Physics Formulas

Motion in straight Line

1. Displacement ≤ Path Length

2. Average Velocity : $(\vec{v}_{avg}) = \frac{Total\ Displacement}{Total\ time} = \frac{\vec{x}_f - \vec{x}_i}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

3. Instantaneous velocity: $(\vec{v}_{ins}) = \lim_{\Delta x \to 0} \frac{\Delta \bar{x}}{\Delta t} = \frac{d\bar{x}}{dt}$

4. Average speed $(v_{avg}) = \frac{Total\ distance}{Total\ time}$

5. Average acceleration $(\vec{a}_{avg}) = \frac{Change\ in\ velocity}{Total\ time} = \frac{Final\ velocity - Initial\ velocity}{Total\ time}$

$$(\vec{a}_{avg}) = \frac{\vec{v}_f - \vec{v}_i}{t_2 - t_1} = \frac{\Delta \bar{v}}{\Delta t}$$

6. Instantaneous acceleration $(\vec{a}_{ins}) = \frac{d\bar{v}}{dt} = \frac{d^2x}{dt^2} = \frac{vdv}{dx}$

⇒ **Position** $\xrightarrow{differentiation}$ **Velocity** $\xrightarrow{differentiation}$ **Acceleration**

$\xleftarrow{Integration}$ $\xleftarrow{Integration}$

⇒ $x - x_0 = \int_{t_0}^{t} vdt$ **(x = final position at 't' and x_0 = Initial position at 't_0')**

⇒ $v - v_0 = \int_{t_0}^{t} adt$ **(v = Final velocity at 't' and v_0 = Initial velocity at 't_0')**

Equations of motion (Only for constant acceleration)

$v = u + at$ .. (i)

$x = x_0 + ut + \frac{1}{2} at^2$ …(ii)

$v^2 = u^2 + 2a(x - x_0)$ ….(iii)

$x - x_0 = \left(\frac{u+v}{2}\right) t$ …..(iv)

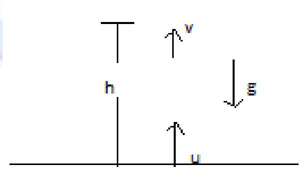
(x_0 = Initial position; x = Final position; u = Initial velocity; v = Final velocity)

