

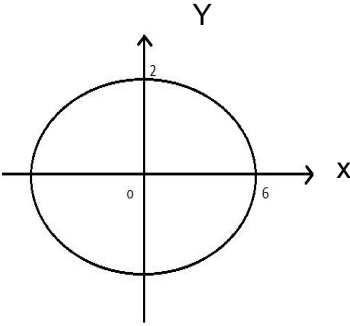
9	0 OR 3	1 1
10	\hat{j}	1
11	$\frac{1}{2} 2\hat{i} \times (-3\hat{j}) = \frac{1}{2} -6\hat{k} = 3 \text{ sq units}$	1
12	$ \hat{a} + \hat{b} ^2 = 1$ $\Rightarrow \hat{a}^2 + \hat{b}^2 + 2\hat{a} \cdot \hat{b} = 1$ $\Rightarrow 2\hat{a} \cdot \hat{b} = 1 - 1 - 1$ $\Rightarrow \hat{a} \cdot \hat{b} = \frac{-1}{2} \Rightarrow \hat{a} \hat{b} \cos \theta = \frac{-1}{2} \Rightarrow \theta = \pi - \frac{\pi}{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$	1
13	1,0,0	1
14	(0,0,0)	1
15	$1 - \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$	1
16	$\left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^7$	1
Section II		
17(i)	(b)	1
17(ii)	(a)	1
17(iii)	(c)	1
17(iv)	(a)	1
17(v)	(d)	1
18(i)	(b)	1
18(ii)	(c)	1
18(iii)	(b)	1
18(iv)	(d)	1
18(v)	(d)	1
Section III		
19	$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right) = \tan^{-1}\left[\frac{\sin\left(\frac{\pi}{2}-x\right)}{1-\cos\left(\frac{\pi}{2}-x\right)}\right]$ $\tan^{-1}\left[\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right]$	$\frac{1}{2}$

	$\tan^{-1} \left[\cot \left(\frac{\pi}{4} - \frac{x}{2} \right) \right] = \tan^{-1} \left[\tan \frac{\pi}{2} - \left(\frac{\pi}{4} - \frac{x}{2} \right) \right]$ $\tan^{-1} \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right] = \frac{\pi}{4} + \frac{x}{2}$	<p>1</p> <p>$\frac{1}{2}$</p>
20	$A^2 = 2A$ $\Rightarrow AA = 2A $ $\Rightarrow A A = 8 A \quad (\because AB = A B \text{ and } 2A = 2^3 A)$ $\Rightarrow A (A - 8) = 0$ $\Rightarrow A = 0 \text{ or } 8$ <p style="text-align: center;">OR</p> $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ $5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}, 7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$ $\Rightarrow A^2 - 5A + 7I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ $\Rightarrow A^{-1}(A^2 - 5A + 7I) = A^{-1}0$ $\Rightarrow A - 5I + 7A^{-1} = 0$ $\Rightarrow 7A^{-1} = 5I - A$ $\Rightarrow A^{-1} = \frac{1}{7} \left(\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \right)$ $\Rightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
21	$\lim_{x \rightarrow 0} \frac{1 - \cos kx}{x \sin x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 \left(\frac{kx}{2} \right)}{x \sin x}$ $= \lim_{x \rightarrow 0} \frac{2 \sin^2 \left(\frac{kx}{2} \right)}{\frac{x^2}{x \sin x}}$ $= \frac{\lim_{x \rightarrow 0} \frac{2 \sin^2 \left(\frac{kx}{2} \right)}{\left(\frac{kx}{2} \right)^2} \times \left(\frac{k}{2} \right)^2}{\lim_{x \rightarrow 0} \frac{\sin x}{x}} = \frac{2 \times 1 \times \frac{k^2}{4}}{1}$	<p>$1 \frac{1}{2}$</p>

