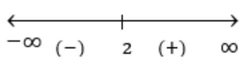
# Marking Scheme

## Mathematics (Term-I)

## Class-XII (Code-041)

| Q.N. | Correct Option | Hints / Solutions |
|---|---|---|
| 1 | d | $sin\left(\frac{\pi}{3}-\left(\frac{-\pi}{6}\right)\right) = sin\left(\frac{\pi}{2}\right) = 1$ |
| 2 | b | $\lim_{x\to o}\left(\frac{1-\cos kx}{x\sin x}\right) = \frac{1}{2}$<br>$\Rightarrow \lim_{x\to o}\left(\frac{2sin^2\frac{kx}{2}}{x\sin x}\right) = \frac{1}{2}$<br>$\Rightarrow \lim_{x\to o} 2\left(\frac{k}{2}\right)^2\left(\frac{sin\frac{kx}{2}}{\frac{kx}{2}}\right)^2\left(\frac{x}{sinx}\right) = \frac{1}{2}$<br>$\Rightarrow k^2$=1$\Rightarrow$ k = $\pm 1$ $but\ k < 0$ $\Rightarrow$ k= -1 |
| 3 | d | $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ |
| 4 | c | As $A$ is singular matrix<br>$\Rightarrow \|A\| = 0$<br>$\Rightarrow 2k^2 - 32 = 0 \ \Rightarrow k = \pm 4$ |
| 5 | b | $f(x) = x^2 - 4x + 6$<br>$f'(x) = 2x - 4$<br>$let\ f'(x) = 0 \Rightarrow x = 2$<br><br><br>as $f'(x) > 0 \ \ \forall \ \ x \in (2, \infty)$<br>$\Rightarrow f(x)$ is Strictly increasing in $(2, \infty)$ |
| 6 | d | as $\|adj\ A\| = \|A\|^{n-1}$, where $n$ is order of the square matrix $A$<br>$= (-4)^2 = 16$ |
| 7 | b | $(1, 2)$ |
| 8 | a | $\left.\begin{matrix} 2a + b = 4 \\ a - 2b = -3 \\ 5c - d = 11 \\ 4c + 3d = 24 \end{matrix}\right\} \Rightarrow \begin{matrix} a = 1 \\ b = 2 \\ c = 3 \\ d = 4 \end{matrix}$<br>$\therefore a + b - c + 2d = 8$ |
| 9 | a | $f(x) = x + \frac{1}{x}, x > 0 \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}, x > 0$<br>As normal to the curve $y = f(x)$ at some point (x, y) is $\perp$ to given line<br>$\Rightarrow \left(\frac{x^2}{1-x^2}\right) \times \frac{3}{4} = -1 \ \ (m_1 . m_2 = -1)$<br>$\Rightarrow \ x^2 = 4 \Rightarrow x = \pm 2$<br>But $x > 0, \therefore x = 2$<br>Therefore point=$\left(2, \ \frac{5}{2}\right)$ |
| 10 | d | $sin\ (tan^{-1}x)$ =$sin\left\{sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right\} \ = \frac{x}{\sqrt{1+x^2}}$ |
| 11 | a | $\{1, 5, 9\}$ |
| 12 | c | $e^x$+$e^y = e^{x+y}$<br>$\Rightarrow e^{-y} + e^{-x} = 1$<br>Differentiating *w.r.t.* $x$: |

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| | | $\Rightarrow\ -e^{-y}\frac{dy}{dx} - e^{-x} = 0 \Rightarrow \frac{dy}{dx} = -e^{y-x}$ |
| **13** | **b** | $3 \times 5$ |
| **14** | **a** | $y = 5cos\,x - 3sin\,x \ \ \Rightarrow \frac{dy}{dx} = -5sin\,x - 3cos\,x$<br>$\Rightarrow\ \frac{d^2y}{dx^2} = -5cos\,x + 3sin\,x \ = \ -y$ |
| **15** | **c** | $adj\ A = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix} \Rightarrow (adjA)' = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ |
