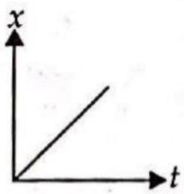
CLASS – 11

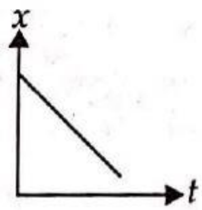
WORKSHEET- MOTION IN STRAIGHT LINE

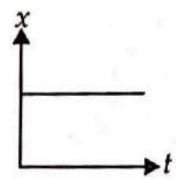
A. INTRODUCTION TO MOTION

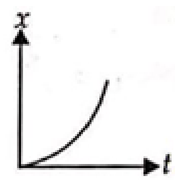
(1 Mark Questions)

1. Which of the following graphs represents the position-time graph of a particle moving with negative velocity?

(a) 

(b) 

(c) 

(d) 

Sol. (b)
The position time graph of a particle with negative velocity is (b).

2. Can a body have a constant speed and still have a velocity?

Sol. Yes if the body is travelling with uniform speed in a circular track its speed remains the same but the velocity is non-uniform as the direction of the body is changing every time.

3. Can a body have zero velocity and still be accelerating?

Sol. A body can have acceleration without zero velocity. For eg. when an object is thrown upwards, then at the maximum height, velocity at body is zero at that instant but it has acceleration due to gravity.

4. Can the direction of velocity of an object change, when acceleration is constant?

Sol. Direction of velocity of a body may change even if the body has a constant acceleration. In projectile motion, the path changes at every point with the change in direction of the velocity. But at every point its acceleration is the acceleration due to gravity and hence, is a constant.

5. Is it possible for a body to accelerated without speeding up or slowing down? If so, give an example.

Sol. Yes, an object in uniform circular motion is accelerating but its speed neither decreases not increases.

6. Under what condition is the average velocity equal to the instantaneous velocity?

Sol. The average velocity is equal to its instantaneous velocity when the body is moving with a uniform velocity.

7. In which of the following examples of motion, can the body be considered approximately a point object:

(a) a railway carriage moving without jerks between two stations.

(b) a monkey sitting on top of a man cycling smoothly on a circular track.

(c) a spinning cricket ball that turns sharply on hitting the ground.

(d) a tumbling beaker that has slipped off the edge of a table

Sol. Consider all the options given and try to make a conclusion, in which given option the object can be considered as a point object. Let us see all the options one by one and then we will try to get to the conclusion that which option can be considered as a point object Let us see first option, A railway carriage moving without jerks between two stations. A railway carriage moving without jerks between two stations, in this option the distance covered by carriage is very small as compared to the distance travelled by carriage between two stations. Therefore, railway carriage can be considered as a point object. Let us see second option, A monkey sitting on top of a man cycling smoothly on a circular track. The size of the monkey is very small as compared to the distance travelled by monkey in the circular track. Therefore, monkeys can be considered as point objects. Let us see third option, A spinning cricket ball that turns sharply on hitting the ground. The size of the ball is comparable to the distance through which the ball turns sharply on hitting the ground. Hence, a cricket ball cannot be considered as a point object. Let us see the fourth option. A tumbling beaker that has slipped off the edge of a table. The tumbling beaker is of comparable size of height of the table through it slips. Hence, the beaker cannot be considered as a point object.

Note: We only consider any object as a point object when the size of the object is very small as compared to the distance travelled by the object. Students should always compare the size of the object to the distance which the object is travelling. While solving problems we consider an object as a point object so that the problem can be solved easily.

8. Read each statement below carefully and state with reasons and examples, if it is true or false: A particle in one-dimensional motion

(a) with zero speed at an instant may have non-zero acceleration at that instant

Sol. True, when a body begins to fall freely under gravity, its speed is zero but it has non-zero acceleration of 9.8 ms^{-1}.

(b) with zero speed may have non-zero velocity,

Sol. False, speed is the magnitude of velocity and the magnitude of non zero velocity cannot be zero

(c) with constant speed must have zero acceleration,

Sol. True, when a particle moves with a constant speed in the same direction, neither the magnitude nor the direction of velocity changes and so acceleration is zero. In case a particle rebounds instantly with the same sped, its acceleration will be infinite which is [physically not possible.

(d) with positive value of acceleration must be speeding up.

Sol. False, if the initial velocity of a body is negative, then even in case of positive acceleration, the body speeds down. A body speeds up when the acceleration acts in the direction of motion.

(2 marks Questions)

9. If displacement of particle is given by $x = t^2 + 5t + 3$. Find
(i) Velocity of the particle at t = 3s and (ii) Average velocity of the particle between t = 1s to t = 3s.

Sol. Give: $x= t^2 + 5t +3$. Since $v(t) = dx/dt$

$v(t) = \frac{d}{dt}(t^2 + 5t + 3) = 2t + 5$

(i) At t = 3s, velocity of the particle is $v(3) = 2\times 3 + 5 = 11$m/s.

(ii) Displacement of the particle between t = 1s to t = 3s = x(3) – x(1)

$= (3^2 + 5 \times 3 + 3) – (1^2 + 5 \times 1 + 3) = (9+18) – (9) = 15$m

Average velocity of the particle between time t = 1s to 3s

$= \frac{\text{Total displacement}}{\text{Total time}} = \frac{18}{3-1} = 9$m/s.

10. The displacement x of a particle along X-axis is given by $3t+7t^2$. Obtain its velocity and acceleration at t = 2s.

Sol. Distance $x = 3 + 8t + 7t^2$

Velocity = dx/dt = 8 + 14t

Velocity, $V_t 2s = 8 +14 \times 2 = 36\ ms^{-1}$

Acceleration, $a = dv/dt = 14\ ms^{-2}$.

(3 Marks Questions)

11. A body travels the first half of the total distance with velocity v_1 and the second half what velocity v_2. Calculate the average velocity.

Sol. Total time taken = 2x. Then, the total time taken $= \frac{x}{v_1} + \frac{x}{v_2} = x\left(\frac{v_1+v_2}{v_1 v_2}\right)$

$\therefore$ Average speed $= \frac{2x}{x\left(\frac{v_1+v_2}{v_1v_2}\right)} = \frac{2v_1v_2}{v_1+v_2}$

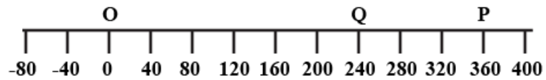
12. A train moves with a speed of 30 km/h in the first 15 min, with another speed of 40km/h the next 15 min, and then with a speed of 60 km/h in the last 30 min. Calculate the average speed of the train for this journey.

Sol. Total distance covered $= 30 \times \frac{15}{60} + 40 \times \frac{15}{60} + 60 \times \frac{30}{60} = 47.5$km

Total time taken = 15 min +15 min + 30 min = 1hr

Average speed $= \frac{47.5}{1} = 47.5$ km/h.

13. A car is moving along X-axis as shown in the figure, it moves from O to P in 18s and returns from P to Q in 6s. What are the average speed of the car in going from (i) O to P and (ii) from O to P and back to Q?



Sol. (i) From O to P, Average velocity = $\frac{\text{Displacement}}{\text{Time interval}} = \frac{360\text{m}}{18\text{s}}$ = 20ms^{-1}

Average speed = $\frac{\text{Path length}}{\text{Time interval}} = \frac{360\text{m}}{18\text{s}}$ = 20ms^{-1}

(ii) From O to P and back to Q

Average velocity = $\frac{\text{Displacement}}{\text{Time interval}} = \frac{\text{OQ}}{18+6} = \frac{+240\text{m}}{24\text{s}}$ = 10ms^{-1}

Average speed = $\frac{\text{Path length}}{\text{Time interval}} = \frac{\text{OP+PQ}}{18+6} = \frac{(360+120)}{24\text{s}}$ m = 20 ms^{-1}

14. The position of an object moving along x-axis is given by $x = a+bt^2$ where a = 8.5m, b = 2.5 m/s^2 and t is measured in seconds. What is its velocity at t = 0s and t = 2s? What is the average velocity between t = 2s and t = 4s?

Sol. Given $x = a + bt^2$

Instantaneous velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(a + bt^2) = 0 + b \times 2t = 2bt$

At t = 0, v = 0; At t = 2s, v = 2× 2.5×2 = 10ms^{-1}; at t = 2s, x = a + 4b; at t = 4s, x = a+16b

Average velocity = $\frac{x_2 - x_1}{t_2 - t_1} = \frac{(a+16b)-(a+4b)}{4-2}$ = 6b = 6 × 2.5 = 15.0 ms^{-1}.