⇒Distance travelled by particle in nth second

$\Delta S_n = u + \frac{a}{2}(2n - 1)$

⇒ **Equation of motion under gravity**

1. Upward motion



$v = u - gt$

$h = ut - \frac{1}{2} gt^2$

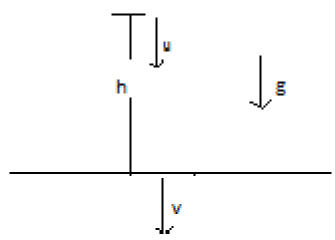
$v^2 = u^2 - 2gh$

Maximum height attain by object $= \frac{u^2}{2g}$

Time taken to reach highest point $= \frac{u}{g}$

Time of flight $= \frac{2u}{g}$

2. Downward motion



$v = u + gt$

$h = ut + 1/2gt^2$

$v^2 = u^2 + 2gh$

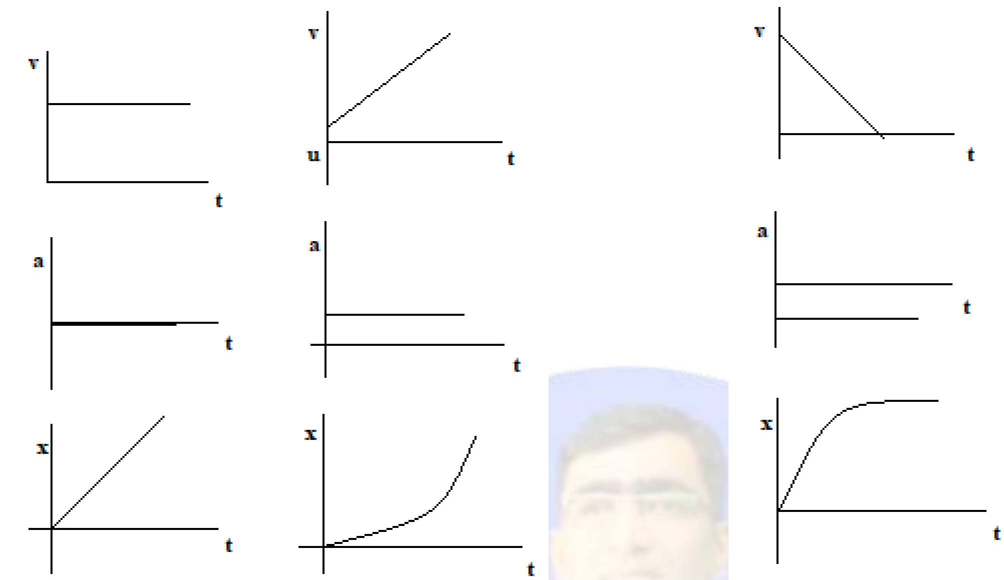
If An object drop a height 'h' then

Time taken to reach the ground $= \sqrt{\frac{2h}{g}}$

Speed by which it hit the ground = $\sqrt{2gh}$

Graph Conversion

Uniform motion | **Uniform accelerated motion** | **Uniform retarded motion**



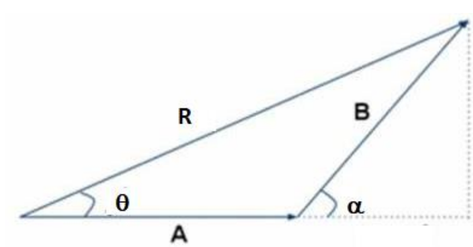
1. Velocity = slope of position-time graph
2. Acceleration = slope of velocity-time graph
3. Displacement = Area under velocity-time graph
4. Change in velocity = area under acceleration time graph
5. $\frac{v^2}{2} - \frac{u^2}{2}$ = Area under a, x graph
6. Acceleration = velocity × slope of v, x graph

Vectors

⇒ $\vec{A} = |\vec{A}|\,\hat{A}$

$\vec{A}$ = Vector Quantity; $|\vec{A}|$ = Magnitude of vector; $\hat{A}$ = Direction of $\vec{A}$ or unit vector

Resultant of Two vectors



R = $|\vec{R}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$

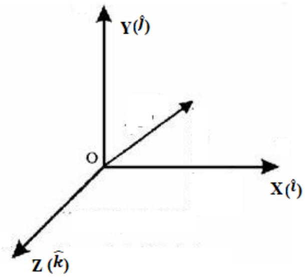
tan α = $\frac{B\sin\theta}{A + B\cos\theta}$

⇒ **A vector in Cartesian Coordinate**

$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

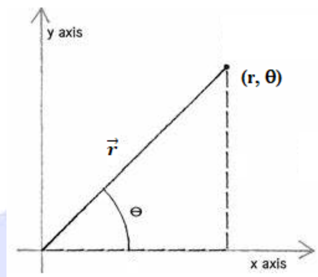
$|\vec{A}| = = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x\hat{\imath}+A_y\hat{\jmath}+A_z\hat{k}}{\sqrt{A_x^2+A_y^2+A_z^2}}$

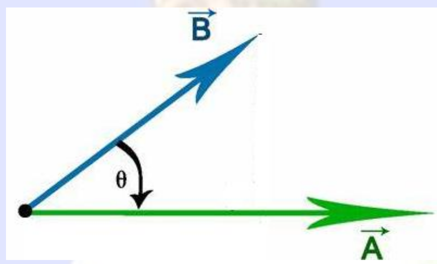


Vector in Polar Form

$\vec{r} = r\, cos\theta\ \hat{\imath}\ \ + r\, sin\theta\ \hat{\jmath}$



Dot Product



$\vec{A}.\vec{B} = \vec{B}.\vec{A}$

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|\, cos\ \theta$

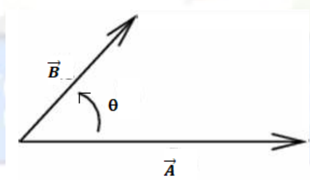
$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

