frac{4}{7C_2} = \frac{4}{21}$ $P(X=4) = \frac{3}{7C_2} = \frac{3}{21} = \frac{1}{7}$ $P(X=5) = \frac{2}{7C_2} = \frac{2}{21}$ $P(X=6) = \frac{1}{7C_2} = \frac{1}{21}$ <table border="1" style="width: 100%; text-align: center;"> <tr> <td>x_i</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> </table>	x_i	1	2	3	4	5	6	<p>$\frac{1}{2}$</p> <p>2</p>
x_i	1	2	3	4	5	6			

	<table border="1"> <tr> <td>p_i</td> <td>$\frac{6}{21}$</td> <td>$\frac{5}{21}$</td> <td>$\frac{4}{21}$</td> <td>$\frac{3}{21}$</td> <td>$\frac{2}{21}$</td> <td>$\frac{1}{21}$</td> </tr> <tr> <td>$p_i x_i$</td> <td>$\frac{6}{21}$</td> <td>$\frac{10}{21}$</td> <td>$\frac{12}{21}$</td> <td>$\frac{12}{21}$</td> <td>$\frac{10}{21}$</td> <td>$\frac{6}{21}$</td> </tr> </table> <p>Mean $= \sum p_i x_i = \frac{6}{21} + \frac{10}{21} + \frac{12}{21} + \frac{12}{21} + \frac{10}{21} + \frac{6}{21} = \frac{56}{21} = \frac{8}{3}$</p> <p style="text-align: center;">OR</p>	p_i	$\frac{6}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{1}{21}$	$p_i x_i$	$\frac{6}{21}$	$\frac{10}{21}$	$\frac{12}{21}$	$\frac{12}{21}$	$\frac{10}{21}$	$\frac{6}{21}$	$\frac{1}{2}$ 1
p_i	$\frac{6}{21}$	$\frac{5}{21}$	$\frac{4}{21}$	$\frac{3}{21}$	$\frac{2}{21}$	$\frac{1}{21}$										
$p_i x_i$	$\frac{6}{21}$	$\frac{10}{21}$	$\frac{12}{21}$	$\frac{12}{21}$	$\frac{10}{21}$	$\frac{6}{21}$										
	<p>Let E_1 = event of selecting a two headed coin E_2 = event of selecting a biased coin, which shows 75% times Head E_3 = event of selecting an unbiased coin. A = event that tossed coin shows head.</p> <p style="text-align: center;">$\therefore P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$</p> <p>$\therefore P(A/E_1) = P(\text{coin showing head given that it is two headed coin})$ $= 1$</p> <p>$P(A/E_2) = P(\text{coin showing head given that it is a biased coin})$ $= \frac{75}{100} = \frac{3}{4}$</p> <p>$P(A/E_3) = P(\text{coin showing head given that it is unbiased coin})$ $= \frac{1}{2}$</p> <p>By Bayes theorem P(getting two headed coin when it is known that it shows Head)</p> $P(E_1/A) = \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$ $= \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times \frac{3}{4} + \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{1}{3} \times 1}{\frac{1}{3} (1 + \frac{3}{4} + \frac{1}{2})} = \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times \frac{9}{4}} = \frac{4}{9}$ <p>Required probability = $\frac{4}{9}$</p>	1 $\frac{1}{2}$ 1 $\frac{1}{2}$														
32	<p>Let tailor A works for x days and tailor B works for y days</p> <p>Objective function :</p> <p>To minimize labour cost $Z = 150x + 200y$ (in ₹)</p> <p>Subject to constraints</p> <p>$6x + 10y \geq 60$ i.e. $3x + 5y \geq 30$</p> <p>$4x + 4y \geq 32$ i.e. $x + y \geq 8$</p> <p>$x \geq 0, y \geq 0$</p> <p>consider equations to draw the graph and then we will shade feasible region</p> $3x + 5y = 30$ $x + y = 8$	$\frac{1}{2}$ $\frac{1}{2}$														



corner points of feasible region are A(10,0), B(5,3) and C(0,8)
Value of Z at these corner points

Point	$Z = 150x + 200y$ (in ₹)
A(10,0)	$=1500+0=1500$
B(5,3)	$=750+600=1350$ (minimum)
C(0,8)	$=0+1600=1600$

So minimum value of Z is ₹1350 when tailor A works for 5 days and tailor B works for 3 days.