15. Explain clearly, with examples, the distinction between :

(a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval:

(b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval].

(c) Show in both (a) and (b) that the second quantity is either greater than or equal to the first. When is the equality sign true? [For simplicity, consider one-dimensional motion only].

Sol. (a) Suppose a body moves from point A and to point B along a straight path and then returns back to point A along the same path. As the body returns back to its initial position A, so magnitude of displacement = 0.

Distance covered = Total length of path covered = AB + BA = AB + AB = 2AB.

(b) In the above example, suppose the body takes time t to complete the whole journey. Then Magnitude of average velocity = Magnitude of displacement/Time taken = 0/t = 0

Average speed = 2AB/t

(c) In example (a) distance covered > magnitude of displacement. In example (b) average speed > magnitude of average velocity.

The sign of equality will hold when the body moves along a straight line path in a fixed direction.

(5 Marks Questions)

16. A body travelling along a straight line traversed one-half of the total distance with velocity v_0. The remaining part of the distance was covered with a velocity v_1, for half the time and with velocity v_2 for the other half of time. Find the mean velocity averaged over the whole time of motion.

Sol. Let total distance = x

Let the time taken to cover one half distance = t_1

Then $t_1 = \frac{x/2}{v_0} = \frac{x}{2v_0}$

Le the time taken for the next x/2 distance = t_2

Then $\frac{x}{2} = v_1.\frac{t_2}{2} + v_2.\frac{t_2}{2} = \frac{(v_1+v_2)t_2}{2}$ or $t_2 = \frac{x}{v_1+v_2}$

Total time take, $t_1 + t_2 = \frac{x}{2v_0} + \frac{x}{v_1+v_2} = \frac{x}{\frac{x(v_1+v_2+2v_0)}{2v_0(v_1+v_2)}} = \frac{2v_0(v_1+v_2)}{x(v_1+v_2+2v_0)}$.

B. UNIFORMLY ACCELERATED MOTION AND EQUATION OF MOTION

(1 Mark Questions)

1. Give example of a motion where x > 0, v < 0, a > 0 at a particular instant.

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(3 Marks Questions)

2. Derive the equations of motions given below: (i) v = u +at (ii) s = ut + ½ at^2 where symbols have their usual meanings.

Sol. (i) Acceleration is defined as a = $\frac{dv}{dt}$ or dv = adt

When time= 0, velocity = u (say); when time= t, velocity v (say)

Integrating equation within the above limits of time and velocity, we get

$\int_u^v dv = \int_0^t dt$

Or $[v]_0^t = a\int_0^t dt = a[t]_0^t$

Or v – u = a(t – 0) or v = u + at

(ii) Velocity is defined as v = $\frac{ds}{dt}$ or ds = vdt = (u +at)dt

When time = 0, distance travelled =0; when time = t, distance travelled = s (say)

Integrating the equation with the above limits of time and distance we get

$\int_0^s ds = \int_0^t (u + at)dt = u\int_0^t dt + a\int_0^t tdt$

Or $[s]_0^s = u[t]_0^t = a\left[\frac{t^2}{2}\right]_0^t$

Or $s - 0 = u(t - 0) + a\left[\frac{t^2}{2} - 0\right]$

Or $s = ut + \frac{1}{2} at^2$

3. Deduce the following relation: $v^2 - u^2 = 2as$, where symbols have their usual meaning.

Sol. By definitions of acceleration and velocity

$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = \frac{dv}{ds} \times v$

or $ads = vdv$

When time = 0, velocity = u, distance travelled = 0; when time = t, velocity = v, distance travelled = s (Say

Integrating the equation within the above limits of velocity and distance we get

$\int_0^s ads = \int_u^v vdv$

Or $a\int_0^s ds = \int_u^v vdv$

Or $a[s]_0^s = \left[\frac{v^2}{2}\right]_u^v$

Or $a[s - 0] = \frac{v^2}{2} - \frac{u^2}{2}$

Or $2as = v^2 - u^2$

Or $v^2 - u^2 = 2as$

4. A car moving along a straight highway with speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?

Sol. Here $u = 126\ kmh^{-1} = 126 \times \frac{5}{18} = 35ms^{-1}$, $v = 0$, $s = 200m$

As $v^2 - u^2 = 2as$

Therefore $0^2 - 35^2 = 2a \times 200$

Or $a = -\frac{35\times35}{2\times200} = -\frac{49}{16} = -3.06\ ms^{-2}$

Therefore retardation = $3.06\ ms^{-2}$

Required time, $t = \frac{v-u}{a} = \frac{0-35}{-49/16} = \frac{35\times16}{49} = \frac{80}{7} = 11.43s$

5. A driver takes 0.20s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of $6.0\ m/s^2$, find the distance travelled by the car after he sees the need to put the brakes.

Sol. During the time of reaction, the car continues to move with uniform speed of 54 kmh^{-1} or 15 ms^{-1}.

Therefore distance covered during 0.20s = $15 \times 0.20 = 3.0m$

For motion with deceleration: $u = 15ms^{-1}$, $v = 0$, $a = 6.0\ ms^{-2}$

As $v^2 - u^2 = 2as$

Therefore $0^2 - 15^2 = 2 \times (-6.0)\ s$

Or s = 225/12 = 18.75m.

6. A bullet travelling with a velocity of 15 m/s penetrates a tree trunk and comes to rest in 0.4m. Find the time taken during the retardation.

Sol. Here u = $16ms^{-1}$. V = 0, s = 0.4m, t = ?

As $v^2 - u^2 = 2as$, so $0^2 - 16^2 = 2a \times 0.4$

Or a = $\frac{16\times16}{2\times0.4}$ = - 320 ms^{-2}

Time, t = $\frac{v-u}{a} = \frac{0-16}{-320}$ = 0.05s.

(5 Marks Questions)

7. Two ends of a train moving with a constant acceleration passes a certain point with velocities u and v. Show that the velocity with which the middle point of the train passes the same point is $\sqrt{\frac{(u^2+v^2)}{2}}$.

Sol. Let x be the total length of the train, V be the velocity fo the middle point of the train while pasign a certain point and a be the uniform acceleration of the train. Taking the motion of the train when middle point is passing from the given point, we have, u = u, v = V, s = x/2.

Using $v^2 = u^2 + 2as$, we have

$V^2 = u^2 + \frac{2ax}{2} = u^2 + ax$ …(i)

Taking the motion of train when the last end of h train is passing from the given point, then u = u, v = v, a = a, s = x

Now we have, $v^2 = u^2 + 2ax$ or ax = $\frac{v^2-u^2}{2}$

Putting this value on (i) we get

$V^2 = u^2 + \frac{v^2-u^2}{2} = \frac{u^2+v^2}{2}$ or V = $\sqrt{\frac{(u^2+v^2)}{2}}$

8. In a car race, car A takes time t second less than car B and finishes the finishing point with a velocity v more than that of the car B. Assuming that the cars start from rest and travel with constant acceleration a_1 and a_2 respectively show that $v = t\sqrt{a_1 a_2}$.

Sol. Let the speed of the two balls (1 and 2) be u_1 and u_2 where $u_1 = 2u$, $u_2 = u$.

If y_1 and y_2 are the distances covered by the balls 1 and 2 respectively, before coming to rest, then

$y_1 = \frac{u_1^2}{2g} = \frac{4u^2}{2g}$ and $y_2 = \frac{u_2^2}{2g} = \frac{u^2}{2g}$

Since $y_1 - y_2$ = 15m, $\frac{4u^2}{2g} - \frac{u^2}{2g}$ = 15m or $\frac{3u^2}{2g}$ = 15m

Or u = $\sqrt{5m \times (2 \times 10ms^{-2})}$ or u = 10 ms^{-1}

Clearly $u_1 = 20ms^{-1}$ and $u_2 = 10ms^{-1}$

As $y_1 = \frac{u_1^2}{2g} = \frac{(20ms^{-1})^2}{2\times10ms^{-2}}$ = 20m, $y_2 = y_1 - 15m = 5m$

If t_2 is the time taken by the ball 2 to cover a distance 5m, then from $y_2 = u_2t - ½ gt_2^2$, 5 = $10t_2 - 5t_2^2$

Or $t_2^2 - 2t_2 + 1 = 0$ or $t_2 = 1s$

Since t_1 (time taken by ball 1 to cover a distance fo 20m) in 2s, time interval between two throws = $t_1 - t_2 = 2s - 1s = 1s$.

