

**Marking Scheme (Mathematics XII 2017-18)**

Sr. No.	Answer	Mark(s)
<b><u>Section A</u></b>		
1.	$[(1,3)] = \{(x, y) \in A \times A : x+3 = y+1\} = \{(x, y) \in A \times A : y-x = 2\} = \{(1,3), (2,4)\}$	[1]
2.	-15	[1]
3.	$\vec{a} = \hat{i}, \vec{b} = \hat{j}$ . (or any other correct answer)	[1]
4.	$(1*2)*3 = 2^2*3 = 2^{12}, 1*(2*3) = 1*2^6 = 2^{64} \therefore (1*2)*3 \neq 1*(2*3)$ . Hence, * is not associative.	[1]
<b><u>Section B</u></b>		
5.	$\Rightarrow 4\sin^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$ $\Rightarrow 3\sin^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}x = \frac{\pi}{6} \Rightarrow x = \sin \frac{\pi}{6} = \frac{1}{2}$	[1] [1]
6.	$\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1} = \frac{1}{9-10} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$ $P = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 13 & 8 \\ 1 & 1 \end{bmatrix}$	[1 + ½] [½]
7.	Let $\cos^{-1}x = \theta$ . Then $\forall x \in \left[\frac{1}{2}, 1\right], \theta \in \left[0, \frac{\pi}{3}\right], x = \cos\theta$ The given expression on LHS $= \theta + \cos^{-1}\left[\frac{\cos\theta}{2} + \frac{\sqrt{3}\sin\theta}{2}\right] = \theta + \cos^{-1}\left[\cos\left(\theta - \frac{\pi}{3}\right)\right] = \theta + \cos^{-1}\left[\cos\left(\frac{\pi}{3} - \theta\right)\right]$ $= \theta + \frac{\pi}{3} - \theta \quad \left(\because 0 \leq \frac{\pi}{3} - \theta \leq \frac{\pi}{3}\right)$ $= \frac{\pi}{3} = RHS$	[½] [1] [½]
8.	Let $y = \frac{1}{x^2}$ . Then $\frac{dy}{dx} = \frac{-2}{x^3}$ . $dy = \left(\frac{dy}{dx}\right)_{x=2} \times \Delta x = \frac{-2}{2^3} \times 0.002 = -0.0005$ . y decreases by 0.0005.	[1/2] [1] [1/2]
9.	$\int e^x \frac{\sqrt{1+\sin 2x}}{1+\cos 2x} dx = \int e^x \frac{\sqrt{(\sin x + \cos x)^2}}{2\cos^2 x} dx$ $= \frac{1}{2} \int e^x \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\cos^2 x}\right) dx = \frac{1}{2} \int e^x (\sec x + \sec x \tan x) dx$ $= \frac{1}{2} e^x \sec x + c \quad \left[\because \int e^x (f(x) + f'(x)) dx = e^x f(x) + c\right]$	[1/2] [1/2] [1]

