Sample Question Paper
Class XII
Session 2022-23
Mathematics (Code-041)

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment (**4 marks each**)** with sub parts.

SECTION A
(Multiple Choice Questions)
Each question carries 1 mark

Q1. If A =[a_{ij}] is a skew-symmetric matrix of order n, then
(a) $a_{ij} = \frac{1}{a_{ji}} \; \forall \; i, j$ (b) $a_{ij} \neq 0 \; \forall \; i, j$ (c)$a_{ij} = 0, where \; i = j$ (d) $a_{ij} \neq 0 \; where \; i = j$

Q2. If A is a square matrix of order 3, $|A'| = -3$, then $|AA'| =$
(a) 9 (b) -9 (c) 3 (d) -3

Q3. The area of a triangle with vertices A, B, C is given by
(a) $|\overrightarrow{AB} \times \overrightarrow{AC}|$ (b) $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$
(b) $\frac{1}{4}|\overrightarrow{AC} \times \overrightarrow{AB}|$ (d) $\frac{1}{8}|\overrightarrow{AC} \times \overrightarrow{AB}|$

Q4. The value of 'k' for which the function f(x) =$\begin{cases} \frac{1-cos4x}{8x^2}, if \; x \neq 0 \\ k, if \; x = 0 \end{cases}$ is continuous at x = 0 is
(a) 0 (b) -1 (c) 1. (d) 2

Q5. If $f'(x) = x + \frac{1}{x}$, then $f(x)$ is
(a) $x^2 + \log|x| + C$ (b) $\frac{x^2}{2} + \log|x| + C$ (c)$\frac{x}{2} + \log|x| + C$ (d) $\frac{x}{2} - \log|x| + C$

Q6. If m and n, respectively, are the order and the degree of the differential equation
$\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4 = 0$, then m + n =
(a) 1 (b) 2 (c) 3 (d) 4

Q7. The solution set of the inequality 3x + 5y < 4 is
(a) an open half-plane not containing the origin.
(b) an open half-plane containing the origin.
(c) the whole *XY*-plane not containing the line 3x + 5y = 4.
(d) a closed half plane containing the origin.

Q8. The scalar projection of the vector $3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ *on the vector* $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ is

(a) $\frac{7}{\sqrt{14}}$ (b) $\frac{7}{14}$ (c) $\frac{6}{13}$ (d) $\frac{7}{2}$

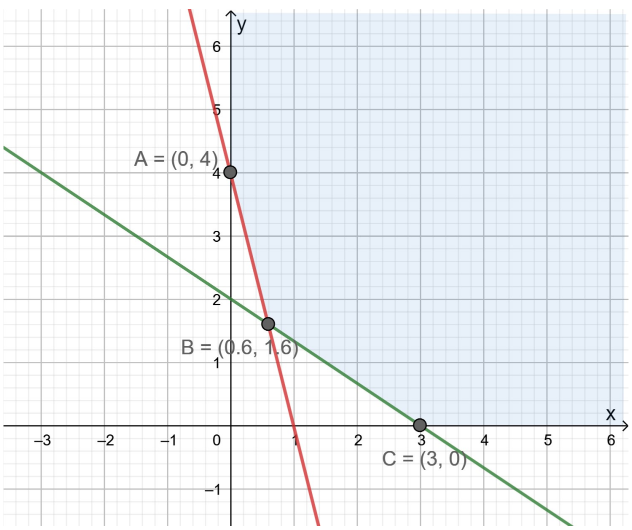
Q9. The value of $\int_2^3 \frac{x}{x^2+1}dx$ is

(a) log4 (b) $log\frac{3}{2}$ (c) $\frac{1}{2}log2$ (d) $log\frac{9}{4}$

Q10. If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$

(a) $A^{-1}B$ (b) $A^{-1}B^{-1}$ (c) BA^{-1} (d) AB

Q11. The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function Z = 4x + 6y occurs at



(a)(0.6, 1.6) *only* (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only

(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0)