9. A car accelerates from rest at a constant rate α for some time, after which it accelerates at a constant rate β to come to rest. If the total time elapsed is t second, then calculate (i) the maximum velocity attained by the car, and (ii) the total distance travelled by the car in terms of α, β and t.

Sol. (i) Let the car accelerate for time t_1 and v be the maximum velocity so attained in time t_1.

As v = u +at

So, $v = 0 + \alpha t^2$ or $t_1 = v/\alpha$ ….(1)

Now starting with the maximum velocity v, the car decelerates the constant rate β and comes to rest in time $(t - t_1)$. Therefore

$0 = v - \beta(t - t_1)$ or $t - t_1 = v/\beta$ …(2)

Adding equations (1) and (2) we get

$t = \frac{v}{\alpha} + \frac{v}{\beta} = v\left(\frac{\alpha+\beta}{\alpha\beta}\right)$ or $v = \frac{\alpha\beta t}{\alpha+\beta}$ …(3)

This gives the maximum velocity attained by the car.

(ii) Distance covered by the car in time t_1 is

$x_1 = 0 + ½ \alpha t_1^2 = ½ \alpha.\frac{v^2}{\alpha^2} = \frac{v^2}{2\alpha}$ [Using (1)]

Distance covered by the car in time $(t - t_1)$ is

$x_2 = v(t - t_1) - ½ (t - t_1)^2 = v.\frac{v^2}{\beta^2} = \frac{v^2}{2\beta}$ [using (2)]

Therefore total time travelled by the car is

$x = x_1 + x_2 = \frac{v^2}{2\alpha} + \frac{v^2}{2\beta} = \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = \frac{\alpha^2\beta^2t^2}{2(\alpha+\beta)}.\frac{(\alpha+\beta)}{\alpha\beta}$ …[using (3)]

or $x = \frac{\alpha\beta t^2}{2(\alpha+\beta)}$

10. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed from some time t and then decelerates at the rate f/2 to come to rest. If the total distance is 5S, then prove that $S = ½ ft^2$.

Sol. For accelerated motion, u = 0, a = f, a = s

As $v^2 - u^2 = 2as$

Therefore $v_1^2 - 0^2 = 2fs$ or $v_1 = \sqrt{2fs}$

For uniform motion: $u = v_1 = \sqrt{2fs}$, t = t

Distance travelled, $s_2 = ut = \sqrt{2fs}t$

For decelerated motion, $u = \sqrt{2fs}$, a = - f/2. V = 0
As $v^2 - u^2 = 2as$.
So, $0^2 - (\sqrt{2fs})^2 = 2 \times (-f/2)s_3$
Distance travelled, $s_3 = 2s$
Given $s + s_2 + s_3 = 5s$
Or $s + \sqrt{2fs}t + 2s = 5s$ or $\sqrt{2fs}t = s$
Or $s = \frac{1}{2} ft^2$.

11. Derive an equation for the distance covered by a uniformly accelerated body in nth second of its motion. A body travels half its path in the last second of its fall from rest, calculate the time of its fall.

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C. EQUATION OF MOTION UNDER GRAVITY

(1 Mark Questions)

1. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with same initial speed. Which one will rise to the greater height?

Sol. Both the balls will rise to the same height. It is because, for a body moving with given initial velocity and acceleration, the distance covered by the body does not depend on the mass of the body.

2. Two balls of different masses (one lighter and other heavier) are thrown vertically upward with same speed. Which one will pass through the point of projection in their downward direction with the greater speed?

Sol. Both the balls will rise to the same height. Because the maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.

(2 marks Questions)

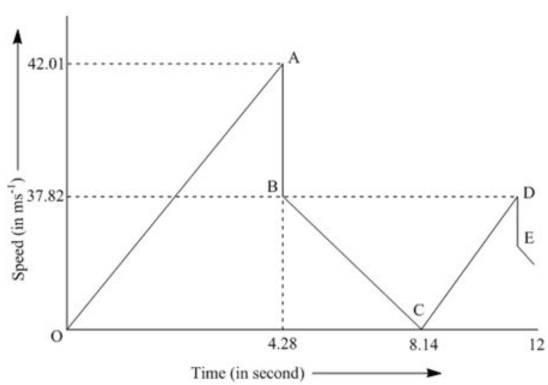
3. A food packet is released from a helicopter which is rising steadily at $2ms^{-1}$. After two seconds (i) what is the velocity of the packet? (ii) How far is it below the helicopter? Take g = 9.8 ms^{-2}.

Sol. Here u = $2ms^{-1}$., g = 9.8 ms^{-2}, t = 2s.
(i) v – u + gt – 2 = 2 – 9.8 × 2 = - $17.6ms^{-1}$
Negative sing shows that the velocity is directed vertically downward.
(ii) Distance covered by the food packet in 2s, s = ut + ½ at^2 = 2 × 2 = ½ × 9.8 ×2^2
= 4 – 19.6 = - 15.6m.
Thus the food packet falls through a distance of 16.5m in 2s but in the mean time the helicopter rises up through a distance = $2ms^{-1}$ × 2s = 4m
Therefore after 2s, the distance of the food packet from the helicopter = 15.6+4 = 19.6m.

(3 Marks Questions)

4. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.

Sol. (i) The time taken by the ball to fall through a height of 90m is obtained as follows:
x = v(0)t + ½ gt^2; 90 = 0 + ½ × $9.8t^2$
or t = $\sqrt{\frac{2\times90}{9.8}} = \frac{30}{7}$s = 4.3s
Now v(t) = v(0) + gt
Therefore v(4.3) = 0 + 9.8 + $\frac{30}{7}$ = 42 ms^{-1}
From time t = 0 to t = 4.3s, v(t) = gt = 9.8t or v(t) ∝ t
In this duration speed increases linearly with time t from 0 to 42 ms^{-1} during the downward motion of the ball and this speed time variation has been shown by straight line OA in figure.

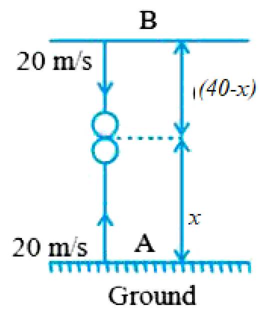


(ii) At first collision with the floor, speed lost by ball = 1/10 × 42 = 4.2 ms^{-1}
Thus the ball rebounds with a speed of 42 – 4.2 = 37.8 ms^{-1}. For the further upward motion, the speed at any instant t is given by v(t) = v(0) – gt = 37.8 – 9.8 × t
Now speed decreases linearly with time and becomes zero after time t = 37.8/9.8 = 3.9s
Thus the ball reaches the highest point against after time t = 4.3+ 3.9 -= 8.2s from the start. Straight line BC represents the speed time graph for this upward motion.

(iii) At highest point, speed of ball is zero, It again starts falling. At any instant t, the sped is given by v(t) = 0 +9.8t.

Again the speed of the ball increases linearly with time t from 0 to 37.8 ms^{-1} (initial speed of the previous upward motion) in the next time interval o f3.9s. Total time taken from the start = 4.3 +3.9 +3.9 = 12.1s. This part of motion is shown by straight line CD.

5. Two balls are thrown simultaneously. A vertically upwards with a speed of 20 m/s from the ground, and B vertically downwards from a height of 40m with the same speed and along the same line of motion. At what points do the two balls collide?
(Take g = 9.8 ms^{-2})



Sol. Suppose the two balls meet at a height of x metre from the ground after time ts from the start.

For upward motion of ball A, u = 20ms^{-1}, g = - 9.8ms^{-2}

$s = ut + ½ gt^2$

$x = 20t – ½ × 9.8t^2 = 20t – 4.9t^2$ …(i)

For downward motion of ball B ,

$40 – x = 20t – ½ × 9.8t^2 = 20t + 4.9t^2$ …(ii)

Adding (i) and (ii), 40 = 10t or t = 1s

From (i), $x = 20 × 1 – 4.9 × (1)^2 = 15.1$m.