$\hat{\imath}.\hat{\jmath} = \hat{\jmath}.\hat{k} = \hat{k}.\hat{\imath} = 0$

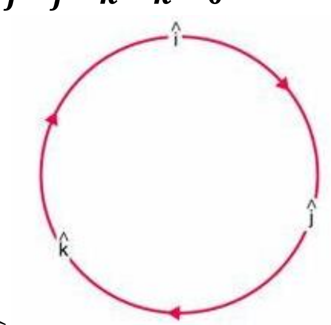
Cross product

$\vec{A} \times \vec{B} = |\vec{A}|\ |\vec{B}| sin\ \theta\ \hat{n}$

$\hat{n}$ = unit vector perpendicular to $\vec{A}$ and $\vec{B}$



$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$



$\hat{\imath} \times \hat{\jmath} = \hat{k}$

$\hat{\jmath} \times \hat{k} = \hat{\imath}$

$k \times \hat{\imath} = \hat{\jmath}$

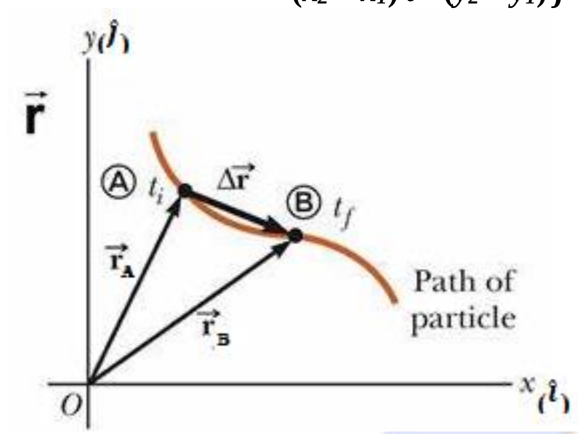
Motion in Plane

⇒ **Position of particle at 'A' = $x_1\hat{\imath} + y_1\hat{\jmath}$**

Position of particle at 'B' = $x_2\hat{\imath} + y_2\hat{\jmath}$

Displacement ($\Delta\vec{r}$) = $\vec{r}_B - \vec{r}_A$

$$= (x_2 - x_1)\,\hat{\imath} + (y_2 - y_1)\,\hat{\jmath}$$

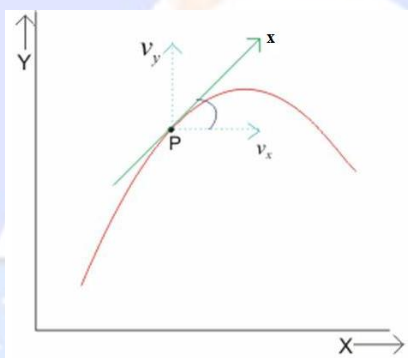


$$\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath}$$

⇒**Average velocity ($\vec{v}_{avg}$)** = $\frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath}$

⇒ **Instantaneous velocity ($\vec{v}_{ins}$)** = $\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath}$

$(\vec{v}_{ins}) = v_x\hat{\imath} + v_y\hat{\jmath}$



⇒ **Speed (v)** = $|\vec{v}_{ins}| = \sqrt{v_x^2 + v_y^2}$

Projectile Motion

Time of flight (T) = $\frac{2u\sin\theta}{g}$

Maximum Height (H_{max}) = $\frac{u^2\sin^2\theta}{2g}$

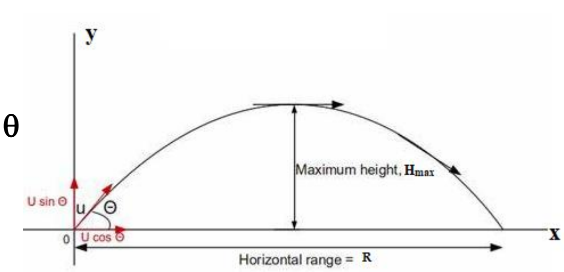
Range (R) = $\frac{u^2\sin 2\theta}{g}$

⇒ **For maximum range, Range θ = 45°**

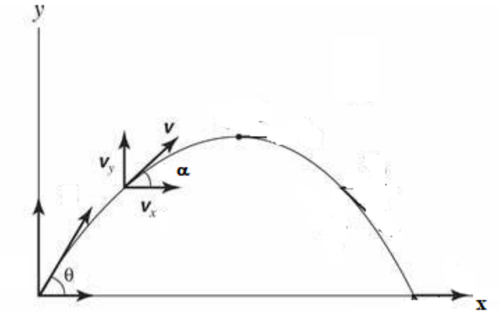
⇒ **Two projection angle for same Range are θ, 90 – θ**

Equation of trajectory, $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

$y = x\tan\theta\,(1 - x/R)$



where R = Range of projectile
Velocity of velocity of projectile at the instant ‘t’ is

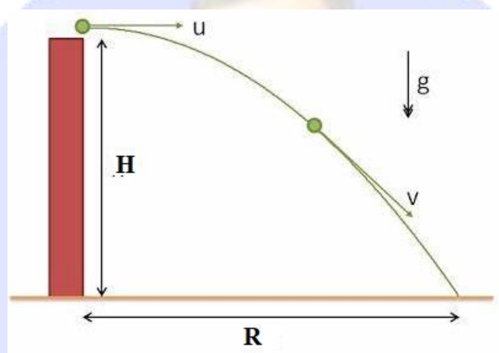


$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$

$\tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$

Oblique Projectile



$R = u\sqrt{\frac{2H}{g}}$

Time of flight = $\sqrt{\frac{2H}{g}}$

Laws of Motion

Linear momentum ($\vec{p}$) = m$\vec{v}$

Change in Linear momentum ($\Delta\vec{p}$)= m$\vec{v}$ – m$\vec{u}$

$\Delta\vec{p}$ = m($\vec{v}$ – $\vec{u}$)