10.	$ax^2 + by^2 = 1 \Rightarrow 2ax + 2byy_1 = 0 \Rightarrow ax + byy_1 = 0 \quad (1)$ $\Rightarrow a + b[yy_2 + y_1^2] = 0 \Rightarrow a = -b[yy_2 + y_1^2] \quad (2)$ Substituting this value, for a in the equation (1), we get, $-b[yy_2 + y_1^2]x + byy_1 = 0 \Rightarrow x[yy_2 + y_1^2] = yy_1$ . Hence verified	[ 1/2 ]  [ 1 ]  [ 1/2 ]
11.	$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} - 2\hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 5,  \vec{b}  = \sqrt{6}$ . The required Projection (vector) of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ \vec{b} ^2} \vec{b}$ $= \frac{5}{6}(\hat{i} - 2\hat{j} + \hat{k})$ .	[ 1/2 ]  [ 1 ]  [ 1/2 ]
12.	$P(B/A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = 0.4 \times 0.6 = 0.24$ $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.24}{0.80} = \frac{3}{10}$	[ 1 ]  [ 1 ]
<b>Section C</b>		
13.	$Let \Delta_1 = \begin{vmatrix} a^3 - 1 & 0 & a - a^4 \\ 0 & a - a^4 & a^3 - 1 \\ a - a^4 & a^3 - 1 & 0 \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$ Where $C_{ij}$ = the cofactor of $a_{ij}$ and $a_{ij}$ = the (i, j)th element of determinant $\Delta$ . $We know that \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} = \Delta^2$ $\therefore \Delta_1 = \Delta^2 = (-4)^2 = 16$	[ 2 ]  [ 1 ]  [ 1 ]
14.	Since, f is differentiable at 1, f is continuous at 1. Hence, $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (2x + 1) = 3$ $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (ax^2 + b) = a + b$ $f(1) = 3$ As f is continuous at 1, we have $a + b = 3 \dots (1)$ $Lf'(1) = \lim_{h \rightarrow 0^+} \frac{f(1-h) - f(1)}{-h} = \lim_{h \rightarrow 0^+} \frac{a(1-h)^2 + b - 3}{-h}$ $= \lim_{h \rightarrow 0^+} \frac{a + ah^2 - 2ah + b - 3}{-h} = \lim_{h \rightarrow 0^+} (-ah + 2a)$ (using (1)) $= 2a$ $Rf'(1) = \lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0^+} \frac{2(1+h) + 1 - 3}{h} = 2$ . As f is differentiable at 1, we have $2a = 2$ , i. e., $a = 1$ and $b = 2$ .	[ 1 ]  [ 1/2 ]  [ 1/2 ]  [ 1/2 ]  [ 1/2 ]  [ 1/2 ]
<b>OR</b>		

	$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \frac{x + \sin x}{\sin(a+1)x}$ $= \lim_{x \rightarrow 0^-} \frac{1 + \frac{\sin x}{x}}{\frac{\sin(a+1)x}{(a+1)x}} = \frac{2}{a+1}$ $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 2 \frac{e^{\sin bx} - 1}{bx}$ $= \lim_{x \rightarrow 0^+} 2 \frac{e^{\sin bx} - 1}{\sin bx} \times \frac{\sin bx}{bx} = 2$ <p><math>f(0) = 2.</math></p> <p>For the function to be continuous at 0, we must have <math>\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^-} f(x) = f(0)</math></p> <p>i.e., we must have <math>\frac{2}{a+1} = 2 \Rightarrow a = 0</math>; b may be any real number other than 0.</p>	<p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p>
15.	$y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = 2\log\left(\frac{x+1}{\sqrt{x}}\right) = 2[\log(x+1) - \frac{1}{2}\log x]$ $y_1 = 2\left[\frac{1}{x+1} - \frac{1}{2} \times \frac{1}{x}\right] = \frac{x-1}{x(x+1)} \quad (1)$ $y_2 = \frac{x(x+1) - (x-1)(2x+1)}{x^2(x+1)^2} = \frac{-x^2 + 2x + 1}{x^2(x+1)^2}$ $\Rightarrow x(x+1)^2 y_2 = \frac{-x^2 + 2x + 1}{x} = \frac{2x - (x+1)(x-1)}{x} = 2 - (x+1)^2 y_1 \quad (\text{using (1)})$ $\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2. \text{ Hence, proved.}$	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>
16.	<p>When <math>y = 0</math>, we have <math>(x-1)(x^2+x+1)(x-2) = 0</math>, i.e., <math>x = 1</math> or <math>2</math>.</p> $\frac{dy}{dx} = x^3 - 1 + (x-2)3x^2 = 4x^3 - 6x^2 - 1$ $\left(\frac{dy}{dx}\right)_{(1,0)} = -3$ $\left(\frac{dy}{dx}\right)_{(2,0)} = 7.$ <p>The required equations of the tangents are <math>y - 0 = -3(x - 1)</math> or, <math>y = -3x + 3</math> and <math>y - 0 = 7(x - 2)</math> or, <math>y = 7x - 14</math>.</p> <p style="text-align: center;"><b>OR</b></p> <p>Domain <math>f = (-1, \infty)</math> <math>f'(x) = \frac{-3}{1+x} + \frac{4}{(2+x)} + \frac{4}{(2+x)^2} = \frac{x(x+4)}{(1+x)(2+x)^2}</math>.</p> $f'(x) = 0 \Rightarrow x = 0 \text{ [} x \neq -4 \text{ as } -4 \notin (-1, \infty)\text{].}$ <p>In <math>(-1, 0)</math>, <math>f'(x) = \frac{(-ve)(+ve)}{(+ve)(+ve)} = -ve</math>. Therefore, <math>f</math> is strictly decreasing in <math>(-1, 0]</math>.</p> <p>In <math>(0, \infty)</math>, <math>f'(x) = +ve</math>. Therefore, <math>f</math> is strictly increasing in <math>[0, \infty)</math>.</p>	<p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[1/2]</p> <p>[2]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>