Hence the two balls will collide after 1s at a height of 15.1m from the ground.

6. A rocket is fired vertically from the ground with a resultant vertical acceleration of 10m/s^2. The fuel is finished in 1 min and it continues to move up. What is the maximum height reached?

Sol. Height covered in 1 min, $s_1 = ut + 1/2gt^2 = 0 + ½ × 10 × (60)^2 = 18000$m

Velocity attained after 1 min, v = u + at = 0 + 10 × 60 = 600 ms^{-1}

After the fuel is finished, u = 600 ms^{-1}, v = 0, $v^2 – u^2 = 2as$

Or $0 – (600)^2 = 2 × s_2$ or $s_2 = \frac{(600)^2}{2×9.8} = 18367.3$m

Maximum height reached, $s_1 + s_2 = 36367.3$m = 36.4 km.

7. A balloon is ascending at the rate of 14 m/s at a height of 98m above the ground when the food packet is dropped from the balloon. After how much time and with that velocity does it reach the ground? Take g = 9.8 ms^{-2}.

Sol. Velocity of balloon = 14 ms^{-1}

Given acceleration = - 9.8 ms^{-2}, distance from the ground = - 9.8m

Final velocity v and time taken to reach the ground = t

$v^2 - u^2 = 2ah = v = \sqrt{2ah + u^2} = \sqrt{2(-9.81)(-98) + 14^2}$ = -46m/s.

Also, v = u + at

$t = \frac{v-u}{a} = \frac{-46-14}{-9.81}$ = 6.11s

At velocity 46m/s and time 6.11s it will reach the ground.

8. A player throws a ball upwards with an initial speed of 29.4 m s^{-1} .

(a) What is the direction of acceleration during the upward motion of the ball ?

(b) What are the velocity and acceleration of the ball at the highest point of its motion ?

(c) Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion.

(d) To what height does the ball rise and after how long does the ball return to the player's hands? (Take g = 9.8 m s^{-2} and neglect air resistance)

Sol. (a) The ball moves under the effect of gravity. The direction foe acceleration due to gravity is always vertically downwards.

(b) At the highest point, velocity of the ball = 0.

Acceleration = Acceleration due to gravity 'g' = 9.8ms^{-2} (vertically downwards)

(c) When the highest point is chosen as the location for x = 0 and t = 0 and vertically downward direction to be the positive direction of x axis: (i) During upward motion: Position is positive, velocity is negative and acceleration is positive. (ii) During upward motion: Position is positive, velocity is positive and acceleration is positive.

(d) For upward motion: u = 29.4 ms^{-1}, g = +9.8 ms^{-2}, v = 0

If s is the height to which the ball rises, then $u^2 - v^2 = 2as$

Or $v^2 - (-29.4)^2 = 2 \times 9.8 \times s$

Or $s = -\frac{(29.4)^2}{2\times 9.8}$ = - 44.1m

Negative sign shows that the distance is covered in upward direction.

If the ball reaches point in time t, then

v = u + at

or 0 = - 29.4 + 9.8t

or $t = \frac{29.4}{9.8}$ = 3s.

9. A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m s^{-1}. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m s^{-1} and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?

Sol. (i) When the lift is stationary: For upward motion of the ball, we have, u = 49 ms^{-1}, g = -9.8 ms^{-2}, v = 0, t = ?

As v = u + at

Therefore 0 = 49 – 9.8t or t = 49/9.8 = 5s
As time of ascent = time of descent. So total time taken = 5+5 = 10s

D. NON-UNIFORM ACCELERATED MOTION

(1 Mark Questions)

1. The velocity of the particle at any time t is given by v = 2t(3 – t)m/s. At what time is its velocity maximum?
(a) 2s (b) 2s (c) 2/3s (d) 3/2s

Sol. (d)

(3 Marks Questions)

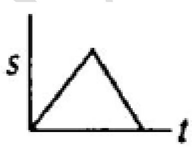
2. The velocity of a particle is given by the equation $v = 2t^2 + 5$ cms^{-1}.Find (i) the change in velocity of the particle during the time interval between t_1 = 2s and t_2 = 4s (ii) the average acceleration during the same interval and (iii) the instantaneous acceleration at t_2 = 4s.

Sol. Given $v = 2t^2 + 5$ cms^{-1}
(i) When t_1 = 2s, $v_1 = 2(2)^2 + 5 = 13ms^{-1}$; when t_2 = 4s, $v_2 = 2(4)^2 + 5 = 37$ cms^{-1}.
Change in velocity = $v_2 - v_1$ = 37 – 13 = 24 cms^1.
(ii) Average acceleration, $a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{37-13}{4-2} = 12cms^{-1}$
(iii) Instantaneous acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(2t^2 + 5) = 4t$
At t = 4s, a = 4 × 4 = $16cms^{-1}$.

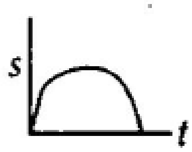
E. GRAPH

(1 Mark Questions)

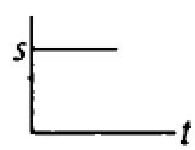
1. Which of the following graphs represents uniform motion? (s is path length).



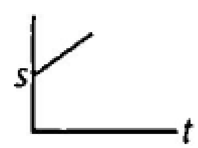
(a)



(b)



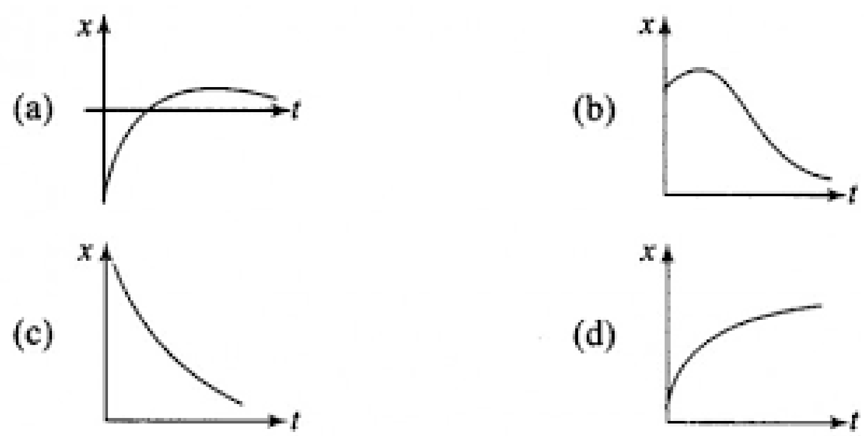
(c)



(d)

Sol. (c)

2. Among the four graphs (Figure), there is only one graph for which average velocity over the time interval (0, T) can vanish for a suitably chosen T. Which one is it?



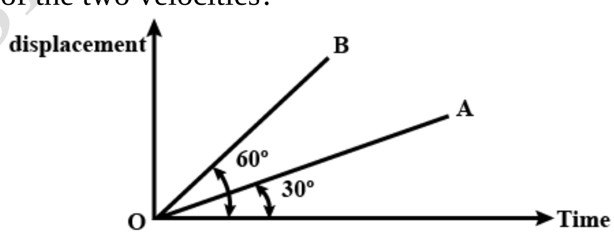
Sol. (b)

(2 marks Questions)

3. Two trains A and B each of length 100m, are running on parallel tracks. One overtakes the other in 20 s moving in the same direction and one crosses the other in 10 s moving in opposite direction. Calculate the velocities of each train.

Sol. Let the velocity of train A be v_A. Let the velocity of train B be v_B. Let $v_A > v_B$. When one overtakes the other
$v_A – v_B = 200/20$ or $v_A – v_B = 10$m/s …(i)
When one crosses the other (opposite direction),
$v_A + v_B = 200/10$ or $v_A + v_B = 20$m/s …(ii)
From equation (i) and (ii), $v_A = 15$, $v_B = 5$
So, the velocity of trains A and B is 15m/s and 5m/s respectively.

4. Two straight lines drawn on the same displacement time graph makes angles 30° and 60° with time axis respectively, as shown in figure. Which line represents greater velocity? What is the ratio of the two velocities?



Sol. Slope of displacement time graph = Velocity of the object
As slope of line OB > slope of line OA
Therefore the line making angle of 60° with time axis represents greater velocity
Ratio of two velocities = $\frac{v_A}{v_B} = \frac{\tan 30°}{\tan 60°} = \frac{1}{3} = 1:3.$

(3 Marks Questions)

5. A woman starts from her home at 9.00 am, walks with a speed of 5km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1}. Choose suitable scales and plot the x-t graph of her motion.