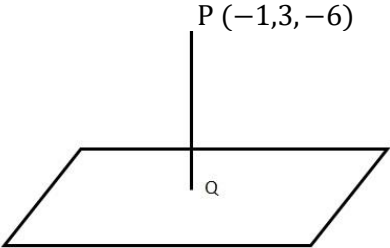
	$\therefore f(x) \text{ is continuous at } x = 0$ $\therefore \lim_{x \rightarrow 0} f(x) = f(0)$ $\Rightarrow \frac{k^2}{2} = \frac{1}{2} \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$	$\frac{1}{2}$
22	$y = x + \frac{1}{x} \Rightarrow \frac{dy}{dx} = 1 - \frac{1}{x^2}$ <p>\therefore normal is perpendicular to $3x - 4y = 7$, \therefore tangent is parallel to it</p> $1 - \frac{1}{x^2} = \frac{3}{4} \Rightarrow x^2 = 4 \Rightarrow x = 2 \quad (\because x > 0)$ <p>when $x = 2$, $y = 2 + \frac{1}{2} = \frac{5}{2}$</p> <p>$\therefore$ Equation of Normal : $y - \frac{5}{2} = -\frac{4}{3}(x - 2) \Rightarrow 8x + 6y = 31$</p>	1 1
23	$I = \int \frac{1}{\cos^2 x (1 - \tan x)^2} dx$ <p>Put, $1 - \tan x = y$</p> <p>So that, $-\sec^2 x dx = dy$</p> $= \int \frac{-1 dy}{y^2} = - \int y^{-2} dy$ $= + \frac{1}{y} + c = \frac{1}{1 - \tan x} + c$ <p style="text-align: center;">OR</p> $I = \int_0^1 x (1 - x)^n dx$ $I = \int_0^1 (1 - x)[1 - (1 - x)]^n dx$ $I = \int_0^1 (1 - x) x^n dx = \int_0^1 (x^n - x^{n+1}) dx$ $I = \left[\frac{x^{n+1}}{n+1} - \frac{x^{n+2}}{n+2} \right]_0^1$ $I = \left[\left(\frac{1}{n+1} - \frac{1}{n+2} \right) - 0 \right] = \frac{1}{(n+1)(n+2)}$	1 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
24	$\text{Area} = 2 \int_0^2 \sqrt{8x} dx$ $= 2 \times 2\sqrt{2} \int_0^2 x^{\frac{1}{2}} dx$	1

	<p>so that $y = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$-----(1)</p> <p>Now, $u = e^{x \sin^2 x}$, Differentiating both sides w.r.t. x, we get</p> $\Rightarrow \frac{du}{dx} = e^{x \sin^2 x} [x(\sin 2x) + \sin^2 x] \quad \text{----- (2)}$ <p>Also , $v = (\sin x)^x$</p> $\Rightarrow \log v = x \log (\sin x)$ <p>Differentiating both sides w.r.t. x, we get</p> $\frac{1}{v} \frac{dv}{dx} = x \cot x + \log (\sin x)$ $\frac{dv}{dx} = (\sin x)^x [x \cot x + \log(\sin x)] \quad \text{----- (3)}$ <p>Substituting from – (2), – (3) in – (1) we get</p> $\frac{dy}{dx} = e^{x \sin^2 x} [x \sin 2x + \sin^2 x] + (\sin x)^x [x \cot x + \log(\sin x)]$	<p>1</p> <p>1</p> <p>$\frac{1}{2}$</p>
31	$\text{RHD} = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{[1+h] - [1]}{h}$ $= \lim_{h \rightarrow 0} \frac{(1-1)}{h} = 0$ $\text{LHD} = \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0} \frac{[1-h] - [1]}{-h} = \lim_{h \rightarrow 0} \frac{0-1}{-h}$ $= \lim_{h \rightarrow 0} \frac{1}{h} = \infty$ <p>Since, RHD \neq LHD Therefore $f(x)$ is not differentiable at $x = 1$</p> <p style="text-align: center;">OR</p> $y = b \tan \theta \Rightarrow \frac{dy}{d\theta} = b \sec^2 \theta \dots (1)$ $x = a \sec \theta \Rightarrow \frac{dx}{d\theta} = a \sec \theta \tan \theta \dots (2)$	<p>1</p> <p>1</p> <p>1</p>