Q12. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, *then the possible* value(s) of 'x' is/are

(a) 3 (b) $\sqrt{3}$ (c) -$\sqrt{3}$ (d) $\sqrt{3}, -\sqrt{3}$

Q13. If A is a square matrix of order 3 and |A| = 5, then $|adjA| =$

(a) 5 (b) 25 (c) 125 (d) $\frac{1}{5}$

Q14. Given two independent events A and B such that P(A) =0.3, P(B) = 0.6 and P$(A' \cap B')$ is

(a) 0.9 (b) 0.18 (c) 0.28 (d) 0.1

Q15. The general solution of the differential equation $ydx - xdy = 0$ *is*

(a) $xy = C$ (b) $x = Cy^2$ (c) $y = Cx$ (d) $y = Cx^2$

Q16. If $y = sin^{-1}x$, then $(1 - x^2)y_2$ *is* equal to

(a) xy_1 (b) xy (c) xy_2 (d) x^2

Q17. If two vectors $\vec{a}$ *and* $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ *and* $\vec{a}.\vec{b} = 4$, *then* $|\vec{a} - 2\vec{b}|$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{6}$ (c) 24 (d) $2\sqrt{2}$

Q18. P is a point on the line joining the points $A(0,5,-2)$ and $B(3,-1,2)$. If the x-coordinate of P is 6, then its z-coordinate is

(a) 10 (b) 6 (c) -6 (d) -10

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

Q19. **Assertion (A)**: The domain of the function $sec^{-1}2x$ is $\left(-\infty, -\frac{1}{2}\right] \cup [\frac{1}{2}, \infty)$

Reason (R): $sec^{-1}(-2) = -\frac{\pi}{4}$

Q20. **Assertion (A)**: The acute angle between the line $\bar{r} = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - \hat{\jmath})$ and the x-axis is $\frac{\pi}{4}$

Reason(R): The acute angle θ between the lines

$\bar{r} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k} + \lambda(a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k})$ and

$\bar{r} = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k} + \mu(a_2\hat{\imath} + b_2\hat{\jmath} + c_2\hat{k})$ is given by $cos\theta = \frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1{}^2+b_1{}^2+c_1{}^2}\sqrt{a_2{}^2+b_2{}^2+c_2{}^2}}$

SECTION B

This section comprises of very short answer type-questions (VSA) of 2 marks each

Q21. Find the value of $sin^{-1}[sin\left(\frac{13\pi}{7}\right)]$

OR

Prove that the function f is surjective, where $f: N \to N$ such that

$$f(n) = \begin{cases} \frac{n+1}{2}, if\ n\ is\ odd \\ \frac{n}{2}, if\ n\ is\ even \end{cases}$$

Is the function injective? Justify your answer.

Q22. A man 1.6 m tall walks at the rate of 0.3 m/sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening?

Q23. If $\vec{a} = \hat{\imath} - \hat{\jmath} + 7\hat{k}$ *and* $\vec{b} = 5\hat{\imath} - \hat{\jmath} + \lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$ are orthogonal.

OR

Find the direction ratio and direction cosines of a line parallel to the line whose equations are

$6x - 12 = 3y + 9 = 2z - 2$

Q24. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, *then prove that* $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$

Q25. Find $|\vec{x}|$ if $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 12$, where $\vec{a}$ is a unit vector.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

Q26. Find: $\int \frac{dx}{\sqrt{3-2x-x^2}}$

Q27. Three friends go for coffee. They decide who will pay the bill, by each tossing a coin and then letting the "odd person" pay. There is no odd person if all three tosses produce the same result. If there is no odd person in the first round, they make a second round of tosses and they continue to do so until there is an odd person. What is the probability that exactly three rounds of tosses are made?

OR

Find the mean number of defective items in a sample of two items drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items. Assume that the items are identical in shape and size.