Sol. The time at which woman leaves for her office is 9A.M.

As she travels with a speed of 5 kmh^{-1} and the distance of office is 2.5km, hence time taken by her to reach office, t = $\frac{\text{distance}}{\text{speed}} = \frac{2.5\text{km}}{5\text{kmh}^{-1}} = \frac{1}{2}$h

Hence time at which she reaches office is 9.30 A.M.

Between 9.30 A.M. to 5.00 P.M. she stays in her office i.e at a distance of 2.5 km from he r home.

For the return journey from office to home:

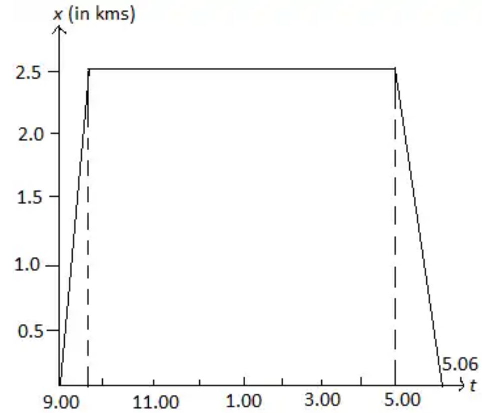
The time at which she leaves her office = 5P.M.

Now she travels a distance fo 2.5km with a sped of 25 kmh^{-1} hence the time taken,

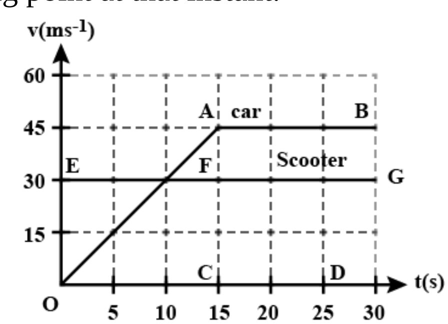
t' = $\frac{2.5\text{km}}{25\text{kmh}^{-1}} = \frac{1}{10}$ = 6 minutes

The time at which she reaches her home = 5.06 P.M.

The x-t graph of the woman's motion is shown in figure. Here 9 A.M. is regarded as the origin for time axis and home as the origin for position axis.



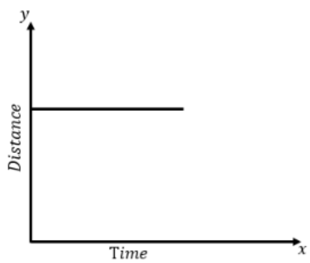
6. As soon as a car just starts from rest in a certain direction, a scooter moving with a uniform speed overtakes the car. Their velocity time graphs are shown in the figure. Calculate (i) the difference between the distances travelled by the car and the scooter in 15s (ii) the time when the car will catch up the scooter and (iii) the distance of car and scooter from the starting point at that instant.

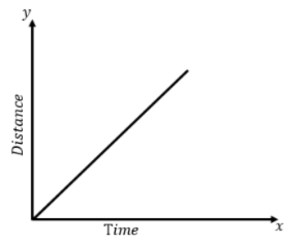


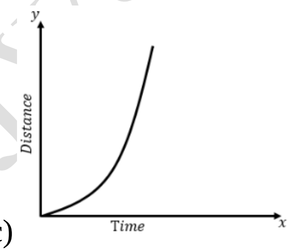
Sol. (i) Distance travelled by the car in 15s = Area of ΔOAC

= ½ × OC × AC = ½ × 15 × 45 = 337.5m
Distance travelled by the scooter in 15s = Area of rect. OEFC = 15×30 = 450m
Difference in the distance travelled = 450 – 337.5 = 112.5 m
(ii) After t = 15s, relative velocity of the car w.r.t. the scooter = 45 – 30 = 15 ms^{-1}
Therefore time in covering a difference of 112.5m = $\frac{112.5m}{15ms^{-1}}$ = 7.5s
So, time after which car will catch up the scooter = 15 +7.5 = 22.5s
(iii) Distance travelled by the scooter in 22.5s = $30ms^{-1}$ × 22.5s = 675m.
So the car catches the scooter when both are at 675m from the starting point.

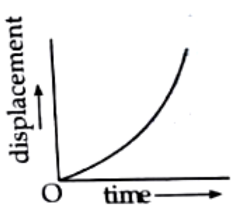
7. Draw the following graphs(expected nature only) between distance and time of an object in case of (a) for a body at rest (b) for a body moving with uniform velocity (c) for a body moving with constant acceleration.

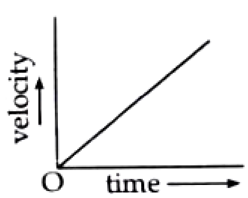
Sol. (a) 

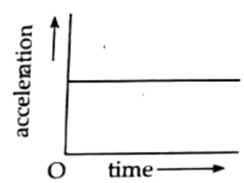
(b) 

(c) 

8. Draw the following graphs (expected nature only) representing motion of an object under free fall. Neglect their resistance. (a) variation of position with respect to time (b) variation of velocity with respect to time (c) variation of acceleration with respect to time.

Sol. (a) 

(b) 

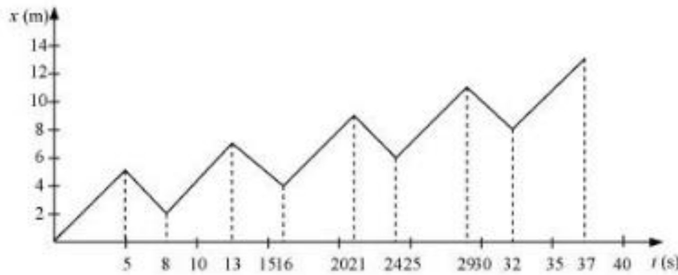
(c) 

9. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1m long and requires 1s. Plot the x – t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13m away from the start.

Sol. (a) Graphical method: Taking the starting point as origin, the positions of the drunkard at various instants of time are given in the following table.

t(s)	0	5	8	13	16	21	24	29	32	37
x(m)	0	5	2	7	4	9	6	11	8	13

The position time graph (x-t) graph for motion of the drunkard is in figure. As is obvious from graph that the drunkard would take 37s to fall in a pit 13m away from the starting point.



(b) Analytic method: In each forward motion of 5 steps and backward motion of 3 steps, net distance covered = 5 – 3 = 2m and time taken = 5+3 = 8s

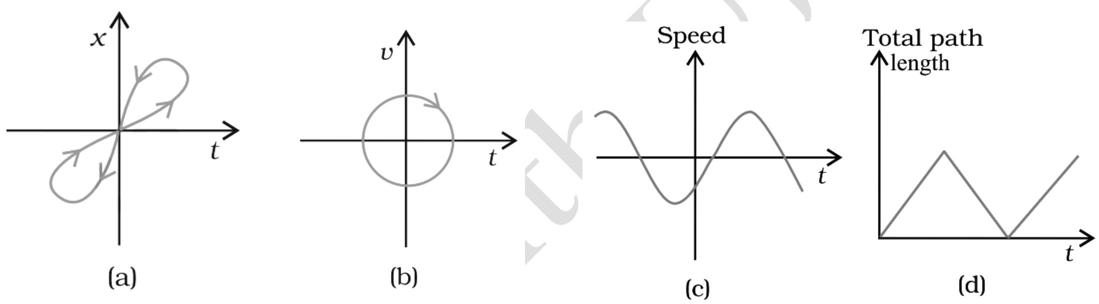
Therefore time required to cover a distance of 8m = $\frac{8}{2}\times 8$ = 32s

Remaining distance of the pit = 13 – 8 = 5m

In next 5s, as he moves 5 steps forward, he falls in the pit.

Therefore total time taken = 32+5 = 37s.

10. Look at the graphs (a) to (d) (Fig.) carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.



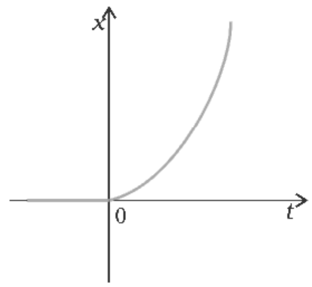
Sol. (a) A line drawn for a given time parallel to position axis will cut the graph at two points which means that at a given instant of time, the particle will have two positions which is not possible. Hence graph (a) is not possible i.e., does not represent one dimensional motion.