	$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b \sec^2 \theta}{a \sec \theta \tan \theta} = \frac{b}{a} \operatorname{cosec} \theta$ <p><i>Differentiating both sides w.r.t. x, we get</i></p> $\frac{d^2y}{dx^2} = \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{d\theta}{dx}$ $= \frac{-b}{a} \operatorname{cosec} \theta \cot \theta \times \frac{1}{a \sec \theta \tan \theta} \quad [\text{using (2)}]$ $= \frac{-b}{a.a} \cot^3 \theta$ $\left. \frac{d^2y}{dx^2} \right _{\theta=\frac{\pi}{6}} = \frac{-b}{a} \left[\cot \frac{\pi}{6} \right]^3 = \frac{-b}{a} (\sqrt{3})^3 = -\frac{3\sqrt{3}b}{a.a}$	$1\frac{1}{2}$ $\frac{1}{2}$
32	$f(x) = \tan x - 4x$ $f'(x) = \sec^2 x - 4$ <p>a) For $f(x)$ to be strictly increasing</p> $f'(x) > 0$ $\Rightarrow \sec^2 x - 4 > 0$ $\Rightarrow \sec^2 x > 4$ $\Rightarrow \cos^2 x < \frac{1}{4} \Rightarrow \cos^2 x < \left(\frac{1}{2}\right)^2$ $\Rightarrow -\frac{1}{2} < \cos x < \frac{1}{2} \Rightarrow \frac{\pi}{3} < x < \frac{\pi}{2}$ <p>b) For $f(x)$ to be strictly decreasing</p> $f'(x) < 0$ $\Rightarrow \sec^2 x - 4 < 0$ $\Rightarrow \sec^2 x < 4$ $\Rightarrow \cos^2 x > \frac{1}{4}$ $\Rightarrow \cos^2 x > \left(\frac{1}{2}\right)^2$ $\Rightarrow \cos x > \frac{1}{2} \left[\because x \in \left(0, \frac{\pi}{2}\right) \right]$ $\Rightarrow 0 < x < \frac{\pi}{3}$	$\frac{1}{2}$ $1\frac{1}{2}$ 1

	<p>Required Area = $\frac{4}{3} \int_0^6 \sqrt{6^2 - x^2} dx$</p>  <p> $= \frac{4}{3} \left[\frac{x}{2} \sqrt{6^2 - x^2} + 18 \sin^{-1} \left(\frac{x}{6} \right) \right]_0^6$ $= \frac{4}{3} \left[18 \times \frac{\pi}{2} - 0 \right] = 12\pi \text{ sq units}$ </p>	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p>
35	<p>The given differential equation can be written as</p> $\frac{dy}{dx} = \frac{y + 2x^2}{x} \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = 2x$ <p>Here $P = -\frac{1}{x}$, $Q = 2x$</p> $\text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x} = \frac{1}{x}$ <p>The solutions is :</p> $y \times \frac{1}{x} = \int \left(2x \times \frac{1}{x} \right) dx$ $\Rightarrow \frac{y}{x} = 2x + c$ $\Rightarrow y = 2x^2 + cx$	<p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p>
36	<p>$A = 1(-1 - 2) - 2(-2 - 0) = -3 + 4 = 1$</p> <p>A is nonsingular, therefore A^{-1} exists</p> $\text{Adj } A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ $\Rightarrow A^{-1} = \frac{1}{ A } (\text{Adj } A) = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$	<p>$\frac{1}{2}$</p> <p>$1\frac{1}{2}$</p>

	<p>The given equations can be written as:</p> $\begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$ <p>Which is of the form $A'X = B$</p> $\Rightarrow X = (A')^{-1}B = (A^{-1})'B$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$ $\Rightarrow x = 0, \quad y = -5, \quad z = -3$ <p style="text-align: center;">OR</p> $AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ $= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ $\Rightarrow AB = 6I$ $\Rightarrow A\left(\frac{1}{6}B\right) = I \Rightarrow A^{-1} = \frac{1}{6}(B)$ <p>The given equations can be written as</p> $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $AX = D, \text{ where } D = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ $\Rightarrow X = A^{-1}D$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$ $x = 2, \quad y = -1, \quad z = 4$	<p style="text-align: center;">$\frac{1}{2}$</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$1\frac{1}{2}$</p> <p style="text-align: center;">$1\frac{1}{2}$</p> <p style="text-align: center;">1</p> <p style="text-align: center;">1</p> <p style="text-align: center;">$1\frac{1}{2}$</p>
37	We have $a_1 = 3\hat{i} + 2\hat{j} - 4\hat{k}$ $b_1 = \hat{i} + 2\hat{j} + 2\hat{k}$	