Impulse ($\vec{I}$) = Change in Linear momentum

= $\vec{F}_{avg}$ × time = Area under Force, time graph

⇒ Force ($\vec{F}$) = $\frac{d\vec{p}}{dt}$

For constant mass, $\vec{F}$ = m$\vec{a}$

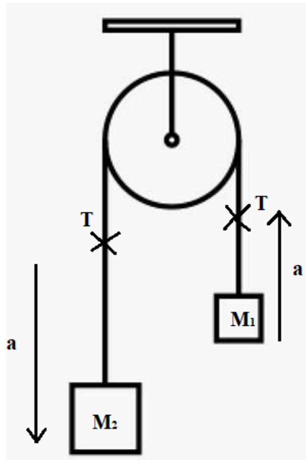
⇒ For variable mass

$\vec{F}_{ext} + \vec{F}_{thrust} = m\frac{d\vec{v}}{dt}$

Where $\vec{F}_{thrust} = \vec{v}_r\frac{dm}{dt}$

$\vec{v}_r$ = velocity of the separated (gained) mass respect to system

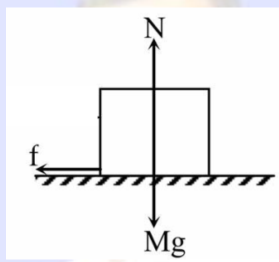
Pulley



$a = \frac{M_2 - M_1}{(M_1 + M_2)} g = \frac{Net\ pulling\ force}{Total\ mass}$

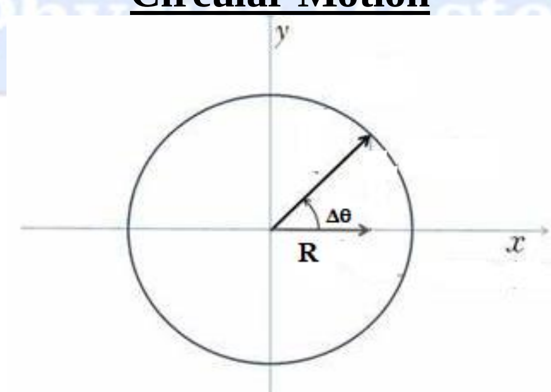
$T = \frac{2M_1 M_2 g}{(M_1 + M_2)}$

Friction (f)



Static friction (f) = applied force
Limiting friction (f_2) = $\mu_s N$
μ_s = coefficient of static friction, N = normal reaction
Kinetic friction (f_k) = $\mu_k N$
μ_k = coefficient of kinetic friction

Circular Motion



$\Delta\vec{\theta}$ = angular displacement

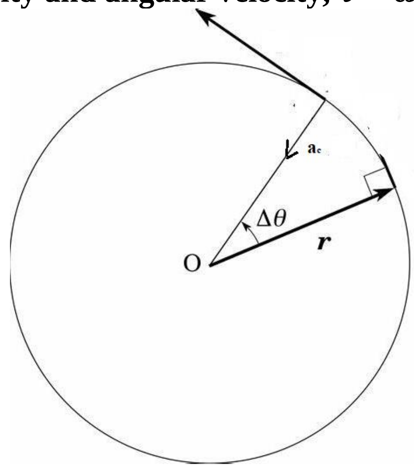
Average angular velocity ($\vec{\omega}_{avg}$) = $\frac{\Delta\vec{\theta}}{\Delta t}$

Instantaneous angular velocity ($\vec{\omega}_{ins}$) = $\frac{d\vec{\theta}}{dt}$

Average angular acceleration ($\vec{\alpha}_{avg}$) = $\frac{\Delta\vec{\omega}}{\Delta t}$

Instantaneous angular acceleration ($\vec{\alpha}_{ins}$) = $\frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\theta}}{dt^2} = \omega\frac{d\omega}{d\theta}$

Relation between Linear velocity and angular velocity, $\vec{v} = \vec{\omega} \times \vec{r}$



Net acceleration of circular motion

$\vec{a} = \vec{a}_c + \vec{a}_t$

$|\vec{a}| = \sqrt{a_c^2 + a_t^2}$

Where a_c = centripetal acceleration

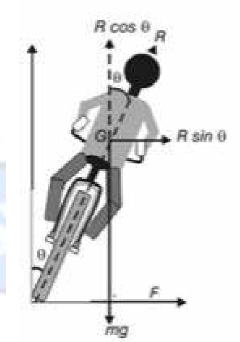
$a_c = v^2/r = \omega^2 r$

And $a_t = dv/dt = \alpha R$ = Tangential acceleration

(i) Uniform circular motion, $a_t = 0$, $a_c \neq 0$

(ii) Non uniform circular motion, $a_t \neq 0$, $a_c \neq 0$

<u>Bending of cyclist on circular road</u>

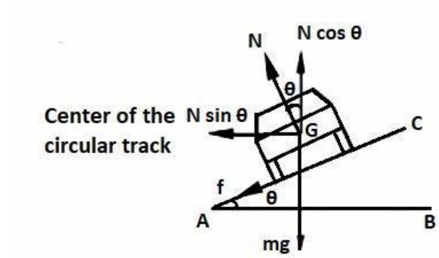


Circular track

$\tan\theta = v^2/rg$

v = sped of cyclist, r = radius of circular track, g = acceleration due to gravity

<u>Banking of road</u>



r = radius of circular track, v = speed of car

$v_{max} = \left[\frac{rg(\mu_s - tan\theta)}{(1+\mu_s tan\theta)}\right]^{1/2}$

Minimum speed to avoid slipping

$v_{min} = \left[\frac{rg(\mu_s - tan\theta}{(1+\mu_s tan\theta)}\right]^{1/2}$

Optimum speed when no wear or tear of tyres (friction absent)

$$v = \sqrt{rg\tan\theta}$$

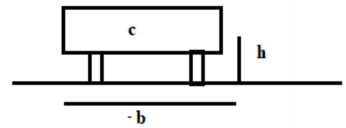
Maximum speed on level circular track

$$v = \sqrt{\mu_s r g}$$

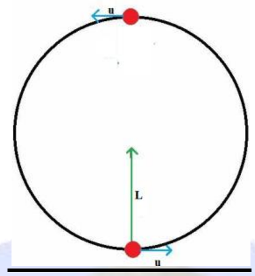
Maximum speed to avoid overturning of car