17.	<p>We have <math>C(x) = x^3 - 45x^2 + 600x, 10 \leq x \leq 20</math>. For the time being we may assume that the function <math>C(x)</math> is continuous in <math>[10, 20]</math>.</p> <p><math>C'(x) = 3x^2 - 90x + 600 = 3(x-10)(x-20)</math></p> <p><math>C'(x) = 0</math> if <math>x = 10</math> or <math>x = 20</math>. But, <math>10, 20 \notin (10, 20)</math>.</p> <p>Therefore, the maximum or the minimum value will occur at the points.</p> <p><math>C(10) = 2500, C(20) = 2000</math>.</p> <p>Hence, the person must place the order for 20 trees and the least amount to be spent = Rs 2000.</p> <p><b>Value: The person cares for a healthy environment despite being economically constrained.</b></p>	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>
18.	$\int \frac{\sec x}{1 + \cos x} dx = \int \frac{\sin x}{\cos x(1 + \sin x)} dx = \int \frac{\sin x \cos x}{(1 + \sin x)^2(1 - \sin x)} dx$ $= \int \frac{t}{(1+t)^2(1-t)} dt \quad [\sin x = t \Rightarrow \cos x dx = dt]$ $\frac{t}{(1+t)^2(1-t)} = \frac{A}{1+t} + \frac{B}{(1+t)^2} + \frac{C}{1-t} \Rightarrow t = A(1+t)(1-t) + B(1-t) + C(1+t)^2$ <p style="text-align: right;">(an identity)</p> <p>Put <math>t = -1, -1 = 2B</math>, i.e., <math>B = -\frac{1}{2}</math>. Put, <math>t = 1, 1 = 4C</math>, i.e., <math>C = \frac{1}{4}</math>. Put <math>t = 0, 0 = A + B + C</math>, which gives <math>A = \frac{1}{4}</math>.</p> <p>Therefore the required integral <math>= \frac{1}{4} \int \frac{1}{1+t} dt + \frac{-1}{2} \int \frac{1}{(1+t)^2} dt + \frac{1}{4} \int \frac{1}{(1-t)} dt</math></p> $= \frac{1}{4} \log 1+t  + \frac{-1}{2} \times \frac{-1}{1+t} - \frac{1}{4} \log 1-t  + c$ $= \frac{1}{4} \log 1 + \sin x  + \frac{1}{2} \times \frac{1}{1 + \sin x} - \frac{1}{4} \log 1 - \sin x  + c$	<p>[1]</p> <p>[1 + 1/2]</p> <p>[1 + 1/2]</p>
19.	<p>The given differential equation is <math>ye^y dx = (y^3 + 2xe^y) dy, y(0) = 1</math></p> <p>or, <math>\frac{ye^y}{(y^3 + 2xe^y)} = \frac{dy}{dx}</math> or, <math>\frac{dx}{dy} + \left(-\frac{2}{y}\right)x = \frac{y^2}{e^y}</math>, which is linear in <math>x</math>.</p> <p>I. F. <math>= e^{\int \frac{-2}{y} dy} = e^{-2 \log y} = \frac{1}{y^2}</math></p> <p>Multiplying both sides by the I. F. and integrating, we get, <math>x \frac{1}{y^2} = \int e^{-y} dy</math></p> $\Rightarrow x \frac{1}{y^2} = -e^{-y} + c \Rightarrow x = -y^2 e^{-y} + cy^2 \text{ (the general solution).}$ <p>When <math>x = 0, y = 1, 0 = -e^{-1} + c \Rightarrow c = \frac{1}{e}</math>. Hence, the required particular solution is</p> $x = -y^2 e^{-y} + \frac{y^2}{e}$ <p style="text-align: center;"><b>OR</b></p>	<p>[1]</p> <p>[1]</p> <p>[1/2]</p> <p>[1]</p> <p>[1/2]</p>