(b) This graph does not represent one dimensional motion because at a given instant of time, the particle will have two values of velocity in positive, as well as in negative direction which is not possible in one dimensional motion.

(c) This is a graph between speed and time and does not represent one dimensional motion as this graph tells that the particle can have the negative speed but the speed can never be negative.

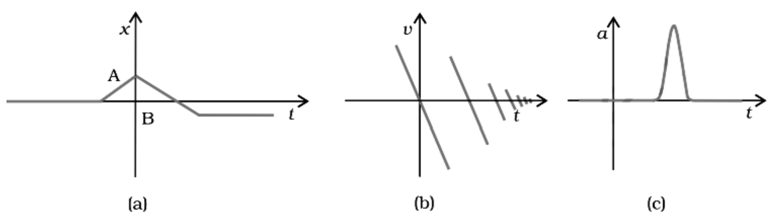
(d) This does not represent one dimensional motion, as this graph tells that the total path length decreases after certain time but total path length of a moving particle can never decrease with time.

11. Figure shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for t < 0 and on a parabolic path for t >0 ? If not, suggest a suitable physical context for this graph.



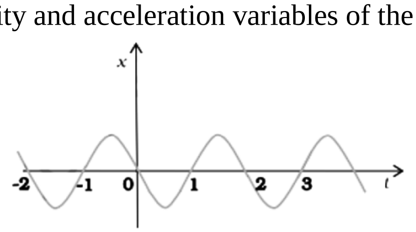
Sol. No, it is wrong to say that the particle moves in a straight line for t < 0 and on a parabolic path t > 0, because a position time (x-t)graph does not represent the trajectory of a moving particle.
The graph can represent the motion of a freely falling particle dropped from a tower when we take its initial position as x = 0 at t = 0.

12. Suggest a suitable physical situation for each of the following graphs (Figure):



Sol. (a) The x-t graph shows that initially x is equal to 0, attains a certain value of x, again x becomes zero and then x increases in opposite direction till it again attains a constant x (i.e., comes to rest). Therefore it may represent a physical situation such as a ball (initially at rest) on being kicked, rebounds from the all with reduced speed and then moves to the opposite wall and then stops.
(b) From the (v-t) graph, it follows that the velocity changes sign again and again with the passage of time and every time losing some speed. Therefore it may represent a physical situation such as a ball falling freely (after being thrown up). On striking the ground rebounds with red iced after each hit against the ground.
(c) The (a-t) graph shows that the body gets accelerated for a short duration only. Therefore it may represent a physical situation such as a ball moving with uniform speed is hit with a bat for a very small time interval.

13. Figure gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at t = 0.3 s, 1.2 s, – 1.2 s.



Sol. The acceleration of a particle executing SHM is given by $a = -\omega^2 x$. where ω is angular frequency and is constant.

At time t= 0.3s:

As is obvious from the graph, $x < 0$. As slope of xt graph is negative, so $v < 0$. As $a = -\omega^2 x$, so $a > 0$.

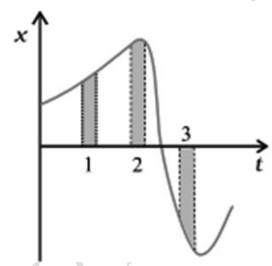
At time t = 1.2s:

As is obvious from the graph, $x > 0$. As slope of xt graph is positive, so $v > 0$. As $a = -\omega^2 x$, so $a < 0$

At time t = - 1.2s

As is obvious from the graph, $x < 0$. As slope fox t graph is positive, so $v > 0$. As $a = -\omega^2 x$, so $a > 0$.

14. Figure gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least ? Give the sign of average velocity for each interval.



Sol. Slope of x-t graph is a small time interval = Average speed in that interval.

As the slope of x-t is greatest in interval 3 and least in interval 1, so the average speed is greatest in interval 3 and least in interval 1.

As the slope of x-t is positive in intervals 1 and 2 and negative in interval 3, so average velocity is positive in intervals 1 and 2 and negative in interval 3.

15. A three – wheeler starts from rest , accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity .plot the distance covered by the vehicle during the n^{th} second (n=1,2,3……..) versus n. what do you expect this plot to be during accelerated motion: a straight line or a parabola?

Sol. Distance traveled in nth second, $s_{nth} = u + \frac{a}{2}(2a + 1)$

AS u = 0, $a = 1ms^{-2}$. So $s_{nth} = 0 + ½ (2n – 1) = ½ (2n – 1)$m

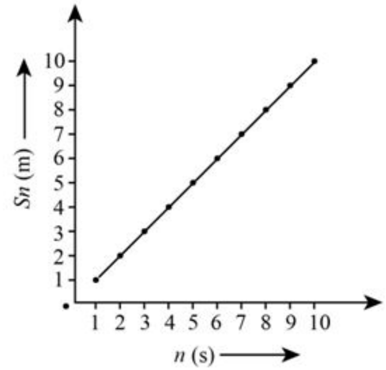
Thus distances travelled by the three wheeler at the end of each second is given by

n(s)	1	2	3	4	5	6	7	8	9	10
S_{nth}(m)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5

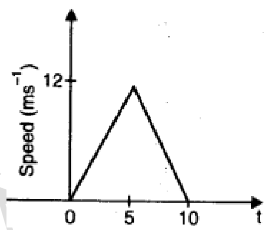
Now velocity of the three wheeler at the end of 10^{th} s is given by

$v = u + at = 0 + 1\times10 = 10ms^{-1}$

Upon n = 10s, the motion is accelerated and the graph between s_{nth} and n is a straight line AB inclined to time axis as shown in figure. After 10^{th} second, the three wheeler moves with uniform velocity of $10ms^{-1}$, so graph is straight line parallel to time axis.



16. The speed-time graph of a particle moving along a fixed direction is shown in Fig. Obtain the distance traversed by the particle between
(a) t = 0 s to 10 s. (b) t = 2 s to 6 s.
What is the average speed of the particle over the intervals in (a) and (b)?

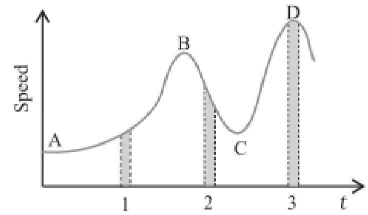


Sol. (i) Distance travelled by the particle t = 0 to 10s is given by
s = ΔOAB = ½ OB × AC = ½ × 10 × 12 \= 60m
Average speed = Total distance covered/Total time taken = 60/10 = $6ms^{-1}$
(ii) Acceleration of the particle during journey OA is given by
v = u +at or 12 – 0 + a × 5 or a = $+2.4ms^{-2}$
Similarly acceleration of the particle during journey AB is given by v = u +at or 0 – 12 + a× 5 or a = 2.4 ms^{-2}
Velocity of the particle after 2s from start will be v = u +at = 0 + 2.4 × 2 = 4.8 ms^{-1}
Therefore distance covered by the particle between t = 2 to 5s (in 3s) is given by
$s_1 = ut + ½ at^2 = 12 × 1 + ½ × (-2.4) × 1^2 = 10.8m$
Total distance travelled in t = 2 to 6s, s = s_1+s_2 = 25.2 + 10.8 = 36m
Average speed in the interval t = 2 to 6s
= Total distance covered/Total time taken = 36/4 = 9 ms^{-1}.

(5 Marks Questions)

17. Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude ?

In which interval is the average speed greatest ? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D ?



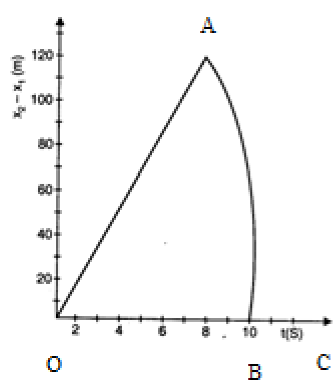
Sol. (i) The magnitude of the average acceleration = Change is speed/Time interval. i.e., average acceleration in a small interval of the time is equal to the slop[e of (v-t)graph in that time interval. As the slop of (v-t) graph is maximum in the interval 2 as compared to intervals 1 and 3, hence the magnitude of average acceleration is greatest in the interval 2.