$$v = \sqrt{\frac{brg}{2h}}$$

where b = width of car, h = height of centre of mass of car from ground



Circular motion in vertical Plane



Minimum speed given to particle to complete the circle, u = $\sqrt{5gL}$

Work, Energy, Power

Work done by constant force, W = $\vec{F}.\overrightarrow{\Delta r}$

$$W = F\Delta r\cos\theta$$

In Cartesian coordinate system

$$W = \vec{F}.\overrightarrow{\Delta r}$$

Where $\overrightarrow{\Delta r} = \vec{r}_2 - \vec{r}_1$

And $\vec{r}_2 = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$

$\vec{r}_1 = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$

Work done by variable force

$W = \int \vec{F}.d\vec{r}$ **= Area under force, displacement graph**

Where $d\vec{r} = dx\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$

<u>Work energy theorem</u>

Total work done = Change in kinetic energy

<u>Kinetic energy (k)</u> = ½ mv^2

Relation between kinetic energy and Linear momentum = $p^2/2m$

<u>Potential energy (U)</u> = - (Work done by conservative force)

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right)$$

⇒ Mechanical enrgy (ME) = K + U

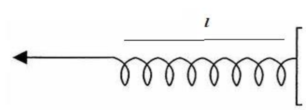
In absence of work done by man conservative force total mechanical energy will be constant

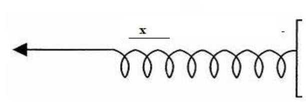
$K_i + U_i = K_f + U_f$

⇒ <u>Spring</u>

Restoring force of spring = -kx

k ⇒ spring constant

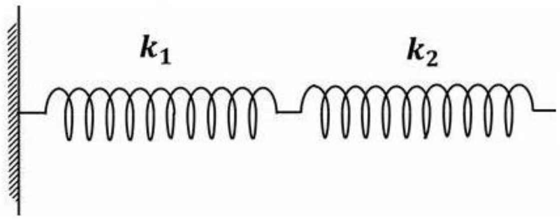




k ∝ 1/natural length of spring

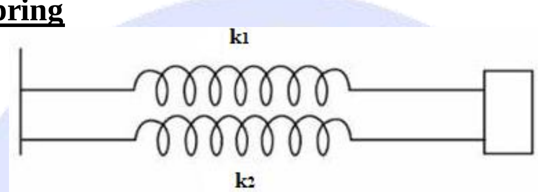
Elastic potential Energy (U) = ½ kx^2

<u>Series combination of spring</u>



Equivalent spring constant (k_{eq}), $1/k_{eq} = 1/k_1 + 1/k_2$

<u>Parallel combination of spring</u>



$k_{eq} = k_1 + k_2$

<u>Power</u>

$$\textit{Average power} = \frac{\textit{total work done}}{\textit{total time}}$$

Instantaneous power (P_{ins}) = dw/dt

Relation between force and power

$$P = \vec{F}.\vec{v}$$

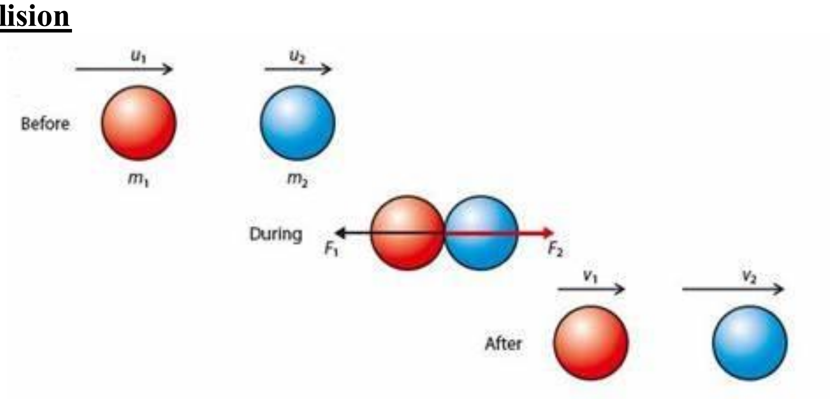
$$P = Fv\cos\theta$$

⇒ **Efficiency (η) = $\frac{\textit{Output power}}{\textit{Input power}}$**

⇒ **Mechanical Advantage (MA) = $\frac{Load}{Effort}$**

⇒ **<u>Conservation of Linear Motion</u> : In absence of external force, Linear momentum of the system remains constant**

⇒ **<u>Newton's law of collision</u>**



$\frac{Velocity\ of\ separation}{Velocity\ of\ approach} = e$

(i) **Head on collision**:

$v_1 = \frac{(m_1 - em_2)}{(m_1+m_2)} u_1 + \frac{(1+e)m_2 u_2}{(m_1+m_2)}$

$v_2 = \frac{(1+e\, m_1)u_1}{(m_1+m_2)} + \frac{(m_2 - em_1)u_2}{(m_1+m_2)}$

$$\text{Loss in KE} = \frac{1}{2}\frac{m_1 m_2}{(m_1+m_2)}(u_1 - u_2)^2(1 - e^2)$$