	<p>The given differential equation is <math>(x - y)dy = (x + 2y)dx</math> or, <math>\frac{dy}{dx} = \frac{x + 2y}{x - y} = \frac{1 + 2\frac{y}{x}}{1 - \frac{y}{x}} = f\left(\frac{y}{x}\right)</math>,</p> <p>hence, homogeneous.</p> <p>Put <math>y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}</math>. The equation becomes <math>v + x\frac{dv}{dx} = \frac{1 + 2v}{1 - v}</math> or, <math>\frac{1 - v}{v^2 + v + 1}dv = \frac{dx}{x}</math></p> <p>or, <math>\frac{-1}{2} \times \frac{2v + 1 - 3}{v^2 + v + 1}dv = \frac{dx}{x}</math> or, <math>\left[\frac{2v + 1}{v^2 + v + 1} + \frac{-3}{v^2 + 2v \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \frac{3}{4}}\right]dv = \frac{-2dx}{x}</math></p> <p>Integrating, we get <math>\int \frac{2v + 1}{v^2 + v + 1}dv + \int \frac{-3}{\left(v + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dv = \int \frac{-2dx}{x}</math></p> <p>or, <math>\log(v^2 + v + 1) - \frac{3 \times 2}{\sqrt{3}} \tan^{-1} \frac{v + \frac{1}{2}}{\frac{\sqrt{3}}{2}} = -2\log x  + c</math></p> <p>or, <math>\log(y^2 + xy + x^2) - 2\sqrt{3} \tan^{-1} \frac{2y + x}{\sqrt{3}x} = c</math> (the general solution).</p>	<p>[ 1 ]</p> <p>[ 1 ]</p> <p>[ 2 ]</p>
<p>20.</p>	<p><math>\vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \vec{0}</math></p> <p><math>\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}</math></p> <p><math>\vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times \vec{0}</math></p> <p><math>\Rightarrow \vec{b} \times \vec{c} = \vec{a} \times \vec{b} \therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}</math></p> <p><math>[\vec{a} \ \vec{b} \ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (\vec{a} \times \vec{b})</math></p> <p><math>= 0</math> [As the scalar triple product of three vectors is zero if any two of them are equal.]</p>	<p>[ 1 ]</p> <p>[ 1 ]</p> <p>[ 1/2 ]</p> <p>[ 1/2 ]</p> <p>[ 1/2 ]</p> <p>[ 1/2 ]</p>
<p>21.</p>	<p>General point on the first line is <math>(\lambda - 2, 2\lambda + 3, 4\lambda - 1)</math>.</p> <p>General point on the second line is <math>(2\mu + 1, 3\mu + 2, 4\mu + 3)</math>.</p> <p>Direction ratios of the required line are <math>\langle \lambda - 3, 2\lambda + 2, 4\lambda - 2 \rangle</math>.</p> <p>Direction ratios of the same line may be <math>\langle 2\mu, 3\mu + 1, 4\mu + 2 \rangle</math>.</p> <p>Therefore, <math>\frac{\lambda - 3}{2\mu} = \frac{2\lambda + 2}{3\mu + 1} = \frac{4\lambda - 2}{4\mu + 2}</math> (1) <math>\Rightarrow \frac{\lambda - 3}{2\mu} = \frac{2\lambda + 2}{3\mu + 1} = \frac{2\lambda - 1}{2\mu + 1} = k</math> (say)</p> <p><math>\Rightarrow \lambda - 3 = 2\mu k, 2\lambda + 2 = (3\mu + 1)k, 2\lambda - 1 = (2\mu + 1)k</math></p> <p><math>\Rightarrow \frac{\lambda - 3}{2} = \mu k, 2\lambda + 2 = 3 \times \frac{\lambda - 3}{2} + k, 2\lambda - 1 = \lambda - 3 + k</math></p> <p><math>\Rightarrow k = \frac{4\lambda + 4 - 3\lambda + 9}{2} = \lambda + 2 \Rightarrow \lambda = 9, \mu = \frac{3}{11}</math>, which satisfy (1).</p> <p>Therefore, the direction ratios of the required line are <math>\langle 6, 20, 34 \rangle</math> or, <math>\langle 3, 10, 17 \rangle</math>.</p> <p>Hence, the required equation of the line is <math>\frac{x - 1}{3} = \frac{y - 1}{10} = \frac{z - 1}{17}</math>.</p>	<p>[ 1/2 ]</p> <p>[ 1/2 ]</p> <p>[ 1/2 ]</p> <p>[ 1/2 ]</p> <p>[ 1 ]</p> <p>[ 1/2 ]</p> <p>[ 1/2 ]</p>