(ii) The average speed is greatest in the interval 3 as peak D is maximum on speed axis.

(iii) v > 0 i.e. positive in all the three intervals. The slope is positive in intervals 1 and 3 so a, i.e., acceleration is positive in these intervals while the slope is negative in interval 2, so acceleration is negative in it. So a > 0, i.e., positive in intervals 1 and 3 and a < 0, i.e., negative in interval 2.

(iv) As slop is zero at points A, B, C and D, so the acceleration is zero at all the four points.

18. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the graph shown in Fig. correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 ms^{-2}. Give the equations for the linear and curved parts of the plot.



Sol. $x(t) = x(0) + v(0)t = \frac{1}{2} gt^2$

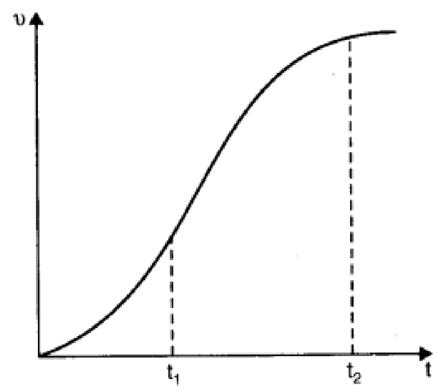
If we take origin for position measurement on the ground, then the positions fo the two stones at any instant t will be

$x_1 = 200+15t - \frac{1}{2} \times 10t^2$ …(i)

$x_2 = 200 + 30t - \frac{1}{2} \times 10t^2$ …(ii)
When the first stone hits the ground, $x_1 = 0$
Or $200 + 15t - 5t^2 = 0$
Or $5t^2 - 15t - 200 = 0$
Or $t^2 - 3t + 40 = 0$
Therefore $t = \frac{3 \pm \sqrt{9+160}}{2} = \frac{3 \pm 13}{2}$ = 8s or – 5s
As time cannot be negative, so t = 8s i.e. the first stone hits the ground after 8s.
From (i) and (ii) the relative position of second stone w.r.t. first is given by
$x_2 - x_1 = 15t$
As there is a linear relationship between $x_2 - x_1$ and t, so the graph is straight line OA upto t = 8s. After t – 8s, only the second stone is in motion. So the graph is parabolic (AB) in accordance with quadratic equation, $x_2 = 200 + 30t - 5t^2$
The second stone will hit the ground when $x_2 = 0$
Or $200 + 30t - 5t^2 = 0$
On solving, t = 10s
After 10s the separation between the balls is zero, which explains the part BC fo the graph.

19. The velocity-time graph of a particle in one-dimensional motion is shown below. Which of the following formula are correct for describing the motion of the particle over the time interval from t_1 tot_2?

(a) $x(t_2) = x(t_1) + v(t_1)(t_2 - t_1) + \frac{1}{2} a (t_2 - t_1)^2$
(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$
(c) $a_{average} = [x(t_2) - x(t_1)]/(t_2 - t_1)$
(d) $a_{average} = [v(t_2) - v(t_1)]/(t_2 - t_1)$
(e) $x(t_2) = x(t_1) + v_{av}(t_2 - t_1) + \frac{1}{2} a_{av} (t_2 - t_1)^2$
(f) $x(t_2) - x(t_1)$ = Area under the v-t curve bounded by t-axis and the dotted lines.



Sol. (a)It is not correct because in the time interval between t_1 and t_2 is not constant.
(b) This relation is also not correct for the reason as n (a).
(c) This relation is correct.
(d) This relation is also correct.
(e) This relation is not correct because average acceleration cannot be used in this relation.
(f) This relation is correct.

F. RELATIVE VELOCITY

(1 Mark Questions)

1. The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. Choose the correct entries in the brackets below ;

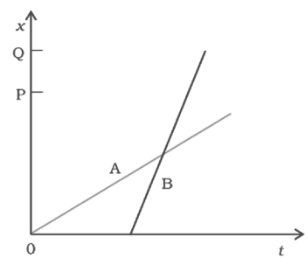
(a) (A/B) lives closer to the school than (B/A) – As OP < OQ, A lives closer to school than B

(b) (A/B) starts from the school earlier than (B/A) – For x = 0, t = 0 for A; while t has some finite value of B. Therefore A starts from the school earlier than B.

(c) (A/B) walks faster than (B/A) – Since the velocity is equal to slope x-t graph in case of uniform motion and slope of x-t graph fro B is greater than that for A, hence B walks faster than A.

(d) A and B reach home at the (same/different) time – It is clear from graph that both A and B reach their respective homes at different time.

(e) (A/B) overtakes (B/A) on the road (once/twice). – B moves later than and his/her speed is greater than A. From the graph it is clear that B overtakes A only once on the road.



2. At a metro station, a girl walks up a stationary escalator in time t_1. If she remains stationary on the escalator, then the escalator take her up in time t_2. The time taken by her to walk up on the moving escalator will be

(a) $(t_1 + t_2)/2$ (b) $t_1t_2/(t_2–t_1)$ (c) $t_1t_2/(t_2+t_1)$ (d) $t_1–t_2$

Sol. (c)

(3 Marks Questions)

3. A jet airplane travelling at the speed 500 km $h^{–1}$ ejects its products of combustion at the speed of 1500 $kmh^{–1}$ relative to the jet plane. What is the speed of the latter with respect to an observer on the ground?

Sol. Here speed of jet airplane, v_1 – 500 $kmh^{–1}$

Let v_2 be the speed of products w.r.t. the ground. Suppose the direction of motion of the jet plane is positive, then the relative velocity of products w.r.t. jet plane is

$v_2 – v_1 = - 1500$

or $v_2 = v_1 – 1500 = 500 – 1500 = - 1000\ kmh^{-1}$

Negative sign shows the direction of products of combustion is opposite to that of the jet plane.

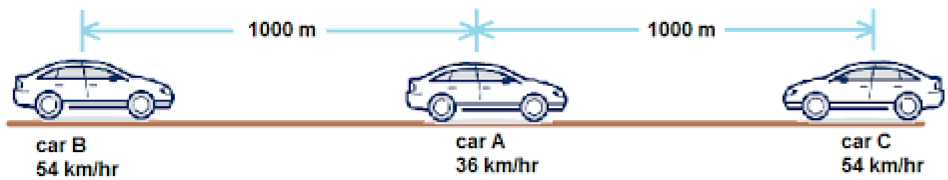
So, speed of products of combustion w.r.t. ground = 1000 kmh^{-1}.

4. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?

Sol. Let x be the distance between the driver of train A and the guard of train B. Initially both trains are moving in the same direction with the same speed of 72 kmh^{-1}. So relative velocity fo B w.r.t. a = $v_B - v_A = 0$. Hence the train B needs to cover a distance with a = $1ms^{-1}$, t = 50s, u = 0
As $s = ut + ½ at^2$
Therefore, $x = 0 \times 50 + ½ \times 1 \times (50)^2 = 1250m$.

5. On a two-lane road, car A is travelling with a speed of 36 km h^{-1}. Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Sol.



At the instant when B decides to overtake Am the speed of three cars are:

$v_A = 36kmh^{-1} = 36 \times \frac{5}{18} = 10ms^{-1}$

$v_B = +54\ kmh^{-1} = +54 \times \frac{5}{18} = +\ 15ms^{-1}$

$v_c = -\ 54\ kmh^{-1} = -\ 54 \times \frac{5}{18} = -\ 15\ ms^{-1}$

Relative velocity of C w.r.t. A, $v_{CA} = v_C - v_A = -\ 15 - 10 = -\ 25\ ms^{-1}$

Therefore time that C requires to just cross A = $\frac{1km}{v_{CA}} = \frac{100m}{25ms^{-1}} = 40s$

In order to avoid the accident, B must overtake A in a time less than 40s. So the car B we have
Relative velocity of car b w.r.t. A, $v_{BA} = v_B - v_A = 15 - 10 = 5ms^{-1}$
Therefore s = 4km, 1000m, u = $5ms^{-1}$, t = 40s
As $s = ut + ½ at^2$
Therefore $100 = 5 \times 40 + ½\ a \times (40)^2$ or $1000 = 200 + 800a$
Or a = 1 ms^{-2}
Thus 1 ms^{-2} is the minimum acceleration that car b requires to avoid an accident.