Oblique collision

u = speed before collision

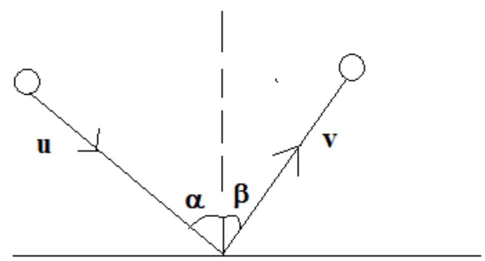
v = speed after collision

$v = u\sqrt{sin^2\alpha + e^2cos^2\alpha}$

$\tan\beta = \tan\alpha / e$

Note: If collision is perpendicular to surface then

$\alpha = 0, \beta = 0$

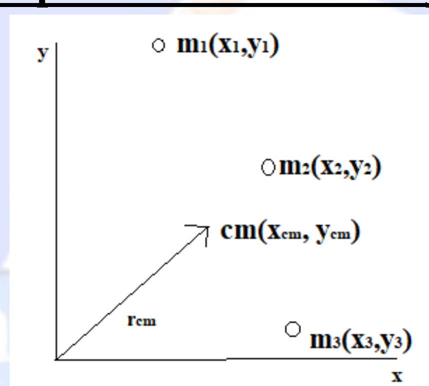


System of particle sand Rotatory Motion

$x_{cm} = \frac{m_1x_1+m_2x_2}{m_1+m_2}$

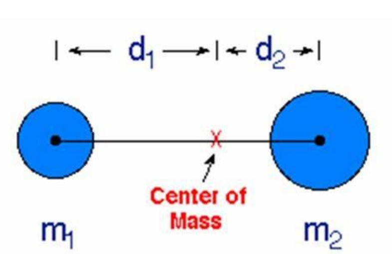
$y_{cm} = \frac{m_1y_1+m_2y_2}{m_1+m_2}$

$\vec{r}_{cm} = \frac{m_1\vec{r_1}+m_2\vec{r_2}}{m_1+m_2}$



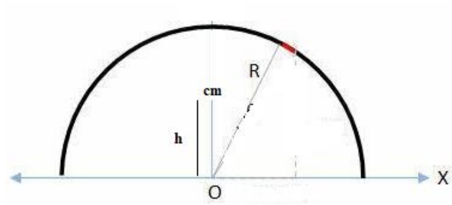
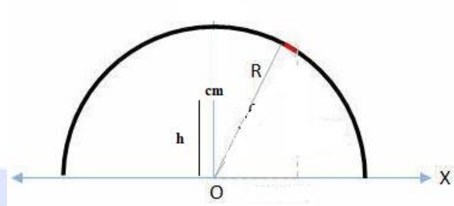
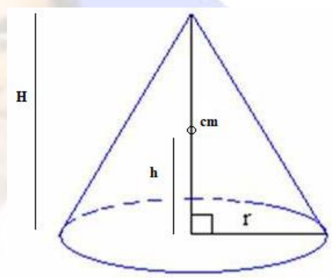
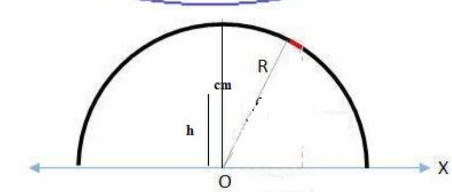
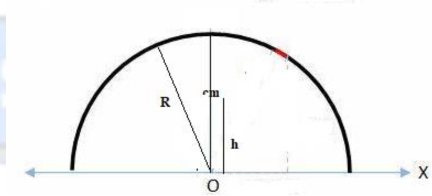
⇒ In case of continuous mass, $x_{cm} = \frac{\int xdm}{\int dm}$, $y_{cm} = \frac{\int ydm}{\int dm}$

$d_1 = \frac{m_2 d}{m_1+m_2}$, $d_2 = \frac{m_1 d}{m_1+m_2}$



Velocity of centre of mass, $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

Acceleration of centre of mass, $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$

<u>**Shape**</u>	<u>**COM**</u>	
Semicircular Ring	$h = 2R/\pi$	
Semicircular Plate	$h = 4R/3\pi$	
Right circular cone	$h = H/4$	
Hemisphere	$h = 3R/8$	
Hemispherical shell	$h = R/2$	

Moment of Inertia(I)

$$I = mr_{\perp}^2$$

$r_{\perp}$ **= perpendicular distance of mass from axis of rotation**

Moment of inertia of distributed masses

$$I = m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2$$

Moment of inertia of continue mass $I = \int dmr^2$

Radius of gyration $(k) = \sqrt{\frac{I}{M}}$

Radius of gyration of system of masses, $k = \sqrt{\frac{r_1^2+r_2^2+\cdots+r_n^2}{n}}$ **where n = number of masses**

Theorem of parallel axis

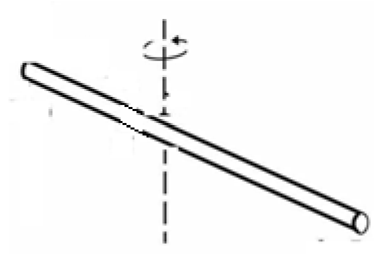
$$I = I_{cm} + Mh^2$$

I_{cm} = moment of inertia about an axis passes through cm and parallel to given axis