22. Let  $E_1 =$  Bag I is chosen,  $E_2 =$  Bag II is chosen,  $E_3 =$  Bag III is chosen,  $A =$  The two balls drawn from the chosen bag are white and red.

$P(E_1) = \frac{1}{3} = P(E_2) = P(E_3)$ ,

$P(A|E_1) = \frac{1}{6} \times \frac{3}{6} \times 2, P(A|E_2) = \frac{2}{4} \times \frac{1}{4} \times 2, P(A|E_3) = \frac{4}{9} \times \frac{2}{9} \times 2.$

By Bayes's Theorem, the required probability =

$$P(E_3|A) = \frac{P(E_3) \times P(A|E_3)}{\sum_{i=1}^3 P(E_i) \times P(A|E_i)} = \frac{\frac{1}{3} \times \frac{4}{9} \times \frac{2}{9} \times 2}{\frac{1}{3} \times \frac{1}{6} \times \frac{3}{6} \times 2 + \frac{1}{3} \times \frac{2}{4} \times \frac{1}{4} \times 2 + \frac{1}{3} \times \frac{4}{9} \times \frac{2}{9} \times 2} = \frac{64}{199}.$$

23. Let  $X$  denotes the random variable. Then  $X = 0, 1, 2.$

$P(X = 0) = \frac{{}^{16}C_2}{{}^{20}C_2} = \frac{60}{95}, P(X = 1) = \frac{{}^4C_1 \times {}^{16}C_1}{{}^{20}C_2} = \frac{32}{95}, P(X = 2) = \frac{{}^4C_2}{{}^{20}C_2} = \frac{3}{95}.$

$x_i$	$p_i$	$x_i p_i$	$x_i^2 p_i$
0	60/95	0	0
1	32/95	32/95	32/95
2	3/95	6/95	12/95
total		38/95	44/95

Mean =  $\sum_{i=1}^3 x_i p_i = \frac{38}{95} = \frac{2}{5}$

Variance =  $\sum_{i=1}^3 x_i^2 p_i - (\sum_{i=1}^3 x_i p_i)^2 = \frac{44}{95} - \frac{4}{25} = \frac{144}{475}.$

**Section D**

24.  $f \circ g : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f \circ g(x) = f(g(x)) = f(x^3 + 5) = 2(x^3 + 5) - 3 = 2x^3 + 7$

Let  $x_1, x_2 \in \mathbb{R}(D_{f \circ g})$  such that

$f \circ g(x_1) = f \circ g(x_2) \Rightarrow 2x_1^3 + 7 = 2x_2^3 + 7 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2.$  Hence,  $f \circ g$  is one-one.