6. Two parallel rail tacks run north south. Train A moves north with a speed of 54 km/h and train B moves south with a speed of 90 km/h. What is the (i) relative velocity of B with respect to A? (ii) relative velocity of ground with respect to B (iii) velocity of a monkey running on the roof of the train A against the motion (with a velocity of 18km/h with respect to the train A) as observed by a man standing on the ground?

Sol. Taking, south to north direction as the positive direction of x axis we have

$v_A = +54km/h = \frac{54\times1000}{3600}ms^{-1} = 15\ ms^{-1}$, $v_B = -\ 90\ km/h = \frac{-90\times1000}{3600}ms^{-1} = -\ 25ms^{-1}$

(i)Relative velocity of b with respect to A = $v_B - v_A$ = -25 – 15 = - 40 ms^{-1}
So to an observer in train A, the train B appears to move with a speed fo 40 ms^{-1} from north to south.
(ii) Relative velocity of ground with respect to B = 0 – v_B = 0 + 25 = 25ms^{-1}
So to an observer in train, B, the earth appears to move with a speed of 25 ms^{-1} from south to north.
(iii) Let velocity of monkey with respect to ground - v_M
Therefore relative velocity of monkey with respect to train A
= $v_M - v_A$= - 18 kmh^{-1} = - 5 ms^{-1}
Or $v_M - v_A$= - 5 – 15 – 5 = 10 ms^{-1}.

7. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

So. Let speed o each bus = v kmh^{-1}
For buses going from town A to B: Relative speed of a bus in the direction of motion of the man (v =- 20)kmh^{-1}
Buses plying go past the cyclist after every 18 min
Therefore distance covered = $(v - 20)\frac{48}{60}$ km
Since a bus leaves the town every T min, so the above distance covered
= $v \times \frac{T}{60}$km, so $(v - 20)\frac{48}{60}$ km = $v \times \frac{T}{60}$ …(i)
Buses going from town B to A: Relative speed of bus in the direction opposite to the motion of the man = (v +20)kmh^{-1}
Buses going in this direction go past the cyclist after every 6 min,
therefore, $(v + 20)\frac{6}{60} = v\times\frac{T}{60}$ …(ii)
Dividing (i) by (ii) we get, $\frac{(v-20)}{(v+20)}\frac{6}{60} = v \times \frac{T}{60}$
Or 3v – 60 = v + 20 or v = 40 kmh^{-1}
From equation (ii), $(40+20)\frac{6}{60} = \frac{40T}{60}$
Or $T = \frac{60\times6}{40}$ = 90 min.

8. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1}. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the
(a) magnitude of average velocity, and
(b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min?

Sol. Case (i) 0 to 30 min: speed = 5kmh^{-1}, distance covered in 30 min = 5 $kmh^{-1} \times \frac{30}{60}$h = 2.5km
Displacement covered = 2.5 km.

(a) Average velocity = $\frac{\text{Displacement}}{\text{time}} = \frac{2.5\text{km}}{30/60\text{h}}$ = 5kmh^{-1}

(b) Average speed = $\frac{\text{Distance}}{\text{time}} = \frac{2.5\text{km}}{30/60\text{h}}$ = 5 kmh^{-1}.

(ii) Case (ii) 0 to 50 min: displacement covered in first 30 min in going to market = 5kmh^{-1} × $\frac{30}{60}$h = 2.5 km.

Displacement covered in next 20 min in coming back to home = 7.5 kmh^{-1} × $\frac{20}{60}$h = 2.5km

Net displacement = 7.5 – 2.5 = 0; Total distance covered = 2.5+2.5 =5km

(a) Average velocity = $\frac{\text{Net displacement}}{\text{Time taken}} = \frac{0}{50/60\text{h}} = 0.$

(b) Average speed = $\frac{\text{Total distance}}{\text{Time taken}} = \frac{5\text{km}}{50/60\text{h}}$ = 6kmh^{-1}.

(iii) Case (iii) 0 to 40 min: Displacement covered in first 30 min in going to market = 2.5km

Displacement covered in next 10 min in coming back to home = 7.5kmh^{-1} × $\frac{10}{60}$h = 1.25km

Net displacement = 2.5 – 1.25 = 1.25km

Total distance travelled = 2.5 + 1.25 = 3.75km.

(a) Average velocity = $\frac{\text{Net displacement}}{\text{Time taken}} = \frac{1.25\text{km}}{40/60\text{h}}$ = 1.875kmh^{-1}

(b) Average speed = $\frac{\text{Total distance}}{\text{Time taken}} = \frac{3.75\text{km}}{40/60\text{h}}$ = 5.625 kmh^{-1}

9. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car?

(5 Marks Questions)

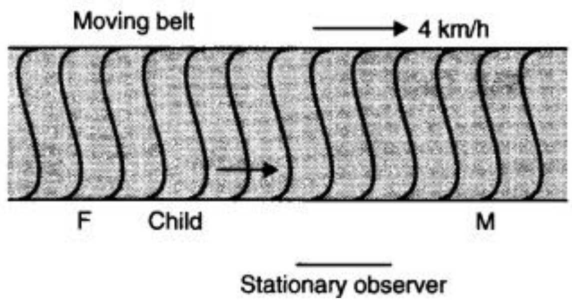
10. On a long horizontally moving belt (Fig.), a child runs to and fro with n speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 a part on the moving belt. The belt moves with a speed of 4 km h^{-1} . For an observe a stationary platform outside, what is the

(a) Speed of the child running in the direction of motion of the belt?

(b) Speed of the child running opposite to the direction of motion of the belt?

(c) Time taken by the child in (a) and (b)?

Which of the answers alter if motion is viewed by one of the parents?



Sol. (i) Speed of the child running in the direcito of motion of the belt = (9+4)kmh^{-1} = 13 kmh^{-1}

(ii) Speed of child running opposite to the direction of motion of the belt = (9 – 4)kmh^{-1} – 5 kmh^{-1}

(iii) Speed of child w.r.t. either parent = 9kmh^{-1} = 2.5ms^{-1}

Distance covered = 50m

Time taken = 50/2.5 = 20s.

G. ASSERTION REASON TYPE QUESTIONS

(a) If both assertion and reason are true and reason is the correct explanation of assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of assertion.
(c) If assertion is true but reason is false **(d) If both assertion and reason are false**
(e) If assertion is false but reason is true

1. Assertion: Displacement of a body may be zero when distance travelled by it is not zero.
Reason: The displacement is the longest distance between initial and final position.

Ans. (c)Assertion is true but reason is false.
The displacement is the shortest distance between initial and final position. When final position of a body coincides with its initial position, displacement is zero, but the distance travelled is not zero.

2. Assertion: The relative velocity between any two bodies moving in opposite direction is equal to sum of velocities of two bodies.
Reason: Sometimes relative velocity between two bodies is equal to difference in velocities of the two bodies.

Ans. (b) Both assertion and reason are true but reason is not the correct explanation of assertion.
When two bodies are moving in opposite directions, relative velocity between them is equal to sum of the velocity of bodies. But if the bodies are moving in same direction their relative velocity is equal to difference in velocity of the bodies.

3. Assertion: A body may be accelerated even when it is moving at uniform speed.
Reason: When direction of motion of the body is changing then body may have acceleration.

Ans. (a) Both assertion and reason are true and reason is the correct explanation of assertion.

The uniform motion of a body means the body is moving with constant speed, but if the direction of motion is changing (such as in uniform circular motion), its velocity changes and thus acceleration is produced in uniform motion.

4. Assertion: A body falling freely many do so with constant velocity.
Reason: A body falls freely, when acceleration of the body is equal to acceleration due to gravity.

Ans. (e) Assertion is false but reason is true.
When a body falling freely, only gravitational force acts on it in vertically downward direction. Due to this downward acceleration the velocity of a body increases and wil be maximum when the body touches the ground. If downward accelerating force is balanced by the upward retarding force, the body falls with constant velocity. The constant velocity is called terminal velocity of body.

5. Assertion: A physical quantity cannot be called as a vector if its magnitude is zero.
Reason: A vector has both magnitude and direction.

Ans. (e) Assertion is false but reason is true.
If a vector quantity has zero magnitude then it is called a null vector. The quantity may have some direction even if the magnitude is zero.

SPACE FOR ROUGH WORK

SPACE FOR NOTES