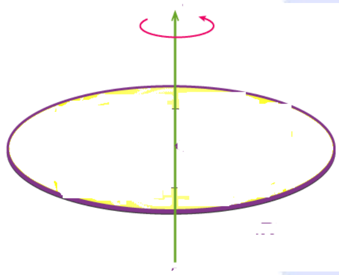
Theorem of perpendicular axis

$$I_z = I_x + I_y$$

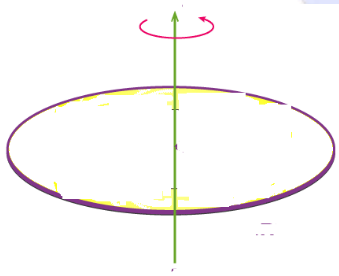
Object	**Moment of Inertia**
Rod	**$ML^2/3$**



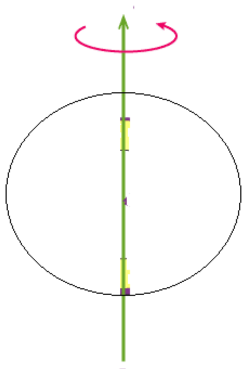
Circular Ring	**MR^2**



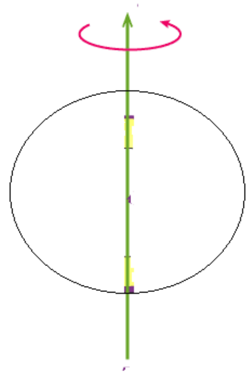
Circular disc	**$MR^2/2$**



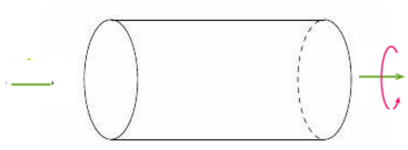
Hollow sphere	**$2/3\ MR^2$**



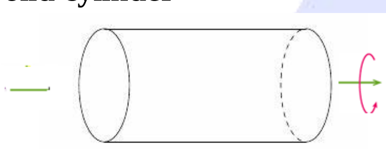
Solid Sphere $\frac{2}{5}MR^2$



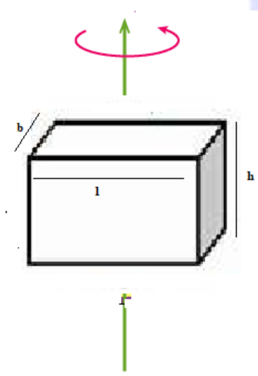
Hollow cylinder MR^2



Solid cylinder $MR^2/2$



Cuboid $I = \frac{M(L^2+b^2)}{12}$



Torque, $\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$

Torque ($\vec{\tau}$**) and angular acceleration (**$\vec{\alpha}$**),** $\vec{\tau} = I\,\vec{\alpha}$

Work done (W) = $\int \vec{\tau}.\,d\vec{\theta}$

Power (P) = $\vec{\tau}.\vec{\omega}$

Angular momentum ($\vec{L}$**) =** $\vec{r} \times \vec{p}$ **where** $\vec{p}$ **= linear momentum**

$$\vec{L} = I\,\vec{\omega}$$

Relation between torque and angular momentum $\vec{\tau} = d\vec{L}/dt$

If $\vec{\tau}$ **(external) = 0, then** $\vec{L}$ **= constant**

$$I_1\vec{\omega}_1 = I_2\vec{\omega}_2$$

Angular Impulse ($\vec{J}$) = $\vec{\tau}.\Delta t$ = change in angular momentum

<u>Equation of Rotatory Motion (constant ‘α’)</u>

$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega.t + 1/2\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

ω_0 = initial angular velocity, ω =final angular velocity after time ‘t’.

<u>Rolling motion</u>

Velocity of a point on rolling object

$$\vec{v} = \vec{v}_0 + \vec{\omega} \times \vec{r}$$

$\vec{v}_0$ = velocity of centre of rotation, $\vec{\omega}$ = angular velocity, $\vec{r}$ = position vector of point ‘P’ with respect to centre of rotation.

Acceleration of a point on rolling object

$$\vec{a} = \vec{a}_0 + \vec{\alpha} \times \vec{r}$$

Total kinetic energy of rolling object

$$E = E_T + E_R$$

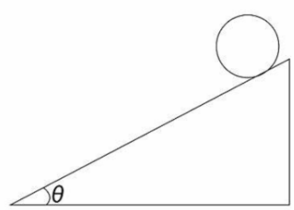
$$E = ½ mv^2 + ½ I\omega^2$$

For pure rolling v = ωR

$$E = 1/2mv^2(1+k^2/R^2) \text{ where } k = \sqrt{\frac{I}{M}}$$

Acceleration of an object rolling from rough inclined plane

$$a = \frac{g\sin\theta}{(1+\frac{K^2}{R^2})}$$



Angular momentum of rolling object

$$\vec{L} = I\,\vec{\omega} + m(\vec{r} \times \vec{v})$$

Where $\vec{\omega}$ = angular velocity of object, $\vec{r}$ = position vector of centre of rotation, $\vec{v}$ = vector of centre of rotation, I = MOI about centre of rotation

<u>Gravitation</u>

Gravitational force between two masses

$$F = GM_1M_2/r^2 \text{ where } G = 6.6\times10^{-11}\ Nm^2\ kg^{-2}$$

Gravitational field intensity ($\vec{E}$)

$$\vec{E} = \vec{F}/m_0$$

Gravitational field intensity due to point mass E = GM/r²

Gravitational potential (V)

$$V = \frac{W_{\infty p}}{m_0}$$

Where $W_{\infty p}$ = work done to move a mass ‘m_0’ from infinity to a given point inside field.