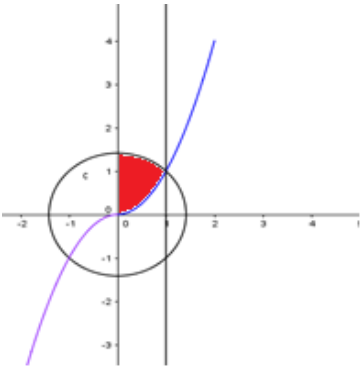
Let  $y \in \mathbb{R}(\text{Codomain}_{f \circ g})$ . Then for any  $x$   $f \circ g(x) = y$  if  $2x^3 + 7 = y$ , i.e., if  $2x^3 = y - 7$ , i.e.,  $x = \sqrt[3]{\frac{y-7}{2}}$ , which  $\in \mathbb{R}(D_{f \circ g})$ . Hence, for every  $y \in \mathbb{R}(\text{Codomain}_{f \circ g}), \exists \sqrt[3]{\frac{y-7}{2}} \in \mathbb{R}(D_{f \circ g})$

such that  $f \circ g(\sqrt[3]{\frac{y-7}{2}}) = y$ . Hence,

$f \circ g$  is onto.

Since,  $f \circ g$  is both one-one and onto, it is invertible.

	<p><math>(f \circ g)^{-1} : \mathbb{R} \rightarrow \mathbb{R}</math> defined by <math>(f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-7}{2}}</math></p> <p><math>(f \circ g)^{-1}(9) = \sqrt[3]{\frac{9-7}{2}} = 1.</math></p> <p style="text-align: center;"><b>OR</b></p> <p>Let <math>a, b \in \mathbb{R}</math> such that <math>a = 0, b \neq 0.</math></p> <p>Then <math>a * b =  a  + b =  0  + b = b, b * a = b, \therefore a * b = b * a</math></p> <p>Let <math>a, b \in \mathbb{R}</math> such that <math>a \neq 0, b = 0.</math></p> <p>Then <math>a * b = a, b * a =  b  + a =  0  + a = a, \therefore a * b = b * a</math></p> <p>Let <math>a, b \in \mathbb{R}</math> such that <math>a = 0, b = 0.</math> Then <math>a * b = a = 0, b * a = b = 0, \therefore a * b = b * a.</math></p> <p>Now we need to check whether <math>*</math> is commutative. One more case is needed to be examined. Let <math>a, b \in \mathbb{R}</math> such that <math>a \neq 0, b \neq 0.</math> Then <math>a * b =  a  + b, b * a =  b  + a</math> and <math>a * b</math> may not be equal to <math>b * a</math>, e.g., <math>(-1) * 2 = 3, 2 * (-1) = 1</math>, hence, <math>(-1) * 2 \neq 2 * (-1)</math>. Thus <math>*</math> is not commutative.</p> <p>The element <math>e \in \mathbb{R}</math> will be the identity element for <math>*</math> if <math>a * e = e * a = a</math> for all <math>a \in \mathbb{R}.</math></p> <p><math>a * e = a</math> provided <math>e = 0</math> and <math>e * a = a</math> provided <math>e = 0</math> (As <math>0 * 0 = 0</math> and <math>0 * a =  0  + a = a</math> for <math>a \neq 0</math>). Hence, <math>0</math> is the identity element for <math>*</math>.</p>	<p>[ 1 ]</p> <p>[ 1/2 ]</p> <p>[ 1 ]</p> <p>[ 1 ]</p> <p>[ 1 ]</p> <p>[ 1 ]</p> <p>[ 2 ]</p>
<p>25.</p>	<p><math> A  = 3(3-6) + (-2)(-12-14) + 1(12+7) = 62 \neq 0.</math></p> <p>Hence, <math>A^{-1}</math> exists. Let <math>c_{ij}</math> represent the cofactor of <math>(i, j)^{\text{th}}</math> element of <math>A</math>. Then,</p> <p><math>c_{11} = -3, c_{12} = 26, c_{13} = 19, c_{21} = 9, c_{22} = -16, c_{23} = 5, c_{31} = 5, c_{32} = -2, c_{33} = -11.</math></p> $\text{adj}A = \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ $A^{-1} = \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix}$ <p>The given system of equations is equivalent to the matrix equation</p> $A'X = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}.$ $\Rightarrow X = (A')^{-1}B = (A^{-1})'B$ $= \frac{1}{62} \begin{bmatrix} -3 & 9 & 5 \\ 26 & -16 & -2 \\ 19 & 5 & -11 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} = \frac{1}{62} \begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ Hence, } x = 1, y = 1, z = 1$ <p style="text-align: center;"><b>OR</b></p>	<p>[ 1 ]</p> <p>[ 2 ]</p> <p>[ 1 ]</p> <p>[ 2 ]</p>