| **16** | **c** | $\frac{x^2}{9} + \frac{y^2}{16} = 1 \Rightarrow \frac{2x}{9} + \frac{2y}{16}\frac{dy}{dx} = 0$<br>$\Rightarrow slope\ of\ normal\ at\ any\ point\ (x,y)\ to\ the\ curve = \frac{-dx}{dy} = \frac{9y}{16x}$<br>As tangent to the curve at the point (x, y) is parallel to *y*-axis<br>$\Rightarrow \frac{9y}{16x} = 0 \Rightarrow y = 0$ and $x = \pm 3$<br>$\therefore\ \ points = (\pm 3, 0)$ |
| **17** | **b** | $\lvert A\rvert = -7$<br>$\therefore \sum_{i=1}^{3} a_{i2}A_{i2} = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} = \lvert A\rvert = -7$ |
| **18** | **d** | $y = log\ (cos\ e^x)$<br>Differentiating w.r.t. $x$:<br>$\frac{dy}{dx} = \frac{1}{cos\,(e^x)}.(-sin\ \ e^x).\ e^x$ (chain rule)<br>$\Rightarrow \frac{dy}{dx} = -e^x\ tan\ e^x$ |
| **19** | **d** | Z is maximum 180 at points *C (15,15)* and *D(0, 20).*<br>⇒ Z is maximum at every point on the line segment *CD* |
| **20** | **c** | $f(x) = 2cosx + x\ \ ,\ \ x \in [0, \frac{\pi}{2}]$<br>$f'(x) = -2sinx + 1$<br>Let $f'(x) = 0 \Rightarrow x = \frac{\pi}{6}\ \ \in [0, \frac{\pi}{2}]$<br>$f(0) = 2$<br>$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6} + \sqrt{3}$<br>$f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$ ⇒ least value of $f(x)$ is $\frac{\pi}{2}$ at $x = \frac{\pi}{2}$ |
| | | **Section-B** |
| **21** | **d** | $let\ f(x_1) = f(x_2)\ such\ that\ x_1 x_2 \in R$<br>$\Rightarrow\ {x_1}^3 = {x_2}^3$<br>$\Rightarrow\ \ x_1 = x_2$<br>⇒ $f$ is one - one<br>$Let\ y \in R(codomain). Then\ for\ any\ x, f(x) = y\ if x^3 = y$<br>$i.e., x = y^{\frac{1}{3}} \in R(domain)$<br>i.e., every element $y\ \in R(codomain)$ has a pre image $y^{\frac{1}{3}}$ in $R$(domain)<br>⇒ $f$ is onto<br>∴ $f$ is one-one and onto |
| **22** | **a** | $x = a\ sec\theta \Rightarrow \frac{dx}{d\theta} = asec\theta tan\theta$<br>$y = btan\ \theta\ \ \Rightarrow\ \frac{dy}{d\theta} = b\ sec^2\theta$<br>$\therefore \frac{dy}{dx} = \frac{b}{a}cosec\theta$<br>$\Rightarrow \frac{d^2y}{dx^2} = \frac{-b}{a}\ cosec\theta.cot\theta.\frac{d\theta}{dx} = \frac{-b}{a^2}cot^3\theta$<br>$\therefore\ \frac{d^2y}{dx^2}\Big]_{\theta=\frac{\pi}{6}} = \frac{-3\sqrt{3}b}{a^2}$ |
| **23** | **c** | $Z$ is minimum -24 at (0, 8) |
| **24** | **a** | $let\ u = sin^{-1}(2x\sqrt{1-x^2}\ )$ |

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| | | and $v = sin^{-1}x,\ -\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$<br>$\Rightarrow sin\, v = x$ ……(1)<br>Using (1), we get :<br>$=sin^{-1}(2sin\, v\, cos\, v)= sin^{-1}(sin2\, v)$<br>$\Rightarrow u = 2v, -\frac{\pi}{2} < 2v < \frac{\pi}{2}$<br>Differentiating u with respect to v, we get: $\frac{du}{dv} = 2$ |