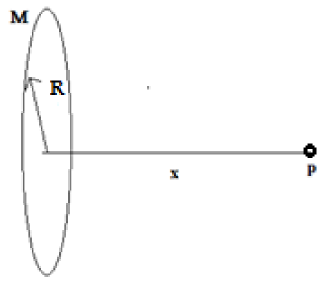
Gravitational potential due to point mass, $V = -\frac{GM}{r}$

Relation between gravitational field intensity (E) and potential (V), E = -dv/dr

Gravitational field intensity (E) and (v) due to ring at axial point ‘p’

$E = GMx/(x^2+R^2)^{3/2}$

$V = -GM/(x^2+R^2)^{1/2}$



<u>Gravitational field intensity (E) and potential (V)</u>

(a) <u>Due to hollow sphere</u>

(i) Inside hollow sphere (r<R)

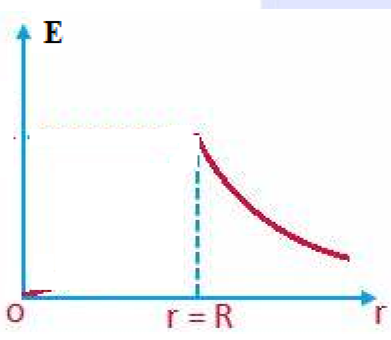
$$E = 0, V = -GM/R$$

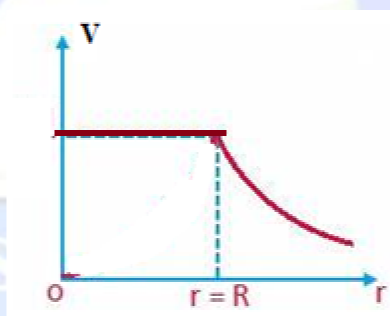
(ii) Outside hollow sphere, (r>R)

$$E = GM/r^2, V = -GM/r$$

(iii) At surface (r = R)

$$E = GM/R^2, V = -GM/R$$





(b) <u>Due to sphere</u>

(i) Internal point (r<R)

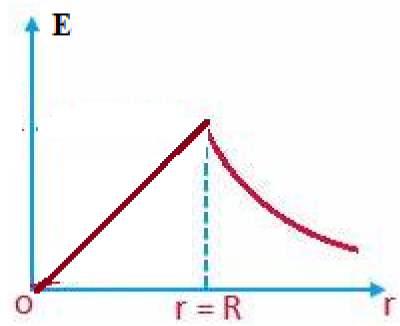
$$E = GMr/R^3 , V = (-GM/2R^3) (3R^2 – r^2)$$

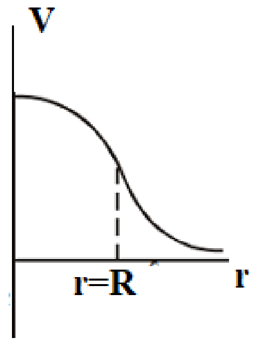
(ii) External point (r>R)

$$E = GM/r^2, V = -GM/r$$

(iii) At surface (r = R)

$$E = GM/R^2, V = -GM/R$$





Gravitational Potential energy (U)

Gravitational potential energy of two point masses placed at separation 'r' will be

$U = -Gm_1m_2/r$

Gravitational potential energy of mass 'm_0' on surface of earth, $U = -GMm_0/R$ where M = mass of earth, m_0 = mass of object, R = Radius of earth.

Change in Gravitational potential energy of mass 'm_0' to move from surface to height 'h'

$$\Delta U = GMm_0/R(R+h)$$

$$\Delta U = m_0gh/(1+h/R)$$

Variation in 'g'

(i) Due to altitude 'h'

$$g' = g/(1+h/R)^2$$

If h<<R the,

$$g' = g(1-(2h/R))$$

(ii) Due to depth 'd'

$$g' = g(1-d/R)$$

(iii) Due to rotation

$$g' = g - \omega^2 R\cos^2\lambda$$

(λ = latitude angle)

Escape velocity (V_e) on surface of earth

$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$ = 11.2 km/s and $g = GM/R^2$

Planet and satellite system

Orbital velocity

$V_0 = \sqrt{\frac{2GM}{R}}$ r = Radius of orbit

Time period (T) = $2\pi r/v_0$

Kinetic energy (k) = $GMm_0/2r$

Potential energy (U) = $-GMm_0/r$

Total energy (U) = $-GMm_0/2r$