To check draw $150x + 200y < 1350$ i.e $3x + 4y < 27$

As there is no region common with feasible region so minimum value is ₹1350

SECTION D

33

$$\text{LHS} = \begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$$

Apply $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$

$$= \begin{vmatrix} (y+z)^2 & x^2 - (y+z)^2 & x^2 - (y+z)^2 \\ y^2 & (z+x)^2 - y^2 & 0 \\ z^2 & 0 & (x+y)^2 - z^2 \end{vmatrix}$$

$$= \begin{vmatrix} (y+z)^2 & (x+y+z)(x-y-z) & (x+y+z)(x-y-z) \\ y^2 & (z+x+y)(z+x-y) & 0 \\ z^2 & 0 & (x+y+z)(x+y-z) \end{vmatrix}$$

Taking $(x+y+z)$ common from C_2 as well as C_3

$$= (x+y+z)^2 \begin{vmatrix} (y+z)^2 & (x-y-z) & (x-y-z) \\ y^2 & (z+x-y) & 0 \\ z^2 & 0 & (x+y-z) \end{vmatrix}$$

Apply $R_1 \rightarrow R_1 - R_2 - R_3$

$$= (x+y+z)^2 \begin{vmatrix} 2yz & -2z & -2y \\ y^2 & (z+x-y) & 0 \\ z^2 & 0 & (x+y-z) \end{vmatrix}$$

1

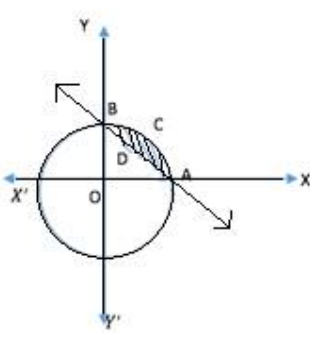
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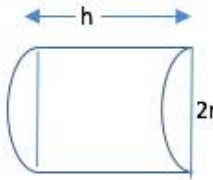
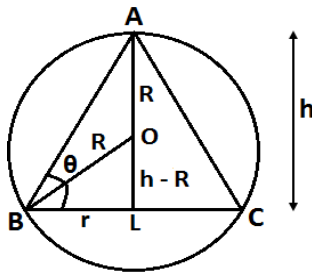
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	<p>Apply $C_2 \rightarrow y C_2$ and $C_3 \rightarrow z C_3$</p> $= \frac{(x+y+z)^2}{yz} \begin{vmatrix} 2yz & -2yz & -2yz \\ y^2 & (yz + yx - y^2) & 0 \\ z^2 & 0 & (zx + zy - z^2) \end{vmatrix}$ <p>Apply $C_2 \rightarrow C_2 + C_1$ and $C_3 \rightarrow C_3 + C_1$</p> $= \frac{(x+y+z)^2}{yz} \begin{vmatrix} 2yz & 0 & 0 \\ y^2 & (yz + yx) & y^2 \\ z^2 & z^2 & (zx + zy) \end{vmatrix}$ <p>expanding along R_1</p> $= \left(\frac{(x+y+z)^2}{yz}\right) 2yz[(yz + yx)(zx + zy) - y^2 z^2]$ $= 2(x + y + z)^2 [xyz^2 + x^2 yz + xy^2 z + y^2 z^2 - y^2 z^2]$ $= 2xyz(x + y + z)^2 (x + y + z)$ $= 2xyz(x + y + z)^3$ <p style="text-align: center;">OR</p>	<p style="text-align: center;"><u>1</u></p> <p style="text-align: center;"><u>1</u></p> <p style="text-align: center;"><u>1</u></p>
	<p>** $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$</p> $ A = 2(-2) - 3(2 - 0) + 4(1 - 0) = -6 \neq 0$ <p style="text-align: center;">$\therefore A^{-1}$ exists</p> <p>Cofactors</p> $A_{11} = -2 \quad A_{12} = -2 \quad A_{13} = 1$ $A_{21} = 2 \quad A_{22} = 4 \quad A_{23} = -2$ $A_{31} = 4 \quad A_{32} = 4 \quad A_{33} = -5$ $Adj A = \begin{bmatrix} -2 & -2 & 1 \\ -2 & 4 & -2 \\ 4 & 4 & -5 \end{bmatrix}'$ $Adj A = \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ $A^{-1} = \frac{Adj A}{ A } = \frac{1}{-6} \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix}$ <p>System of equations can be written as $AX = B$</p> <p>Where $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$</p> <p>Now $AX = B$</p> $\Rightarrow X = A^{-1}B$ $\Rightarrow X = \frac{1}{-6} \begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & 4 \\ 1 & -2 & -5 \end{bmatrix} \begin{bmatrix} 17 \\ 3 \\ 7 \end{bmatrix}$	<p style="text-align: center;">1</p> <p style="text-align: center;">2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$\frac{1}{2}$</p>