| **25** | **d** | $AB = 6I \Rightarrow B^{-1} = \frac{1}{6}A$ |
| **26** | **b** | $f'(x) = 6(x^2 - x - 6) = 6(x-3)(x+2)$<br>As $f'(x) < 0\ \forall\ x \in (-2,3)$<br>$\Rightarrow$ $f(x)$ is strictly decreasing in $(-2,3)$ |
| **27** | **a** | $tan^{-1}\left(\frac{\sqrt{1+cos\,x}+\sqrt{1-cosx}}{\sqrt{1+cos\,x}-\sqrt{1-cos\,x}}\right)$<br>$= tan^{-1}\left(\frac{-\sqrt{2}cos\frac{x}{2}+\sqrt{2}sin\frac{x}{2}}{-\sqrt{2}cos\frac{x}{2}-\sqrt{2}sin\frac{x}{2}}\right)$ , $\pi < x < \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$<br>$= tan^{-1}\left(\frac{cos\frac{x}{2}-sin\frac{x}{2}}{cos\frac{x}{2}+sin\frac{x}{2}}\right) = tan^{-1}\left(\frac{1-tan\frac{x}{2}}{1+tan\frac{x}{2}}\right) = tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$<br>$= \frac{\pi}{4} - \frac{x}{2}, -\frac{\pi}{4} > \frac{\pi}{4} - \frac{x}{2} > -\frac{\pi}{2}$ |
| **28** | **c** | $A^2 = 2A$<br>$\Rightarrow \left\|A^2\right\| = \|2A\|$<br>$\Rightarrow \|A\|^2 = 2^3\|A\|$ as $\|kA\| = k^n\|A\|\ for\ a\ square\ matrix\ of\ order\ n$<br>$\Rightarrow$ either $\|A\| = 0\ or\ \|A\| = 8$<br>But A is non-singular matrix<br>$\therefore \|A\| = 8^2 = 64$ |
| **29** | **b** | $f'(x) = 1 - sin\,x \Rightarrow f'(x) \geq 0\ \ \forall x \in R$<br>$\Rightarrow no\ value\ of\ b\ exists$ |
| **30** | **c** | $a = b - 2\ and\ b > 6$<br>$\Rightarrow (6,8) \in R$ |
| **31** | **a** | $f(x) = \{\frac{x}{-x} = -1\ ,\quad x < 0 \quad -1\ ,\quad x \geq 0$<br>$\Rightarrow f(x) = -1\ \forall\ x \in R$<br>$\Rightarrow f(x) is\ continous\ \forall\ x \in R\ as\ it\ is\ a\ constant\ function$ |
| **32** | **b** | $kA = \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$<br>$\Rightarrow k = -6, a = -4\ and\ b = -9$ |
| **33** | **c** | Corner points of feasible region — $Z = 30x + 50y$<br>(5,0) — 150<br>(9,0) — 270<br>(0,3) — 150<br>(0,6) — 300<br>Minimum value of $Z$ occurs at infinitely many points |
| **34** | **c** | $f'(x) = \frac{-2x^2-10x+100}{\sqrt{100-x^2}}$<br>$f'(x) = 0 \Rightarrow x = -10\ or\ 5$, But $x > 0 \Rightarrow x = 5$<br>$f''(x) = \frac{2x^3-300x-1000}{(100-x)^{\frac{3}{2}}} \Rightarrow f''(5) = \frac{-30}{\sqrt{75}} < 0$<br>$\Rightarrow$ Maximum area of trapezium is $75\sqrt{3}\ cm^2$ when x = 5 |
| **35** | **d** | $(I - A)^3 - 7A = I + A + 3A + 3A - 7A = I$ |
| **36** | **c** | $\frac{-\pi}{2} < y < \frac{\pi}{2}$ |

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| **37** | **b** | Since, distinct elements of A have distinct f-images in B. Hence, f is injective and every element of B does not have its pre-image in A, hence f is not surjective. <br> $\therefore f\ is\ injective\ and\ is\ not\ surjective.$ |