	$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A (R_1 \leftrightarrow R_2)$ $\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix} A (R_2 \rightarrow R_2 - 2R_1)$ $\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -2 & 4 & 1 \end{bmatrix} A (R_3 \rightarrow R_3 - 2R_2)$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} A (R_2 \rightarrow R_2 + R_3, R_1 \rightarrow R_1 - R_3)$ $\Rightarrow A^{-1} = \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix}$ $X = [1 \ 0 \ 1]A^{-1} = [1 \ 0 \ 1] \begin{bmatrix} 2 & -3 & -1 \\ -1 & 2 & 1 \\ -2 & 4 & 1 \end{bmatrix} = [0 \ 1 \ 0]$	<p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p> <p>[1]</p>
<p>26.</p>	$y = x x  = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ <p>Solving <math>y = x^2, x^2 + y^2 = 2</math> simultaneously, <math>y + y^2 - 2 = 0 \Rightarrow (y+2)(y-1) = 0 \Rightarrow y = 1</math>  (<math>y = x^2</math> lies in quadrant I).  <math>\Rightarrow x = 1</math></p>  <p>The required area = the shaded area = <math>\int_0^1 (\sqrt{2-x^2} - x^2) dx</math></p> $= \frac{1}{2} [x\sqrt{2-x^2} + 2\sin^{-1} \frac{x}{\sqrt{2}}]_0^1 - \frac{1}{3} [x^3]_0^1 = (\frac{1}{6} + \frac{\pi}{4}) \text{ sq units.}$	<p>[1]</p> <p>[1/2]</p> <p>[1]</p> <p>[2]</p> <p>[1+1/2]</p>



27.

$$\text{The given definite integral } = I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2 - \cos 2x} dx + \frac{\pi}{4} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2 - \cos 2x} dx$$

$$f(x) = \frac{x}{2 - \cos 2x}, f(-x) = \frac{-x}{2 - \cos 2x} = -f(x).$$

$$\text{Hence, } f \text{ is odd. Therefore, } \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2 - \cos 2x} dx = 0 \quad [1]$$

$$g(x) = \frac{1}{2 - \cos 2x}, g(-x) = \frac{1}{2 - \cos 2x} = g(x).$$

$$\text{Hence, } g \text{ is even. Thus } \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2 - \cos 2x} dx = 2 \int_0^{\frac{\pi}{4}} \frac{1}{2 - \cos 2x} dx. \quad [1]$$

$$\text{Hence, } I = \frac{\pi}{2} \int_0^{\frac{\pi}{4}} \frac{1}{2 - \cos 2x} dx = \frac{\pi}{2} \int_0^{\frac{\pi}{4}} \frac{1}{1 + 2 \sin^2 x} dx$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1 + \tan^2 x + 2 \tan^2 x} dx = \frac{\pi}{2} \int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1 + 3 \tan^2 x} dx = \frac{\pi}{2} \int_0^1 \frac{1}{1 + 3t^2} dt \quad [\tan x = t \Rightarrow \sec^2 x dx = dt] \quad [2]$$

$$= \frac{\pi}{2} \times \frac{1}{3} \int_0^1 \frac{1}{\left(\frac{1}{\sqrt{3}}\right)^2 + t^2} dt = \frac{\pi}{6} \sqrt{3} [\tan^{-1} \sqrt{3}t]_0^1$$

$$= \frac{\pi}{6} \sqrt{3} \left[\frac{\pi}{3}\right] = \frac{\sqrt{3}\pi^2}{18} \quad [2]$$