Q28. Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{tanx}}$

OR

Evaluate: $\int_0^4 |x - 1|\, dx$

Q29. Solve the differential equation: $ydx + (x - y^2)dy = 0$

OR

Solve the differential equation: $xdy - ydx = \sqrt{x^2 + y^2}\, dx$

Q30. Solve the following Linear Programming Problem graphically:

Maximize Z = 400x + 300y subject to $x + y \leq 200, x \leq 40, x \geq 20, y \geq 0$

Q31. Find $\int \frac{(x^3+x+1)}{(x^2-1)} dx$

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

Q32. Make a rough sketch of the region $\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$ and find the area of the region using integration.

Q33. Define the relation R in the set $N \times N$ as follows:

For (a, b), (c, d) ∈ $N \times N$, (a, b) R (c, d) iff ad = bc. Prove that R is an equivalence relation in $N \times N$.

OR

Given a non-empty set X, define the relation R in P(X) as follows:
For A, B $\in P(X), (A, B) \in R$ iff $A \subset B$. Prove that R is reflexive, transitive and not symmetric.

Q34. An insect is crawling along the line $\bar{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ and another insect is crawling along the line $\bar{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$. At what points on the lines should they reach so that the distance between them is the shortest? Find the shortest possible distance between them.

OR

The equations of motion of a rocket are:
$x = 2t, y = -4t, z = 4t,$ where the time t is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket? At what distances will the rocket be from the starting point O(0, 0, 0) and from the following line in 10 seconds?
$\vec{r} = 20\hat{\imath} - 10\hat{\jmath} + 40\hat{k} + \mu(10\hat{\imath} - 20\hat{\jmath} + 10\hat{k})$

Q35. If A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the following system of equations
$2x - 3y + 5z = 11, 3x + 2y - 4z = -5, x + y - 2z = -3$

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

Q36. **Case-Study 1**: Read the following passage and answer the questions given below.

The temperature of a person during an intestinal illness is given by
$f(x) = -0.1x^2 + mx + 98.6, 0 \leq x \leq 12$, m being a constant, where f(x) is the temperature in °F at x days.

(i) Is the function differentiable in the interval (0, 12)? Justify your answer.
(ii) If 6 is the critical point of the function, then find the value of the constant m.

(iii) Find the intervals in which the function is strictly increasing/strictly decreasing.

OR

(iii) Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.

Q37. **Case-Study** 2: Read the following passage and answer the questions given below.

In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

(i) If the length and the breadth of the rectangular field be 2x and 2y respectively, then find the area function in terms of x.

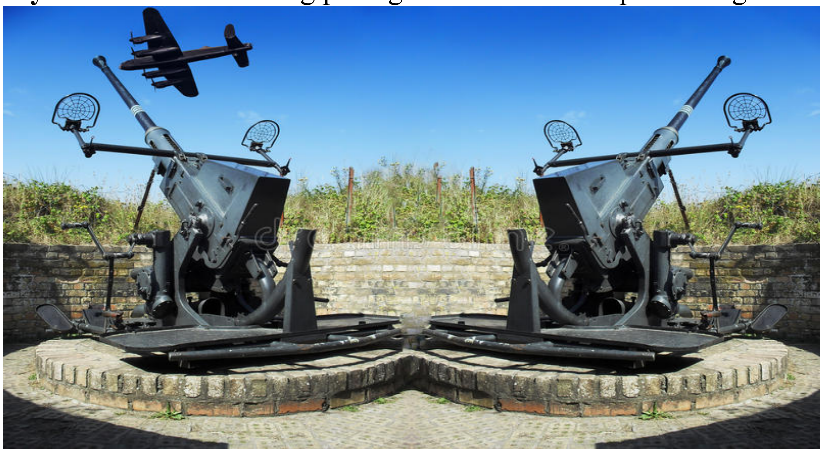
(ii) Find the critical point of the function.

(iii) Use First derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

OR

(iii) Use Second Derivative Test to find the length 2x and width 2y of the soccer field (in terms of a and b) that maximize its area.

Q38. **Case-Study** 3: Read the following passage and answer the questions given below.



There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B?