| **38** | **b** | $\|A\| = 7\ ,\ adjA = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ <br> $\therefore 14A^{-1} = 14 \times \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$ |
| **39** | **b** | $y = x^3 - 11x + 5\ \Rightarrow \frac{dy}{dx} = 3x^2 - 11$ <br> Slope of line $y = x - 11\ is\ 1\ \Rightarrow 3x^2 - 11 = 1 \Rightarrow x = \pm 2$ <br> ∴ point is (2, -9) as (-2, 19) does not satisfy the equation of the given line |
| **40** | **c** | $A^2 = 3I$ <br> $\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\ \Rightarrow 3 - \alpha^2 - \beta\gamma = 0$ |
| **Section C** | | |
| **41** | **a** | As $Z$ is maximum at (30, 30) and (0, 40) <br> $\Rightarrow 30a + 30b = 40b \Rightarrow b - 3a = 0$ |
| **42** | **b** | $y = mx + 1 \ldots..(1)$ and $y^2 = 4x \ldots\ldots(2)$ <br> Substituting (1) in (2) : $(mx+1)^2 = 4x$ <br> $\Rightarrow m^2x^2 + (2m-4)x + 1 = 0 \ldots\ldots..(3)$ <br> As line is tangent to the curve <br> ⇒ line touches the curve at only one point <br> $\Rightarrow (2m-4)^2 - 4m^2 = 0 \Rightarrow m = 1$ |
| **43** | **c** | Let $f(x) = [x(x-1)+1]^{\frac{1}{3}},\quad 0 \le x \le 1$ <br> $f'(x) = \frac{2x-1}{3(x^2-x+1)^{\frac{2}{3}}}$ let $f'(x) = 0\quad \Rightarrow\ x = \frac{1}{2} \in [0,1]$ <br> $f(o) = 1, f\left(\frac{1}{2}\right) = \left(\frac{3}{4}\right)^{\frac{1}{3}}$ and $f(1) = 1$ <br> ∴ Maximum value of $f(x)$ is 1 |
| **44** | **b** | Feasible region is bounded in the first quadrant |
| **45** | **d** | $\|A\| = 2 + 2sin^2\alpha$ <br> As $-1 \le sin\alpha \le 1, \forall\ 0 \le \alpha \le 2\pi$ <br> $\Rightarrow 2 \le 2 + 2sin^2\alpha \le 4 \Rightarrow \|A\| \in [2,4]$ |
| **46** | **d** | Fuel cost per hour = $k(speed)^2$ <br> $\Rightarrow 48 = k.16^2 \Rightarrow k = \frac{3}{16}$ |
| **47** | **b** | Total cost of running train (let C) =$\frac{3}{16}v^2t + 1200t$ <br> Distance covered = 500km ⇒ $time = \frac{500}{v}hrs$ <br> Total cost of running train 500 km= $\frac{3}{16}v^2\left(\frac{500}{v}\right) + 1200\left(\frac{500}{v}\right)$ <br> $\Rightarrow\ C = \frac{375}{4}v + \frac{600000}{v}$ |
| **48** | **c** | $\frac{dC}{dv} = \frac{375}{4} - \frac{600000}{v^2}$ <br> Let $\frac{dC}{dv} = 0\quad \Rightarrow v = 80\ km/h$ |
| **49** | **c** | Fuel cost for running 500 km $\frac{375}{4}v = \frac{375}{4} \times 80 = Rs.7500/-$ |
| **50** | **d** | Total cost for running 500 km = $\frac{375}{4}v + \frac{600000}{v}$ <br> $= \frac{375\times80}{4} + \frac{600000}{80} = Rs.15000/-$ |