**OR**Let  $f(x) = 3x^2 - 2x + 4$ . Then the given definite integral =

$$\int_{-2}^2 f(x) dx = \lim_{\substack{n \rightarrow \infty \\ h \rightarrow 0}} h \sum_{r=1}^n f(-2 + rh), \text{ where } nh = 4. \quad [1]$$

$$f(-2 + rh) = 3(-2 + rh)^2 - 2(-2 + rh) + 4 = 3r^2h^2 - 14rh + 20$$

$$\sum_{r=1}^n f(-2 + rh) = 3h^2 \sum_{r=1}^n r^2 - 14h \sum_{r=1}^n r + 20n = 3h^2 \times \frac{n(n+1)(2n+1)}{6} - 14h \times \frac{n(n+1)}{2} + 20n \quad [1]$$

$$\int_{-2}^2 f(x) dx = \lim_{\substack{n \rightarrow \infty \\ h \rightarrow 0}} h \left[ 3h^2 \times \frac{n(n+1)(2n+1)}{6} - 14h \times \frac{n(n+1)}{2} + 20n \right] \quad [2 + 1/2]$$

$$= \lim_{\substack{n \rightarrow \infty \\ h \rightarrow 0}} \left[ 3 \times \frac{nh(nh+h)(2nh+h)}{6} - 14 \times \frac{nh(nh+h)}{2} + 20nh \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{4(4+h)(8+h)}{2} - 7 \times 4(4+h) + 20 \times 4 \right] = 32 \quad [1 + 1/2]$$

28.

The general point on the given line  $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z+1}{-9}$

is  $(\lambda + 1, 3\lambda + 2, -9\lambda - 1)$ .

The direction ratios of the line parallel to the plane  $x - y + 2z - 3 = 0$  intersecting the given line and passing through the point  $(-2, 3, -4)$  are  $\langle \lambda + 3, 3\lambda - 1, -9\lambda + 3 \rangle$

and  $(\lambda + 3)1 + (3\lambda - 1)(-1) + (-9\lambda + 3)2 = 0 \Rightarrow \lambda = \frac{1}{2}$ .

The point of intersection is  $(\frac{3}{2}, \frac{7}{2}, \frac{-11}{2})$ .

The required distance =  $\sqrt{(\frac{3}{2} + 2)^2 + (\frac{7}{2} - 3)^2 + (\frac{-11}{2} + 4)^2} = \frac{\sqrt{59}}{2}$  unit.

[1]

[1]

[1]

[1]

[1]

[1]

29.

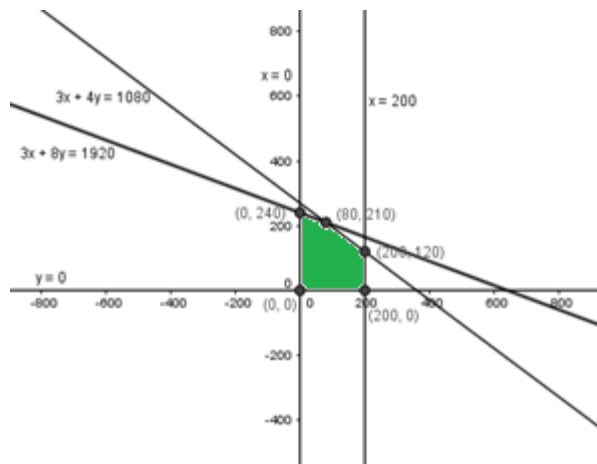
Let  $x$  = the number of units of Product 1 to be produced daily

$y$  = the number of units of Product 2 to be produced daily

To maximize  $P = (9 - 1.2)x + (8 - 0.9)y = 7.8x + 7.1y$

subject to the constraints:

$$\frac{x}{4} + \frac{y}{3} \leq 90, \text{ or } 3x + 4y \leq 1080, \frac{x}{8} + \frac{y}{3} \leq 80, \text{ or } 3x + 8y \leq 1920, x \leq 200, x \geq 0, y \geq 0.$$



[1]

[2]

[2]

At the point	P
(0, 0)	0
(200, 120)	2412
(0, 240)	1704
(200, 0)	1560
(80, 210)	2115

[1]

The maximum profit = Rs. 2412.