	$\Rightarrow X = \frac{1}{-6} \begin{bmatrix} -34 - 6 + 28 \\ -34 + 12 + 28 \\ 17 - 6 - 35 \end{bmatrix}$ $\Rightarrow X = \frac{1}{-6} \begin{bmatrix} -12 \\ 6 \\ -24 \end{bmatrix}$ $\Rightarrow X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $\Rightarrow x = 2, \quad y = -1, \quad z = 4$	$1\frac{1}{2}$
34	$x^2 + y^2 = 1 \dots\dots\dots(1)$ $x + y = 1 \dots\dots\dots(2)$ solving (1) and(2) $x^2 + (1 - x)^2 = 1$ $x^2 + x^2 - 2x + 1 = 1$ $2x^2 - 2x = 0$ $2x(x - 1) = 0$ $x = 0 \text{ or } x = 1$  Required area = shaded area ACBDA = area(OACBO) – area(OADBO) $= \int_0^1 (y_{circle} - y_{line}) dx$ $\int_0^1 \sqrt{1 - x^2} dx - \int_0^1 (1 - x) dx$ $= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2} \sin^{-1} x \right]_0^1 - \left[x - \frac{x^2}{2} \right]_0^1$ $\left[\left(0 + \frac{1}{2} \cdot \frac{\pi}{2} \right) - 0 \right] - \left[\left(1 - \frac{1}{2} \right) \right]$ $\left(\frac{\pi}{4} - \frac{1}{2} \right)$ square units	1 1 1 $1\frac{1}{2}$ $1\frac{1}{2}$
35	Let r be the radius and h be the height of half cylinder Volume $= \frac{1}{2} \pi r^2 h = V(\text{constant}) \dots\dots\dots(1)$	$\frac{1}{2} (fig)$

	<div style="text-align: center;">  </div> <p>Total surface area of half cylinder is</p> $S=2\left(\frac{1}{2}\pi r^2\right) + \pi r h + 2rh \dots\dots\dots(2)$ <p>From (1) put the value of h in (2)</p> $S=(\pi r^2) + \pi r \left(\frac{2V}{\pi r^2}\right) + 2r \left(\frac{2V}{\pi r^2}\right)$ $S=(\pi r^2) + \left(\frac{1}{r}\right) \left[\frac{4V}{\pi} + 2V\right]$ $\frac{ds}{dr} = (2\pi r) + \left(\frac{-1}{r^2}\right) \left[\frac{4V}{\pi} + 2V\right] \dots\dots\dots(3)$ <p>For maxima/minima $\frac{ds}{dr} = 0$</p> $\Rightarrow (2\pi r) + \left(\frac{-1}{r^2}\right) \left[\frac{4V}{\pi} + 2V\right] = 0$ $\Rightarrow (2\pi r) = \left(\frac{1}{r^2}\right) \left[\frac{4V}{\pi} + 2V\right]$ $\Rightarrow \pi r^3 = V \left[\frac{2 + \pi}{\pi}\right]$ $\Rightarrow V = \frac{\pi^2 r^3}{\pi + 2} \dots\dots\dots(4)$ <p>From (1) and (4)</p> $\Rightarrow \frac{1}{2} \pi r^2 h = \frac{\pi^2 r^3}{\pi + 2}$ $\Rightarrow \frac{h}{2r} = \frac{\pi}{\pi + 2}$ $\Rightarrow \text{height: diameter} = \pi : \pi + 2$ <p>Differentiating (3) with respect to r</p> $\frac{d^2s}{dr^2} = (2\pi) + \left(\frac{2}{r^3}\right) \left[\frac{4V}{\pi} + 2V\right] = \text{positive (as all quantities are +ve)}$ <p>so S is minimum when</p> $\text{height: diameter} = \pi : \pi + 2$ <p>OR</p>	<p>$1 \frac{1}{2}$</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p>
	<p>Let $2r$ be the base and h be the height of triangle, which is inscribed in a circle of radius R</p> <p>Area of triangle $= \frac{1}{2} (\text{base})(\text{height})$</p> $A = \frac{1}{2} (2r)(h) = rh \dots\dots\dots(1)$ <div style="text-align: center;">  </div> <p>Area being positive quantity, A will be maximum or minimum if A^2 is</p>	<p>1</p> <p>$\frac{1}{2}$ (fig)</p>