	$a_2 = 5i - 2j$ $b_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 4\hat{j} + 4\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix} = \hat{i}(12 - 4) - \hat{j}(6 - 6) + \hat{k}(2 - 6)$ $\vec{b}_1 \times \vec{b}_2 = 8\hat{i} + 0\hat{j} - 4\hat{k} = 8\hat{i} - 4\hat{k}$ $\therefore (\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1) = 16 - 16 = 0$ <p>\therefore The lines are intersecting and the shortest distance between the lines is 0.</p> <p>Now for point of intersection</p> $3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k}) = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$ $\Rightarrow 3 + \lambda = 5 + 3\mu \quad \text{--- -- (1)}$ $2 + 2\lambda = -2 + 2\mu \quad \text{--- -- (2)}$ $-4 + 2\lambda = 6\mu \quad \text{--- -- (3)}$ <p>Solving (1) and (2) we get, $\mu = -2$ and $\lambda = -4$</p> <p>Substituting in equation of line we get</p> $\vec{r} = 5i - 2j + (-2)(3i + 2j - 6k) = -i - 6j - 12k$ <p>Point of intersection is $(-1, -6, -12)$</p> <p style="text-align: center;">OR</p> <p>Let P be the given point and Q be the foot of the perpendicular.</p> <p>Equation of PQ $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+6}{-2} = \lambda$</p> <div style="text-align: center;">  </div> <p>Let coordinates of Q be $(2\lambda - 1, \lambda + 3, -2\lambda - 6)$</p> <p>Since Q lies in the plane $2x + y - 2z + 5 = 0$</p> $\therefore 2(2\lambda - 1) + (\lambda + 3) - 2(-2\lambda - 6) + 5 = 0$ $\Rightarrow 4\lambda - 2 + \lambda + 3 + 4\lambda + 12 + 5 = 0$	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>$1\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>
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$$\Rightarrow 9\lambda + 18 = 0 \quad \Rightarrow \lambda = -2$$

\therefore coordinates of Q are (-5, 1, -2)

$$\begin{aligned} \text{Length of the perpendicular} &= \sqrt{(-5 + 1)^2 + (1 - 3)^2 + (-2 + 6)^2} \\ &= 6 \text{ units} \end{aligned}$$

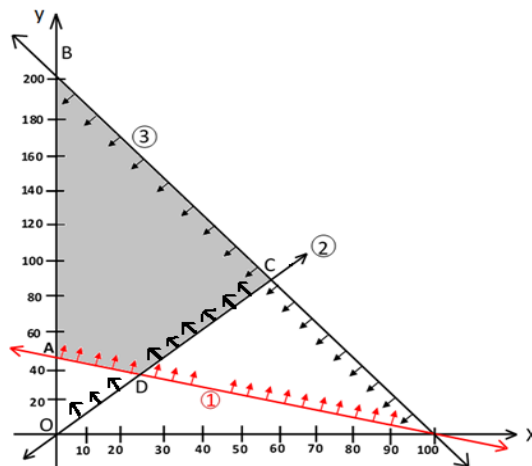
1
1
1

38

$$\text{Max } Z = 3x + y$$

Subject to

$$\begin{aligned} x + 2y &\geq 100 && \text{-----} && (1) \\ 2x - y &\leq 0 && \text{-----} && (2) \\ 2x + y &\leq 200 && \text{-----} && (3) \\ x &\geq 0, && y &\geq 0 \end{aligned}$$



3

Corner Points	$Z = 3x + y$
A (0, 50)	50
B (0, 200)	200
C (50, 100)	250
D (20, 40)	100

1

$$\text{Max } z = 250 \text{ at } x = 50, \quad y = 100$$

1

OR

(i)

Corner points	$Z = 3x - 4y$
O(0,0)	0
A(0,8)	-32
B(4,10)	-28
C(6,8)	-14
D(6,5)	-2
E(4,0)	12

$Max\ Z = 12\ at\ E(4,0)$

$Min\ Z = -32\ at\ A(0,8)$

(ii) Since maximum value of Z occurs at B(4,10) and C(6, 8)

$$\therefore 4p + 10q = 6p + 8q$$

$$\Rightarrow 2q = 2p$$

$$\Rightarrow p = q$$

Number of optimal solution are infinite

$1\frac{1}{2}$

1

2

$\frac{1}{2}$