	<p>maximum or minimum.</p> $Z = A^2 = r^2 h^2 \dots \dots \dots (2)$ <p>Now In triangle OLB $BL^2 = OB^2 - OL^2$ In $\triangle OBD$ $Z = A^2 = r^2 h^2 \quad r^2 = R^2 - (h - R)^2 \Rightarrow r^2 = 2hR - h^2$ put in (2)</p> $Z = h^2(2hR - h^2)$ $\Rightarrow Z = (2h^3R - h^4)$ $\Rightarrow \frac{dZ}{dh} = 6h^2R - 4h^3 \dots \dots \dots (3)$ <p>For maxima/minima $\frac{dZ}{dh} = 0$ $\Rightarrow 6h^2R - 4h^3 = 0$ $\Rightarrow 6R = 4h(h \neq 0)$</p> $\Rightarrow h = \frac{3R}{2}$ <p>differentiating (3) w.r.t. h</p> $\Rightarrow \frac{d^2Z}{dh^2} = 12hR - 12h^2$ $\Rightarrow \left. \frac{d^2Z}{dh^2} \right _{h=\frac{3R}{2}} = 12\left(\frac{3R}{2}\right)R - 12\left(\frac{3R}{2}\right)^2$ $= 18R^2 - 27R^2 = -ve$ <p>so $Z=A^2$ is maximum when $h = \frac{3R}{2}$ $\Rightarrow A$ is maximum when $h = \frac{3R}{2}$</p> <p>when $h = \frac{3R}{2}, r^2 = 2hR - h^2 = 2R \cdot \frac{3R}{2} - \left(\frac{3R}{2}\right)^2$</p> $r^2 = \frac{3R^2}{4}$ $r = \frac{\sqrt{3}R}{2}$ $\tan \theta = \frac{h}{r} = \frac{\frac{3R}{2}}{\frac{\sqrt{3}R}{2}} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$ <p>Triangle ABC is equilateral triangle</p>	<p style="text-align: center;">1 2</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>
36	<p>Let $P(x, y, z)$ be any point on the plane in which $A(2, 1, 2)$ and $B(4, -2, 1)$ lie. $\therefore \vec{AP}$ and \vec{AB} lie on required plane.</p> <p>Also required plane is perpendicular to given plane $\vec{r} \cdot (\hat{i} - 2\hat{k}) = 5$ \therefore normal to given plane $\vec{n}_1 = (\hat{i} - 2\hat{k})$ lie on required plane.</p> <p>$\Rightarrow \vec{AP}, \vec{AB}$ and \vec{n}_1 are coplanar.</p> <p>Where $\vec{AP} = (x - 2)\hat{i} + (y - 1)\hat{j} + (z - 2)\hat{k}$ $\vec{AB} = 2\hat{i} - 3\hat{j} - \hat{k}$ \Rightarrow Scaler triple product $[\vec{AP} \quad \vec{AB} \quad \vec{n}_1] = 0$</p>	<p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p>

	$\Rightarrow \begin{vmatrix} x-2 & y-1 & z-2 \\ 2 & -3 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 0$ $\Rightarrow (x-2)(6-0) - (y-1)(-4+1) + (z-2)(0+3) = 0$ $\Rightarrow 6x - 12 + 3y - 3 + 3z - 6 = 0$ $\Rightarrow 2x + y + z = 7 \dots \dots \dots (1)$ <p>Line passing through points $L(3,4,1)$ and $M(5,1,6)$ is</p> $\Rightarrow \frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = \lambda \dots \dots \dots (2)$ <p>\Rightarrow General point on the line is $Q(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$</p> <p>As line (2) crosses plane (1) so point Q should satisfy equation(1)</p> $\therefore 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) = 7$ $4\lambda + 6 - 3\lambda + 4 + 5\lambda + 1 = 7$ $6\lambda = -4$ $\lambda = -\frac{2}{3}$ $Q\left(-\frac{4}{3} + 3, 2 + 4, -\frac{10}{3} + 1\right) = Q\left(\frac{5}{3}, 6, -\frac{7}{3}\right)$	<p>1</p> <p>1</p> <p>1</p>
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